

## Image Distances

This document describes the steps in estimating the distances between a given point and the burner tip in the images.

### Conversion between image and world coordinates

The relation between a real world coordinate set and the image coordinate set is described by Crimini et al. [1] and sketched in Figure 1. Figure 1a shows the translation in 2D, and Figure 1b shows the translation in 1D, between a world line and image line. This is the system of interest in this application where it is desired to determine the axial distance along the world line (distance to burner tip) based on the recorded images.

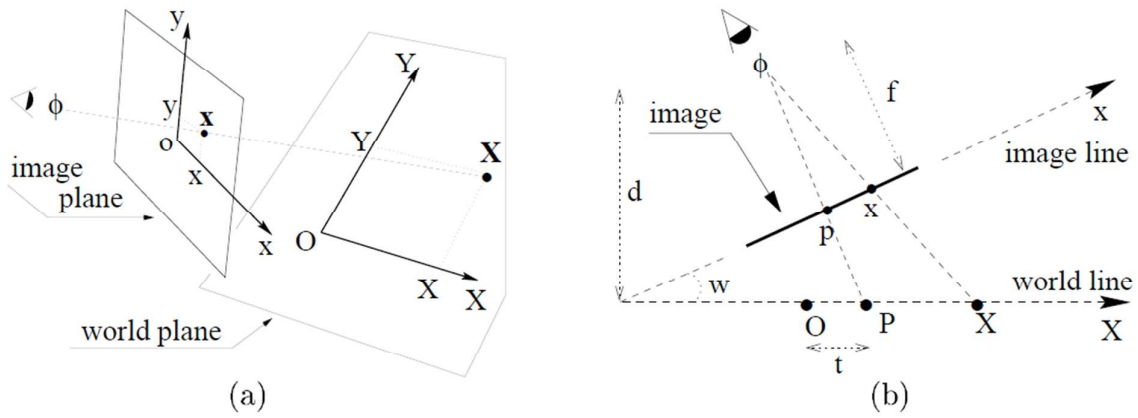


Figure 1: a) Plane Camera model: The point X in real world plane is imaged as point x in the image plane. The coordinate system is (X,Y) in the world plane and (x,y) in the image plane. b) One-dimensional camera model.  $\phi$  is the camera,  $f$  is the focal length (camera distance to image line), P and X are real world coordinates projected to the image coordinates p and x,  $t$  is the world ordinate,  $w$  is the angle between world line and image line,  $d$  is the distance from world line to camera [1].

Crimini et al. [1] show that the relation between a real world point, X, and the image point, x, is given as:

$$X = \frac{\alpha x + t}{\mu x + 1} \quad \text{E 1}$$

Here  $t$  is the ordinate as shown in Figure 1b and  $\alpha$  and  $\mu$  are calculated by:

$$\alpha = \frac{d}{f * \cos^2 w} - \frac{t}{f} * \tan(w) \quad \text{E 2}$$

$$\mu = -\frac{\tan(w)}{f} \quad \text{E 3}$$

Here  $d$  is the distance from the world line to the camera,  $w$  is the angle between the world and image lines, and  $f$  is the focal length of the camera, as also shown in Figure 1b.

In the following it will be described how focal length,  $f$ , distance,  $d$ , and angle,  $w$ , are found in order to convert between image and real world coordinates.

### Finding the camera focal length - Camera Calibration

The focal length of the camera was estimated by a number of images where a person of 1.8 m, was moving away from the camera in increments of 2 m as indicated by the images in Figure 2.



Figure 2: Images for calibration of camera. Person moves from 4 to 16 m in increments of 2 m.

Based on the images it was possible to measure what the distance 1.8 m corresponded to in pixels on the image, depending on the distance to the camera. This made it possible to construct the graph in Figure 3, which shows the conversion factor between height in meters and pixels in the image at various distances from the camera.

The equation that relates the real height (RH) with the image height (IH) is the pinhole projection formula [2]:

$$\frac{IH}{f} = \frac{RH}{d} \quad \text{E 4}$$

Here  $d$  is the distance to the object and  $f$  is the focal length. Rewriting the equation gives a linear relationship between distance,  $d$ , and the image height,  $IH$ :

$$\frac{1}{f * RH} * d = \frac{1}{IH} \quad \text{E 5}$$

The data extracted from the images of Figure 2 are shown in Figure 3 and plotted according to E 5, which gives a straight-line plot. The slope of the plot is given as  $1/(f * RH) = 3.58 * 10^{-4}$ . Since RH is 1.8 m,  $f$  can be calculated as 1510 pixel.

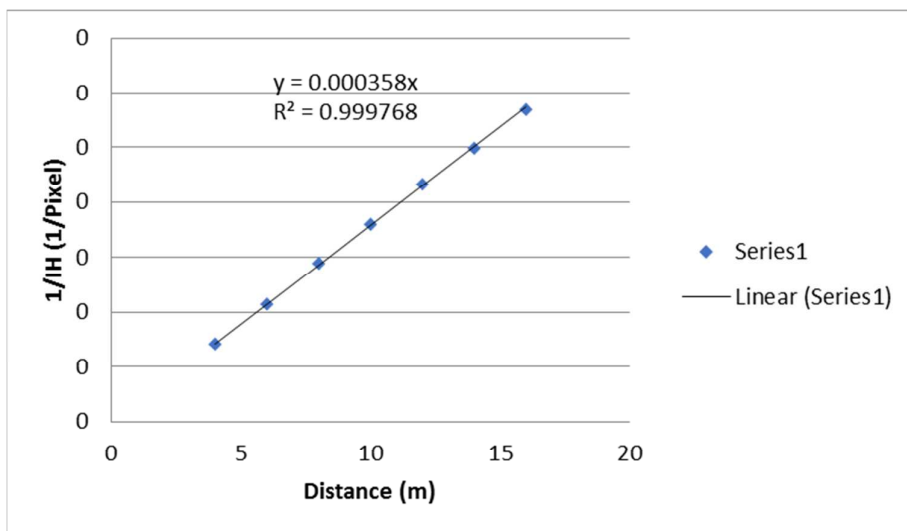


Figure 3: Relationship between distance and image height.

### Finding distance to burner

It is known that the burner plus refractory is 0.82 m in diameter as shown in Figure 4. The burner refractory are in four pieces and the gaps between the pieces are can be clearly seen in the images of the burner. The distance between these gaps is 0.58 m. In Figure 4 this distance has been measured to be 499 pixels in the image. Applying the pinhole camera formula, the distance between burner and camera is calculated as:

$$d = \frac{f}{IH} RH = \frac{1510 \text{ pixel}}{499 \text{ pixel}} * 0.58 \text{ m} = 1.76 \text{ m}$$

With the radius of the kiln being approx. 2.5 m, this appears to be realistic.

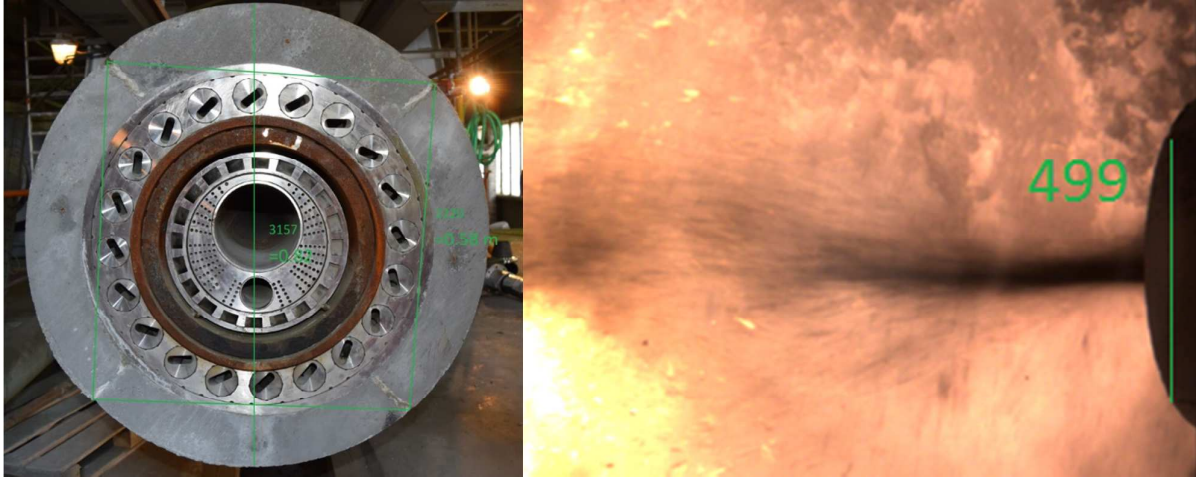


Figure 4: Burner image with measurements.

### Angle between image and world lines

The images are normally recorded at an angle as shown in Figure 4, which means that the world and image lines are not aligned and the burner points “inwards” in the image.

Depending on the angle between camera and burner the view will change as sketched in Figure 5, which shows a small part of a cylinder, such as the kiln burner. A protruding section, called  $p$  in Figure 6, will be seen when the burner is not parallel to the image plane. In Figure 5, this section has been highlighted with the green lines on the left of each drawing. By measuring the width of this section, the burner angle can be estimated.

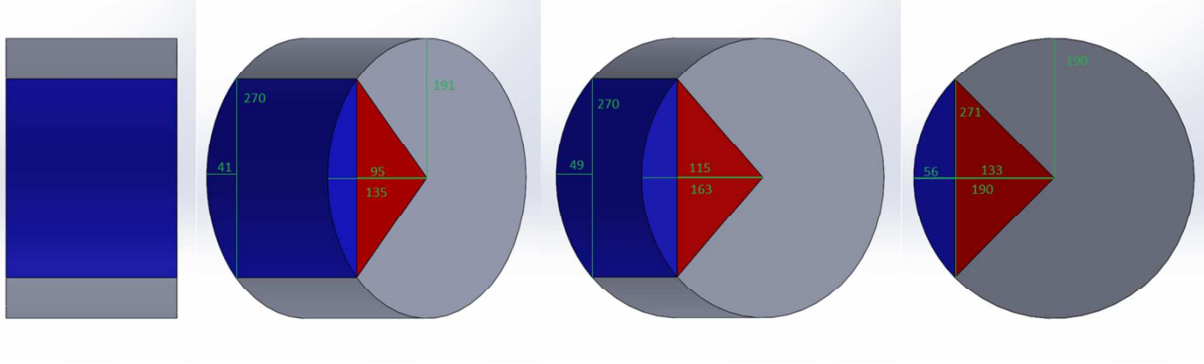


Figure 5: Sketch of the view of the burner at angles  $\theta = 0, 45, 60$ , and  $90^\circ$  compared to the camera plane.

In the burner images the vertical distance,  $h$ , between the gaps in the refractory can be measured. This section covers  $90^\circ$  and is sketched with blue and red in Figure 5. The measured distance should be independent on the burner angle as exemplified in Figure 5 where the distance is  $h=270$  pixels. Based on this value the corresponding radius can be calculated. By observing the sketch in Figure 6 it is seen that the radius can be calculated as:

$$r = \sqrt{\left(\frac{h}{2}\right)^2 + \left(\frac{h}{2}\right)^2} = \sqrt{\frac{h^2}{4} + \frac{h^2}{4}} = \frac{\sqrt{2}h}{2}$$

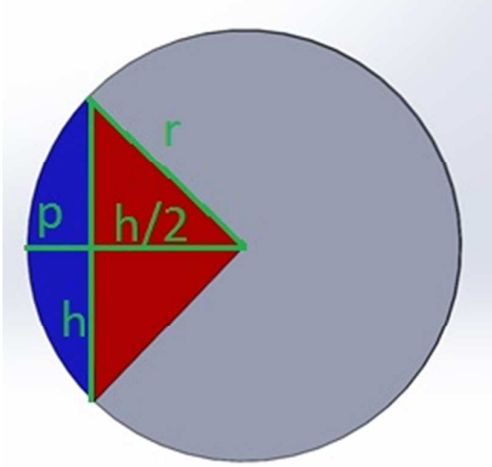


Figure 6: Sketch of image measurements with burner angle  $\omega = 90^\circ$ .

A new angle,  $\theta$ , is now defined as:

$$\theta = 90^\circ - w$$

Thus, the burner angle  $w=90^\circ$  corresponds to the angle  $\theta=0^\circ$ . In this case, observing Figure 6, it is realized that the distance,  $p$ , is calculated as:

$$p = r - \frac{h}{2}$$

It is also known that the distance  $p$  will be 0 when the angle  $\omega$  is  $90^\circ$ . Assuming that there is a linear relationship between the distance  $p$ , given by applying the cosine of the angle  $\theta$ , leads to:

$$p = \cos(\theta) * r - \cos(\theta) * \frac{h}{2}$$

Isolating for the angle  $\theta$  gives:

$$\theta = \cos^{-1}\left(\frac{p}{r - \frac{h}{2}}\right)$$

Thus the burner angle  $\omega$  is found as:

$$w = 90^\circ - \cos^{-1}\left(\frac{p}{r - \frac{h}{2}}\right) = 90^\circ - \cos^{-1}\left(\frac{p}{\frac{\sqrt{2}h}{2} - \frac{h}{2}}\right)$$

Applying the calculations to the example measurements shown in Figure 5 gives angles of 0, 47, 61, and  $89^\circ$ , which is close to the actual position.

## Applied Example

Two examples of applying the above described calculations to estimate distances are shown in this section. To assess the accuracy of the calculations, some images of a permanent marker with a ruler attached were made from different angles as shown in Figure 7. The pen has a cylindrical shape like the burner in the cement kiln and the ruler can be used to compare the calculated distance estimates with actual distances. The images were taken with a camera with a focal length of 3518 pixel.



Figure 7: Two example images used to assess the precision of the distance estimates.

First the distances indicated in Figure 8 are found. These allow for the estimation of the distance between the camera and the pen and the angle of the pen according to the calculations described above. The real distance between the two black lines was measured to be 1.1 cm.



Figure 8: Distanced measured to calculate distance from camera to pen and angle of the pen.

For the first image of Figure 8 the distance to the pen is calculated as:

$$d = \frac{f}{IH} * RH = \frac{3518 \text{ px}}{139 \text{ px}} * 1.1 \text{ cm} = 27.8$$

E 6

The real distance was measured to 30 cm, giving a small deviation between the real and the measured distances.



The next step was to calculate the angle between the world and image planes. This is calculated as

$$w = 90^\circ - \cos^{-1}\left(\frac{d}{\frac{\sqrt{2}h}{2} - \frac{h}{2}}\right) = 90^\circ - \cos^{-1}\left(\frac{13 \text{ px}}{\frac{\sqrt{2} * 139 \text{ px}}{2} - \frac{139 \text{ px}}{2}}\right) = 26.8^\circ \quad \text{E 7}$$

With these two measured the alpha and mu parameters can be determined to:

$$\alpha = \frac{d}{f * \cos^2 w} - \frac{t}{f} * \tan(w) = \frac{27.8}{3518 * \cos^2 26.8^\circ} - 0 = 9.9 * 10^{-3} \quad \text{E 8}$$

$$\mu = -\frac{\tan(w)}{f} = -\frac{\tan(26.8^\circ)}{3518} = -1.4 * 10^{-4} \quad \text{E 9}$$

It is now possible to convert between image and real world coordinates. For instance, in the first image in Figure 7 the 10 cm point is located at the image x-coordinate 333 and the tip of the pen is located at 1255. The distance between the two points in the real world is then estimated by the equation:

$$X = \frac{\alpha(1255 - 333) + 0}{\mu(1255 - 333) + 1} = 10.6 \quad \text{E 10}$$

Thus, the point 10 cm is estimated to be 10.6 cm from the tip of the pen, which is a rather small error.

In Figure 9 several distance markers are drawn on top of the image. They are seen to compare quite well with the actual distances indicated by the ruler in the image.

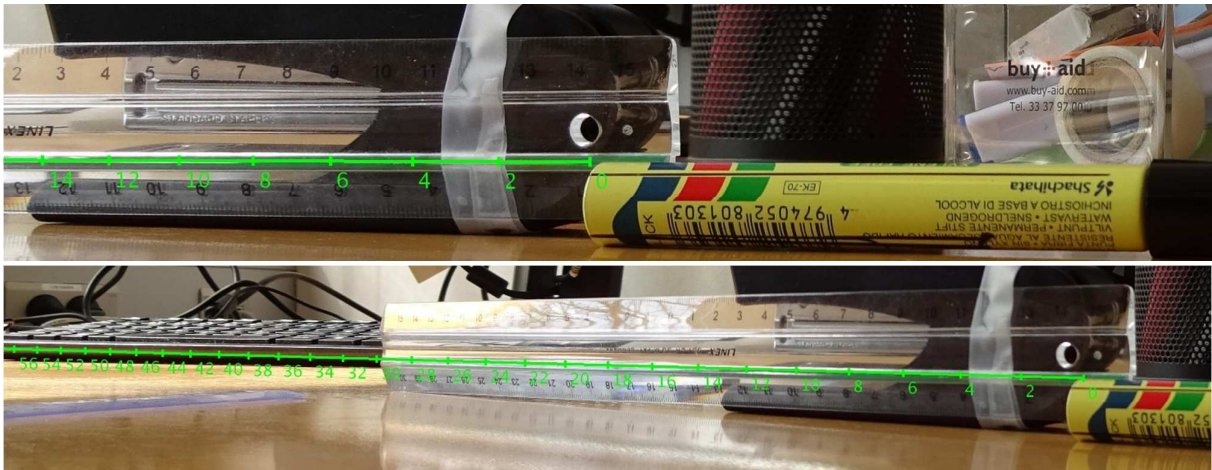


Figure 9: Examples of applying the described measurements to determine the distances in the image.

## Bibliography

- [1] A. Criminisi, I. Reid, A. Zisserman, A plane measuring device, *Image and Vision Computing*. 17 (1999) 625–634. doi:10.1016/S0262-8856(98)00183-8.
- [2] P. Sturm, Pinhole Camera Model, in: K. Ikeuchi (Ed.), *Computer Vision - A Reference Guide*, 1st ed., Springer, New York, 2014: pp. 610–613. doi:10.1007/978-1-84882-256-6.