Supporting Information for:

# Embedded Cluster Model for $\mathrm{Al}_{2} \mathrm{O}_{3}$ and $\mathrm{AlPO}_{4}$ Surfaces Using Point Charges and Periodic Electrostatic Potential 

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## Derivation of One-electron Integral of Periodic Electrostatic Potential

The one way to incorporate the electrostatic potential by the bulk is to use electrostatic potential that is calculated using periodic PC distribution obtained by slab calculation. Here, we describe how to construct such electrostatic potential embedding method.

Because Gaussian basis functions are employed in almost all cluster models, we need to calculate one-electron integral of periodic point charge distribution, using Gaussian basis functions. Under the periodic boundary condition (PBC), one-electron orbital $\psi_{i, \mathbf{k}}(\mathbf{r})$ is represented by a Bloch function, as follows:

$$
\begin{equation*}
\psi_{i, \mathbf{k}}(\mathbf{r})=u_{i, \mathbf{k}}(\mathbf{r}) e^{i \mathbf{k} \cdot \mathbf{r}} \tag{S1}
\end{equation*}
$$

where $i, \mathbf{k}$, and $u_{i, \mathbf{k}}$ are a band index, a wave vector, and a periodic part of the Bloch function, respectively. The periodic part $u_{i, \mathbf{k}}$ is represented by a linear combination of periodic Gaussian basis functions $\phi_{A n, \mathbf{k}}^{\mathrm{PBC}}$; see eqs. S2 and S3;

$$
\begin{align*}
& u_{i, \mathbf{k}}(\mathbf{r})=\sum_{A n} C_{A n i, \mathbf{k}} \phi_{A n, \mathbf{k}}^{\mathrm{PBC}}(\mathbf{r})  \tag{S2}\\
& \phi_{A n, \mathbf{k}}^{\mathrm{PBC}}(\mathbf{r})=\sum_{\mathbf{R}} \phi_{A n}(\mathbf{r}-\mathbf{R}) e^{i \mathbf{k} \cdot \mathbf{R}} \tag{S3}
\end{align*}
$$

where $\mathbf{R}$ is a lattice vector, $C_{A n i, \mathbf{k}}$ is a coefficient of linear combination, and $\phi_{A n}$ is a contracted Gaussian basis function at $\mathbf{r}_{\mathrm{A}}=\left(x_{\mathrm{A}}, y_{\mathrm{A}}, z_{\mathrm{A}}\right)$, and $A$ and $n$ are indices of atom and basis function at $\mathbf{r}_{\mathrm{A}}$, respectively. The basis function $\phi_{A n}$ is given by Cartesian Gaussian function $\varphi_{\text {Aal }}$;

$$
\begin{equation*}
\phi_{A n}(\mathbf{r})=\sum_{a} N_{a l} c_{a n} \varphi_{A a l}(\mathbf{r}) \tag{S4}
\end{equation*}
$$

where $N_{a l}$ is a normalization constant, $c_{a n}$ is a contraction coefficient, and $l$ is a total angular momentum. The Cartesian Gaussian function $\varphi_{A a l}$ is given by eq. S5;

$$
\begin{align*}
\varphi_{A a l}(\mathbf{r}) & =\left(x-x_{A}\right)^{l_{x}}\left(y-y_{A}\right)^{l_{y}}\left(z-z_{A}\right)^{l_{z}} \exp \left(-\alpha_{a}\left|\mathbf{r}-\mathbf{r}_{A}\right|^{2}\right) \\
& =\left(x-x_{A}\right)^{l_{x}}\left(y-y_{A}\right)^{l_{y}}\left(z-z_{A}\right)^{l_{z}} g_{A a}(\mathbf{r}) \tag{S5}
\end{align*}
$$

where $l_{x}, l_{y}$, and $l_{z}$ are Cartesian angular momenta of $x, y$, and $z$ components, respectively, and $g_{A a}$ is a primitive Gaussian function at $\mathbf{r}_{\mathrm{A}}$.

Here, we employed the super-cell approach, ${ }^{\text {S1 }}$ where each cluster is positioned in one unit cell with very large lattice vectors, and the centers of basis functions are placed in the same cell; these are possible without loss of generality. Because the cluster and its periodic images are separated very well from each other under this condition, overlap between basis functions of the clusters located at different cells is negligibly small;

$$
\begin{equation*}
\phi_{A n}(\mathbf{r}-\mathbf{R}) \phi_{B m}\left(\mathbf{r}-\mathbf{R}^{\prime}\right) \approx 0 \quad\left(\mathbf{R} \neq \mathbf{R}^{\prime}\right) \tag{S6}
\end{equation*}
$$

Owing to the sufficient separation of the clusters, band dispersion disappears, and only the $\Gamma$-point $(\mathbf{k}=(0,0,0))$ sampling over the first Brillouin zone is necessary.

Under such conditions, the one-electron orbital $\psi_{i}$ of the cluster model is represented by eq. S7;

$$
\begin{equation*}
\psi_{i}(\mathbf{r})=\sum_{A n} C_{A n i} \phi_{A n}^{\mathrm{PBC}}(\mathbf{r}) \tag{S7}
\end{equation*}
$$

The basis function $\phi_{A n}^{\mathrm{PBC}}$ can be represented with Cartesian Gaussian function (eq. S5) by eq. 8 ;

$$
\begin{align*}
\phi_{A n}^{\mathrm{PBC}}(\mathbf{r}) & =\sum_{a} N_{a l} c_{a n} \varphi_{A a l}^{\mathrm{PBC}}(\mathbf{r}) \\
& =\sum_{a} N_{a l} c_{a n} \sum_{\mathbf{R}} \varphi_{A a l}(\mathbf{r}-\mathbf{R}) \tag{S8}
\end{align*}
$$

The next task is to evaluate one-electron integral of periodic potential;

$$
\begin{align*}
\left\langle\phi_{A n}^{\mathrm{PBC}}\right| \hat{V}^{\mathrm{PBC}}\left|\phi_{B m}^{\mathrm{PBC}}\right\rangle & =\frac{1}{V^{\mathrm{SC}}} \int_{V^{\mathrm{SC}}} d \mathbf{r} \phi_{A n}^{\mathrm{PBC}}(\mathbf{r}) V^{\mathrm{PBC}}(\mathbf{r}) \phi_{B m}^{\mathrm{PBC}}(\mathbf{r}) \\
& =\sum_{a} \sum_{b} N_{a l} N_{b l^{\prime}} c_{a n} c_{b m} \frac{1}{V^{\mathrm{SC}}} \int_{V^{\mathrm{SC}}} d \mathbf{r} \varphi_{A a l}^{\mathrm{PBC}}(\mathbf{r}) V^{\mathrm{PBC}}(\mathbf{r}) \varphi_{B b l^{\prime}}^{\mathrm{PBC}}(\mathbf{r}) \tag{S9}
\end{align*}
$$

where $V^{\mathrm{SC}}$ is a volume of the super-cell and $\int_{V^{S C}} \mathrm{~d} \mathbf{r}$ indicates integration over the super-cell. Because the overlap of basis functions between different cells can be neglected (see eq. S6), the integral in eq. S9 can be represented through Fourier transformation as follows:

$$
\begin{equation*}
\int_{V^{\mathrm{SC}}} d \mathbf{r} \varphi_{A a l}^{\mathrm{PBC}}(\mathbf{r}) V^{\mathrm{PBC}}(\mathbf{r}) \varphi_{B b l^{\prime}}^{\mathrm{PBC}}(\mathbf{r})=\sum_{\mathbf{G}} V^{\mathrm{PBC}}(\mathbf{G}) \int_{\text {all }} d \mathbf{r} \varphi_{A a l}(\mathbf{r}) \varphi_{B b l^{\prime}}(\mathbf{r}) e^{i \mathbf{G} \cdot \mathbf{r}} \tag{S10}
\end{equation*}
$$

where $V^{\mathrm{PBC}}(\mathbf{G})$ is a Fourier transform of $V^{\mathrm{PBC}}(\mathbf{r}), \mathbf{G}$ is a reciprocal lattice vector, and $\int_{\text {all }} \mathbf{d r}$ indicates integration over all real-space.

The integral in the right-hand side in eq. S10 is a complex conjugate of Fourier transform of the product of two Cartesian Gaussian functions. To evaluate the one-electron integral of periodic potential, therefore, the Fourier transform of the product of two Cartesian Gaussian functions is required. Because both of the product of two Gaussian functions and the Fourier transform of Gaussian function are Gaussian function, the Fourier transform of the product of two Gaussian functions is also the Gaussian function as follows:

$$
\begin{equation*}
\int_{\text {all }} d \mathbf{r} g_{A a}(\mathbf{r}) g_{B b}(\mathbf{r}) e^{-i \mathbf{G} \cdot \mathbf{r}}=\left(\frac{\pi}{\alpha_{p}}\right)^{\frac{3}{2}} F_{A a B b} e^{-i \mathbf{G} \cdot \mathbf{r}_{P}} \exp \left(-\frac{|\mathbf{G}|^{2}}{4 \alpha_{p}}\right) \tag{S11}
\end{equation*}
$$

where

$$
\alpha_{p}=\alpha_{a}+\alpha_{b}, \quad F_{A a B b}=\exp \left(-\frac{\alpha_{a} \alpha_{b}}{\alpha_{a}+\alpha_{b}}\left|\mathbf{r}_{A}-\mathbf{r}_{B}\right|^{2}\right), \text { and } \mathbf{r}_{P}=\frac{\alpha_{a} \mathbf{r}_{A}+\alpha_{b} \mathbf{r}_{B}}{\alpha_{a}+\alpha_{b}}
$$

Though the Fourier transform of the product of two Cartesian Gaussian functions cannot be provided as a simple form, it can be represented as a recursion formula within a binomial expansion; details are presented in Appendix A (page S8)).

Hereafter, we focus on the electrostatic potential that is defined by eq. S12;

$$
\begin{equation*}
V^{\mathrm{ES}}(\mathbf{r})=\int_{\mathrm{all}} d \mathbf{r}^{\prime} \frac{n\left(\mathbf{r}^{\prime}\right)}{\left|\mathbf{r}-\mathbf{r}^{\prime}\right|}+\sum_{\mathbf{R}} \sum_{C} \frac{Z_{C}}{\left|\mathbf{r}-\mathbf{r}_{C}-\mathbf{R}\right|} \tag{S12}
\end{equation*}
$$

where $n(\mathbf{r})$ is electron density at $\mathbf{r}$ and $Z_{C}$ is a nuclear charge of the $C$-th nucleus at $\mathbf{r}_{C}$ in the unit cell. Using Poisson's equation, $V^{\mathrm{ES}}(\mathbf{r})$ is transformed to eq. S13;

$$
\begin{equation*}
V^{\mathrm{ES}}(\mathbf{G})=\frac{4 \pi}{|\mathbf{G}|^{2}}\left(n(\mathbf{G})+\sum_{C} Z_{C} e^{-i \mathbf{G} \cdot \mathbf{r}_{C}}\right) \tag{S13}
\end{equation*}
$$

where $n(\mathbf{G})$ is a Fourier transform of $n(\mathbf{r})$. The one-electron integral of the electrostatic potential can be evaluated using eqs. S10 and S13; however, the Fourier series expansion in eq. S10 is, in general, not complete because the finite number of wave-vectors $\mathbf{G}$, which is usually determined by a cut-off energy, is not sufficient to incorporate high frequency components of the nuclear point-charge potential in reciprocal-space. To avoid this problem, the Ewald summation method, which is developed to evaluate electrostatic interaction between PCs in the PBC, is applied to evaluation of electrostatic interaction between PCs and one-electron orbital represented by the Gaussian basis functions in the PBC.

In the Ewald summation method, the electrostatic potential of PCs is represented as the sum of two short-range terms and one long-range term as follows:

$$
\begin{gather*}
V^{\mathrm{ES}}(\mathbf{r})=V_{\mathrm{sr} 1}^{\mathrm{ES}}(\mathbf{r})+V_{\mathrm{sr} 2}^{\mathrm{ES}}(\mathbf{r})+V_{\mathrm{lr}}^{\mathrm{ES}}(\mathbf{r})  \tag{S14}\\
V_{\mathrm{sr} 1}^{\mathrm{ES}}(\mathbf{r})=\sum_{\mathbf{R}, C}^{\left|\mathbf{r}-\mathbf{r}_{C}-\mathbf{R}\right| \leq r_{\mathrm{cut}}} \frac{Z_{C}}{\left|\mathbf{r}-\mathbf{r}_{C}-\mathbf{R}\right|}  \tag{S15}\\
V_{\mathrm{sr} 2}^{\mathrm{ES}}(\mathbf{r})=\sum_{\mathbf{R}, C}^{\left|\mathbf{r}-\mathbf{r}_{C}-\mathbf{R}\right| \leq r_{\mathrm{cut}}} \frac{-Z_{C}}{\left|\mathbf{r}-\mathbf{r}_{C}-\mathbf{R}\right|} \operatorname{erf}\left(\frac{1}{\sqrt{2} \sigma}\left|\mathbf{r}-\mathbf{r}_{C}-\mathbf{R}\right|\right)  \tag{S16}\\
V_{\mathrm{lr}}^{\mathrm{ES}}(\mathbf{r})=\sum_{\mathbf{G}}^{|\mathbf{G}|^{2} / 2 \leq E_{\mathrm{cut}}} \frac{4 \pi}{|\mathbf{G}|^{2}} \sum_{C} Z_{C} e^{-i \mathbf{G} \cdot \mathbf{r}_{C}} \exp \left(-\frac{\sigma^{2}|\mathbf{G}|^{2}}{2}\right) e^{i \mathbf{G} \cdot \mathbf{r}} \tag{S17}
\end{gather*}
$$

where $r_{\text {cut }}$ is a cut-off parameter in real-space, erf is an error function, $\sigma$ is a convergence parameter of the Ewald sum, and $E_{\text {cut }}$ is a cut-off energy in reciprocal space. Because the short-range terms have no reciprocal space contribution, one-electron integral of $V^{\mathrm{ES}}$ srl can be evaluated in the same manner as nuclear-attraction integral, which is implemented in standard ab initio program. One-electron integral of $V^{\mathrm{ES}}{ }_{\text {sr2 }}$ can be evaluated in a similar manner to two-electron integral; see Appendix B in page S 9 . Finally, the one-electron integral of $V^{\mathrm{ES}}{ }_{\mathrm{sr} 2}$ for the Gaussian functions is represented by eq. S18;

$$
\begin{align*}
\int_{\text {all }} d \mathbf{r} g_{A a}(\mathbf{r}) V_{\mathrm{sr} 2}^{\mathrm{ES}}(\mathbf{r}) g_{B b}(\mathbf{r})= & \left(\frac{\pi}{\alpha_{p}}\right)^{\frac{3}{2}} F_{A a B b} \sum_{\mathbf{R}, C}^{\left|\mathbf{r}-\mathbf{r}_{C}-\mathbf{R}\right| \leq r_{\text {cut }}} \frac{-Z_{C}}{\left|\mathbf{r}_{P}-\mathbf{r}_{C}-\mathbf{R}\right|} \\
& \times \operatorname{erf}\left(\sqrt{\frac{\alpha_{p} \alpha_{c}}{\alpha_{p}+\alpha_{c}}}\left|\mathbf{r}_{P}-\mathbf{r}_{C}-\mathbf{R}\right|\right) \tag{S18}
\end{align*}
$$

where $\alpha_{c}=1 / 2 \sigma^{2}$. The one-electron integral of $V^{\mathrm{ES}}{ }_{\text {sr2 }}$ for the Cartesian Gaussian functions is presented in Appendix B (page S9). The one-electron integral of $V^{\mathrm{ES}}{ }_{\text {Ir }}$ can be evaluated using eqs. S10 and S17.

In this way, the electrostatic interaction between the periodic PC distribution and the one-electron orbital of the embedded cluster model can be correctly evaluated in reasonable computational cost using the super-cell approach, where no approximation is employed except for very large lattice vectors for super-cells. The determination of cut-off energy in Ewald summation method and the dependency on the super-cell size are discussed in page S11.

## Appendices for Derivation of One-electron Integral of

## PE Potential

## Appendix A: Fourier transform of product of two Cartesian Gaussian functions

The Fourier transform of product of two Cartesian Gaussian functions is represented as

$$
\begin{equation*}
\int_{\mathrm{all}} d \mathbf{r} \varphi_{A a l}(\mathbf{r}) \varphi_{B b l^{\prime}}(\mathbf{r}) e^{-i \mathbf{G} \cdot \mathbf{r}}=F_{A a B b} s_{P p l_{x} l_{x}^{\prime}}\left(G_{x}\right) s_{P p l_{y} l_{y}^{\prime}}\left(G_{y}\right) s_{P p l_{z} l_{z}}\left(G_{z}\right) \tag{A1}
\end{equation*}
$$

Here, the $x$-component, $s_{P p l_{x} l_{x}^{\prime}}\left(G_{x}\right)$, is defined by

$$
\begin{equation*}
s_{P p l_{x} l_{x}^{\prime}}\left(G_{x}\right)=\int_{-\infty}^{+\infty} d x\left(x-x_{A}\right)^{l_{x}}\left(x-x_{B}\right)^{l_{x}^{\prime}} \exp \left(-\alpha_{p}\left|x-x_{P}\right|^{2}\right) e^{-i G_{x} x} \tag{A2}
\end{equation*}
$$

The $y$ - and $z$-components, $s_{P p l_{y} l_{y}^{\prime}}\left(G_{y}\right)$ and $s_{P p l_{z} l_{z}^{\prime}}\left(G_{z}\right)$, respectively, are similarly defined. Using a binomial expansion, $s_{P p l_{x} l_{x}^{\prime}}\left(G_{x}\right)$ can be represented as follows:

$$
\begin{equation*}
s_{P p l_{x} l_{x}^{\prime}}\left(G_{x}\right)=\sum_{k=0}^{l_{x}} \sum_{k^{\prime}=0}^{l_{x}^{\prime}}\binom{l_{x}}{k}\binom{l_{x}^{\prime}}{k^{\prime}}\left(x_{P}-x_{A}\right)^{l_{x}-k}\left(x_{P}-x_{B}\right)^{l_{x}^{\prime}-k^{\prime}} \varphi_{P p k+k^{\prime}}\left(G_{x}\right) \tag{A3}
\end{equation*}
$$

Here, $\varphi_{P p k+k^{\prime}}\left(G_{x}\right)$ is the x-component of Fourier transform of Cartesian Gaussian function;

$$
\begin{equation*}
\varphi_{P p k+k^{\prime}}\left(G_{x}\right)=\int_{-\infty}^{+\infty} d x\left(x-x_{P}\right)^{k+k^{\prime}} \exp \left(-\alpha_{p}\left|x-x_{P}\right|^{2}\right) e^{-i G_{x} x} \tag{A4}
\end{equation*}
$$

which can be derived using the following recursion relations in a similar manner to eq. 46 of Ref. S2:

$$
\begin{align*}
\varphi_{P p 0}\left(G_{x}\right) & =\left(\frac{\pi}{\alpha_{p}}\right)^{\frac{1}{2}} \exp \left(-\frac{G_{x}^{2}}{4 \alpha_{p}}\right) e^{-i G_{x} x_{P}} \\
\varphi_{P p 1}\left(G_{x}\right) & =-\frac{i G_{x}}{2 \alpha_{p}} \varphi_{P p 0}\left(G_{x}\right) \\
& \vdots \\
\varphi_{P p l_{x}}\left(G_{x}\right) & =\frac{l_{x}-1}{2 \alpha_{p}} \varphi_{P p l_{x}-2}\left(G_{x}\right)-\frac{i G_{x}}{2 \alpha_{p}} \varphi_{P p l_{x}-1}\left(G_{x}\right) \tag{A5}
\end{align*}
$$

The y- and z-components of Fourier transform of Cartesian Gaussian function are similarly derived.

## Appendix B: One electron integral of $V_{\mathrm{sr} 2}^{\mathrm{ES}}$ in Ewald summation method

In the Ewald summation method, the electrostatic potential of point charges is represented as the sum of two short-range terms and one long-range term, as represented in eq. S14. The second short-range term $V_{\mathrm{sr} 2}^{\mathrm{ES}}$ (eq. S16) can be expressed as follows:

$$
\begin{align*}
V_{\mathrm{sr} 2}^{\mathrm{ES}}(\mathbf{r}) & =\sum_{\mathbf{R}, C}^{\left|\mathbf{r}-\mathbf{r}_{C}-\mathbf{R}\right| \leq r_{\text {cut }}} \frac{-Z_{C}}{\left|\mathbf{r}-\mathbf{r}_{C}-\mathbf{R}\right|} \operatorname{erf}\left(\frac{1}{\sqrt{2} \sigma}\left|\mathbf{r}-\mathbf{r}_{C}-\mathbf{R}\right|\right) \\
& =\sum_{\mathbf{R}, C}^{\left|\mathbf{r}-\mathbf{r}_{C}-\mathbf{R}\right| \leq r_{\text {cut }}}\left(-Z_{C}\right)\left(\frac{1}{2 \pi \sigma^{2}}\right)^{\frac{3}{2}} \int_{\text {all }} d \mathbf{r}^{\prime} \frac{\exp \left(-\frac{1}{2 \sigma^{2}}\left|\mathbf{r}^{\prime}-\mathbf{r}_{C}-\mathbf{R}\right|^{2}\right)}{\left|\mathbf{r}-\mathbf{r}^{\prime}\right|} \tag{B1}
\end{align*}
$$

As presented in eq. B1, $V_{\text {sr2 }}^{\mathrm{ES}}$ of the Ewald summation method can be regarded as the electrostatic potential formed by the Gaussian charge densities (for more details of the Ewald summation method, see Ref. S3). Thus, one-electron integral of $V_{\mathrm{sr} 2}^{\mathrm{ES}}$ can be evaluated in a similar manner to evaluation of two-electron integral which is carried out by the two Gaussian basis functions with the same electron coordinate ( $\mathbf{r}^{\prime}$ ), the same position $\left(\mathbf{r}_{C}+\mathbf{R}\right)$, the same Gaussian width $(\sqrt{2} \sigma)$, the same normalization constant $\left(\left(1 / 2 \pi \sigma^{2}\right)^{3 / 4}\right)$, and no Cartesian angular momenta $\left(l_{x}=l_{y}=l_{z}=0\right)$. Using a Hermite polynomial expansion with Rodrigues' formula, ${ }^{\text {S4 }}$ the one-electron integral of $V_{\mathrm{sr} 2}^{\mathrm{ES}}$ for the

Cartesian Gaussian functions is presented as

$$
\begin{align*}
\int_{\text {all }} d \mathbf{r} \varphi_{A a l}(\mathbf{r}) V_{\mathrm{sr} 2}^{\mathrm{ES}}(\mathbf{r}) \varphi_{B b l^{\prime}}(\mathbf{r})= & F_{A a B b} \sum_{\mathbf{R}, C}^{\left|\mathbf{r}-\mathbf{r}_{C}-\mathbf{R}\right| \leq r_{\mathrm{cut}}}\left(\frac{\alpha_{c}}{\pi}\right)^{\frac{3}{2}} \frac{-Z_{C} \pi^{\frac{5}{2}}}{\alpha_{p} \alpha_{c} \sqrt{\alpha_{p}+\alpha_{c}}} \\
& \times \sum_{s_{x} s_{x}^{\prime} t_{x} t_{x}^{\prime} u_{x} v_{x}} I_{x} \sum_{s_{y} s_{y}^{\prime} t_{y} t_{y}^{\prime} u_{y} v_{y}} I_{y} \sum_{s_{z} s_{z}^{\prime} t_{z} t_{z}^{\prime} u_{z} v_{z}} I_{z} \\
& \times 2 B_{\nu}\left(\frac{\alpha_{p} \alpha_{c}}{\alpha_{p}+\alpha_{c}}\left|\mathbf{r}_{P}-\mathbf{r}_{C}-\mathbf{R}\right|^{2}\right) \tag{B2}
\end{align*}
$$

Here, $\alpha_{c}=1 / 2 \sigma^{2}, B_{\nu}$ is the $\nu$-th order Boys function as

$$
\begin{equation*}
B_{\nu}(T)=\int_{0}^{1} d t t^{2 \nu} \exp \left(-T t^{2}\right) \tag{B3}
\end{equation*}
$$

and

$$
\begin{align*}
\sum_{s_{x}^{\prime} s_{x}^{\prime} t_{x}^{\prime} t_{x}^{\prime} u_{x} v_{x}} I_{x}= & \frac{(-1)^{l_{x}+l^{\prime} x} l_{x}!l_{x}^{\prime}!}{p^{l_{x}+l_{x}^{\prime}}} \sum_{s_{x}=0}^{\left[l_{x} / 2\right]} \sum_{s_{x}^{\prime}=0}^{\left[l_{x}^{\prime} / 2\right]} \sum_{t_{x}=0}^{l_{x}-2 s_{x}} \sum_{t_{x}^{\prime}=0}^{l_{x}^{\prime}-2 s_{x}^{\prime}\left[\left(t_{x}+t_{x}^{\prime}\right) / 2\right]} \sum_{u_{x}=0} \frac{(-1)^{t_{x}^{\prime}+u_{x}}\left(t_{x}+t_{x}^{\prime}\right)!}{4^{s_{x}+s_{x}^{\prime}+u_{x}} S_{x}!s_{x}^{\prime}!t_{x}!t_{x}^{\prime}!u_{x}!} \\
& \times \frac{a^{t_{x}^{\prime}-s_{x}-u_{x}} b^{t_{x}-s_{x}^{\prime}-u_{x}} p^{2\left(s_{x}+s_{x}^{\prime}\right)+u_{x}}\left(r_{A x}-r_{B x}\right)^{t_{x}+t_{x}^{\prime}-2 u_{x}}}{\left(l_{x}-2 s_{x}-t_{x}\right)!\left(l_{x}^{\prime}-2 s_{x}^{\prime}-t_{x}^{\prime}\right)!\left(t_{x}+t_{x}^{\prime}-2 u_{x}\right)!} \\
& \times \sum_{v_{x}=0}^{\left[\mu_{x} / 2\right]} \frac{(-1)^{v_{x}} \mu_{x}!(p c /(p+c))^{\mu_{x}-v_{x}}\left(r_{P x}-r_{C x}\right)^{\mu_{x}-2 v_{x}}}{4^{v_{x}} v_{x}!\left(\mu_{x}-2 v_{x}\right)!} \tag{B4}
\end{align*}
$$

where $\mu_{x}=l_{x}+l_{x}^{\prime}-2\left(s_{x}+s_{x}^{\prime}\right)-\left(t_{x}+t_{x}^{\prime}\right), \nu=\mu_{x}+\mu_{y}+\mu_{z}-\left(v_{x}+v_{y}+v_{z}\right)$, and $I_{y}$ and $I_{z}$ are similarly defined in terms of $y$ - and $z$-components, respectively. Boys function can be evaluated as

$$
\begin{equation*}
B_{\nu}(T)=\frac{(2 \nu)!}{2 \nu!}\left[\frac{\sqrt{\pi}}{4^{\nu} T^{\nu+1 / 2}} \operatorname{erf}(\sqrt{T})-e^{-T} \sum_{k=0}^{\nu-1} \frac{(\nu-k)!}{4^{k}(2 \nu-2 k)!T^{k+1}}\right] \tag{B5}
\end{equation*}
$$

when $T>0$, and $B_{\nu}(T)=1 /(2 \nu+1)$ when $T=0 .{ }^{\mathrm{S} 4}$

# Effects of Cut-off Energy in the Ewald Summation Method and Super-cell Size on Interactions of $\mathbf{R h}_{2}$ with $\mathrm{Al}_{2} \mathrm{O}_{3}$ and $\mathrm{AlPO}_{4}$ 

Effects of cut-off energy in the Ewald summation method on interactions of $\mathrm{Rh}_{2}$ with $\mathrm{Al}_{2} \mathrm{O}_{3}$ and $\mathrm{AlPO}_{4}$ :

Because the long-range term of electrostatic potential in the Ewald summation method (eq. S17) quickly converges in reciprocal-space, the cut-off energy $E_{\text {cut }}$, which determines the number of wave-vectors used in Fourier expansion, can be significantly reduced. Figure S 1 shows the convergence behavior of total energies of the $\mathrm{Al}_{2} \mathrm{O}_{3}$ cluster model with respect to cut-off energy value in the presence and absence of the Ewald summation method. Here, a $2 \times 2 \times 1$ super-cell $(33 \times 34 \times 36 \AA)$, a convergence parameter of $\sigma=1$ $\AA$, and a real-space cut-off criteria, $\left(\alpha_{p} \alpha_{c} /\left(\alpha_{p}+\alpha_{c}\right)\right)\left|\mathbf{r}_{P}-\mathbf{r}_{C}-\mathbf{R}\right|^{2} \leq r_{\text {cut }}=20 \ln 10$, were used; the cut-off criteria is same as the default used in GAMESS ${ }^{\text {S5 }}$ for calculation of one-electron integrals. In the absence of the Ewald summation method, the total energy varies by more than 50 eV when the cut-off range is 400 to 800 eV . In the presence of the Ewald summation method, on the other hand, the total energy converges rapidly with respect to the cut-off energy; for instance, it varies within 0.001 eV when the cut-off energy is larger than 150 eV . Therefore, the cut-off energy of 150 eV was used in this work.

## Effect of super-cell size on interactions of $\mathrm{Rh}_{2}$ with $\mathrm{Al}_{2} \mathrm{O}_{3}$ and

## $\mathrm{AlPO}_{4}$ :

In the embedding method incorporating periodic electrostatic potential developed in this work, two-electron integrals (Coulomb and exchange integrals) between clusters located at different cells are not considered. Therefore, the large super-cell should be employed so as that the two-electron integrals between clusters located at different cells are negligible.


Figure S1: Total energy of $\mathrm{Al}_{2} \mathrm{O}_{3}$ cluster model as a function of cut-off energy with and without the Ewald summation method (blue dashed- and red solid-lines, respectively).

Table S1 shows dependency of interaction energy and HOMO-LUMO gap on size of super-cell in the $\mathrm{Al}_{2} \mathrm{O}_{3}$ and $\mathrm{AlPO}_{4}$ cluster models. For the $\mathrm{Al}_{2} \mathrm{O}_{3}$ cluster model, the interaction energy and HOMO-LUMO gap of the $2 \times 2$ super-cell agree with those of the larger ones within 0.02 eV . For the $\mathrm{AlPO}_{4}$ one, those values of the $3 \times 3$ super-cell agree with those of the larger ones within 0.01 eV . Therefore, we employed these super-cell sizes in this work.

Table S1: The minimum atomic distances ( $d_{\text {min }}$ ) between clusters located at different cells, interaction energies ( $E_{\text {int }}$ ) of $\mathrm{Rh}_{2}$ with $\mathrm{Al}_{2} \mathrm{O}_{3}$ and $\mathrm{AlPO}_{4}$, and HOMO-LUMO gaps ( $\varepsilon_{\text {gap }}$ ) of distorted surfaces at various sizes of super-cells.

| Size of super-cell |  | $1 \times 1$ | $2 \times 2$ | $3 \times 3$ | $4 \times 4$ | $5 \times 5$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $d_{\text {min }}$ | ( $\AA$ ) | $\mathrm{Rh}_{2} / \mathrm{Al}_{2} \mathrm{O}_{3}$ |  |  |  |  |
|  |  | 5.46 | 22.3 | 39.1 | 55.9 | 72.6 |
|  |  |  |  | PBE |  |  |
| $E_{\text {int }}$ | (eV) | ${ }^{a}$ | -6.02 | -6.02 | -6.02 | -6.02 |
| $\varepsilon_{\text {gap }}$ | (eV) | ${ }^{a}$ | 1.82 | 1.82 | 1.82 | 1.82 |
|  |  | B3LYP |  |  |  |  |
| $E_{\text {int }}$ | (eV) | -5.43 | $-5.50$ | -5.50 | -5.50 | $-5.50$ |
| $\varepsilon_{\text {gap }}$ | (eV) | 2.44 | 3.51 | 3.50 | 3.49 | 3.49 |
|  | (A) | $\mathrm{Rh}_{2} / \mathrm{AlPO}_{4}$ |  |  |  |  |
| $d_{\text {min }}$ |  | 1.69 | 18.2 | 34.9 | 51.6 | 68.2 |
|  |  |  |  | PBE |  |  |
| $E_{\text {int }}$ | (eV) | $a$ | -5.74 | -5.71 | -5.70 | -5.70 |
| $\varepsilon_{\text {gap }}$ | (eV) | $a$ | $0.63{ }^{\text {b }}$ | $0.63{ }^{\text {c }}$ | $0.63{ }^{\text {c }}$ | $0.63{ }^{\text {c }}$ |
|  |  |  |  | B3LYP |  |  |
| $E_{\text {int }}$ | (eV) | $a$ | -5.57 | -5.54 | $-5.53$ | $-5.53$ |
| $\varepsilon_{\text {gap }}$ | (eV) | ${ }^{a}$ | $2.32^{\text {b }}$ | $2.32^{\text {c }}$ | $2.32^{\text {c }}$ | $2.32^{\text {c }}$ |

${ }^{a}$ SCF calculations do not converge.
${ }^{b} \varepsilon_{\text {gap }}=\varepsilon_{\text {LUMO }}-\varepsilon_{\text {НОМО-4 }}$.
${ }^{c} \varepsilon_{\text {gap }}=\varepsilon_{\text {LUMO }}-\varepsilon_{\text {HOMO-2 }}$.

Table S2: Effects of choice of atomic charge on interaction energies ( $E_{\mathrm{int}} ; \mathbf{e V}$ ) of $\mathrm{Rh}_{2}$ with $\mathrm{Al}_{2} \mathrm{O}_{3}$ and $\mathrm{AlPO}_{4}$.

|  | VLP ${ }^{\text {a }}$ |  | PE ${ }^{\text {b }}$ |  |
| :---: | :---: | :---: | :---: | :---: |
|  | Bader $^{\text {c }}$ | Formal ${ }^{\text {d }}$ | Bader ${ }^{\text {c }}$ | Formal ${ }^{\text {d }}$ |
|  | $\begin{gathered} \mathrm{Rh}_{2} / \mathrm{Al}_{2} \mathrm{O}_{3} \\ \mathrm{PBE} \end{gathered}$ |  |  |  |
| $E_{\text {int }}$ | $\begin{gathered} -5.99 \\ (-4.93)^{\mathrm{e}} \end{gathered}$ | $\begin{gathered} -6.79 \\ (-5.20)^{\mathrm{e}} \end{gathered}$ | $\begin{gathered} -6.02 \\ (-4.95)^{\mathrm{e}} \end{gathered}$ | $\begin{gathered} -6.72 \\ (-5.23)^{\mathrm{e}} \end{gathered}$ |
| $E_{\text {int }}$ | $\begin{gathered} -5.51 \\ (-4.53)^{\mathrm{e}} \end{gathered}$ | $\begin{gathered} -6.18 \\ (-4.81)^{\mathrm{e}} \end{gathered}$ | $\begin{gathered} -5.47 \\ (-4.47)^{\mathrm{e}} \end{gathered}$ | $\begin{gathered} -6.16 \\ (-4.76)^{\mathrm{e}} \end{gathered}$ |
|  | $\begin{gathered} \mathrm{Rh}_{2} / \mathrm{AlPO}_{4} \\ \text { PBE } \end{gathered}$ |  |  |  |
| $E_{\text {int }}$ | $\begin{gathered} -5.55 \\ (-5.09)^{\mathrm{e}} \end{gathered}$ | $\begin{gathered} -5.44 \\ (-4.99)^{\mathrm{e}} \end{gathered}$ | $\begin{gathered} -5.71 \\ (-5.25)^{\mathrm{e}} \end{gathered}$ | $\begin{gathered} -5.62 \\ (-5.16)^{\mathrm{e}} \end{gathered}$ |
| $E_{\text {int }}$ | $\begin{gathered} -5.43 \\ (-4.95)^{\mathrm{e}} \\ \hline \end{gathered}$ | $\begin{gathered} -5.32 \\ (-4.85)^{\mathrm{e}} \\ \hline \end{gathered}$ | $\begin{gathered} -5.52 \\ (-5.11)^{\mathrm{e}} \end{gathered}$ | $\begin{gathered} -5.45 \\ (-5.03)^{\mathrm{e}} \\ \hline \end{gathered}$ |

${ }^{\text {a }}$ A number of point charges is $1451940\left(920 \times 920 \times 15 \AA^{3}\right)$ for the $\mathrm{Al}_{2} \mathrm{O}_{3}$ embedded models with very large number of point charges (VLP), and that is $1016310\left(970 \times 920 \times 15 \AA^{3}\right)$ for the $\mathrm{AlPO}_{4}$ ones with VLP.
${ }^{\mathrm{b}} \mathrm{PE}$ indicates periodic electrostatic potential.
${ }^{c}$ The Bader charges calculated by the slab calculations were condidered in the calculation.
${ }^{\mathrm{d}}$ Formal charges were considered in the calcilation; +3 for $\mathrm{Al},-2$ for O , and +5 for P.
${ }^{e}$ In parentheses are the interaction energies after correction of basis set superposition error.

Table S3: Basis set effects on interaction energies ( $E_{\mathrm{int}} ; \mathrm{eV}$ ) of $\mathrm{Rh}_{2}$ with $\mathrm{Al}_{2} \mathrm{O}_{3}$ and $\mathrm{AlPO}_{4}$.

| Al | VLP ${ }^{\text {a }}$ |  |  | PE ${ }^{\text {b }}$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  | LANL2DZ ${ }^{\text {c }}$ | SDD ${ }^{\text {d }}$ | cc-pVDZ ${ }^{\text {e }}$ | LANL2DZ ${ }^{\text {c }}$ | SDD ${ }^{\text {d }}$ |
| P | LANL2DZ ${ }^{\text {c }}$ | SDD ${ }^{\text {d }}$ | cc-pVDZ ${ }^{\text {e }}$ | LANL2DZ ${ }^{\text {c }}$ | SDD ${ }^{\text {d }}$ |
| O | D95V ${ }^{\text {f }}$ | DZP ${ }^{\text {g }}$ | cc-pVDZ ${ }^{\text {e }}$ | D95V ${ }^{\text {f }}$ | DZP ${ }^{\text {g }}$ |
| Rh | LANL2DZ ${ }^{\text {c }}$ | SDD ${ }^{\text {d }}$ | SDD ${ }^{\text {d }}$ | LANL2DZ ${ }^{\text {c }}$ | SDD ${ }^{\text {d }}$ |
| $E_{\text {int }}$ |  |  | $\mathrm{Rh}_{2} / \mathrm{Al}_{2} \mathrm{O}_{3}$ PBE |  |  |
|  | $\begin{gathered} -5.99 \\ (-4.93)^{\mathrm{h}} \end{gathered}$ | $\begin{gathered} -6.98 \\ (-5.09)^{\mathrm{h}} \end{gathered}$ | $\begin{gathered} -8.19 \\ (-5.07)^{\mathrm{h}} \end{gathered}$ | $\begin{gathered} -6.02 \\ (-4.95)^{\mathrm{h}} \end{gathered}$ | $\begin{gathered} -7.00 \\ (-5.11)^{\mathrm{h}} \end{gathered}$ |
|  |  |  | B3LYP |  |  |
| $E_{\text {int }}$ |  | $-6.44$ | $-7.57$ |  | $-6.46$ |
|  | $(-4.53)^{\mathrm{h}}$ | $(-4.74)^{\mathrm{h}}$ | $(-4.64)^{\mathrm{h}}$ | $(-4.47)^{\mathrm{h}}$ | $(-4.68)^{\mathrm{h}}$ |
| $E_{\text {int }}$ |  |  | $\begin{gathered} \mathrm{Rh}_{2} / \mathrm{AlPO}_{4} \\ \mathrm{PBE} \end{gathered}$ |  |  |
|  | $-5.55$ | -5.95 | $-6.08$ | $-5.71$ | -6.13 |
|  | $(-5.09)^{\mathrm{h}}$ | $(-5.13)^{\mathrm{h}}$ | $(-5.13)^{\mathrm{h}}$ | $(-5.25)^{\mathrm{h}}$ | $(-5.40)^{\mathrm{h}}$ |
| $E_{\text {int }}$ |  |  | B3LYP |  |  |
|  | $-5.43$ | $-5.96$ | -6.10 | $-5.52$ | $-5.96$ |
|  | $(-4.95)^{\mathrm{h}}$ | $(-5.13)^{\mathrm{h}}$ | $(-5.14)^{\mathrm{h}}$ | $(-5.11)^{\mathrm{h}}$ | $(-5.23)^{\mathrm{h}}$ |

${ }^{\text {a }}$ A number of point charges is $1451940\left(920 \times 920 \times 15 \AA^{3}\right)$ for the $\mathrm{Al}_{2} \mathrm{O}_{3}$ embedded models with very large number of point charges (VLP), and that is $1016310\left(970 \times 920 \times 15 \AA^{3}\right)$ for the $\mathrm{AlPO}_{4}$ ones with VLP.
${ }^{\mathrm{b}} \mathrm{PE}$ indicates periodic electrostatic potential.
${ }^{c}$ Los Alamos basis sets and effective core potentials (ECPs) with $d$-polatization function. ${ }^{\text {S5-S7 }}$
${ }^{\text {d }}$ Stuttgart/Dresden basis sets and ECPs. ${ }^{\text {S }}$-S10
${ }^{e}$ Dunning's correlation consistent basis sets. ${ }^{\text {S11,S12 }}$
${ }^{\mathrm{f}}$ Huzinaga-Dunning valence double-zeta basis sets. ${ }^{\mathrm{S} 13}$
${ }^{\mathrm{g}}$ Huzinaga-Dunning double-zeta basis sets with $d$-polarization function. ${ }^{\text {S13 }}$
${ }^{\mathrm{h}}$ In parentheses are the interaction energies after correction of basis set superposition error.

In the case of $\mathrm{Rh}_{2} / \mathrm{Al}_{2} \mathrm{O}_{3}$, the $E_{\text {int }}$ without BSSE correction increases considerably, as the quality of basis sets increases. In the case of $\mathrm{Rh}_{2} / \mathrm{AlPO}_{4}$, the basis set effects on the $E_{\text {int }}$ is moderate. In both cases, the $E_{\text {int }}$ after BSSE correction depends little on the basis sets, suggesting that the basis set effects arise from the BSSE. Because the $E_{\text {int }}$ without BSSE correction calculated with the LANL2DZ is the closest to the $E_{\text {int }}$ (no-BSSE), we employed LANL2DZ here for discussing HOMO, LUMO, DOS etc.

Table S4: Cluster size effects on frontier orbital energies ( $\varepsilon_{\text {номо }}$ and $\varepsilon_{\text {Lumo }}$; eV ) and band gaps of distoetd $\mathrm{Al}_{2} \mathrm{O}_{3}$ and $\mathrm{AlPO}_{4}$. ${ }^{\text {a }}$

|  | VLP ${ }^{\text {b }}$ | PE ${ }^{\text {c }}$ | VLP ${ }^{\text {b }}$ | PE ${ }^{\text {c }}$ |
| :---: | :---: | :---: | :---: | :---: |
|  | distorted $\mathrm{Al}_{2} \mathrm{O}_{3}{ }^{\text {a }}$ |  | distorted $\mathrm{Al}_{2} \mathrm{O}_{3}-\mathrm{L}^{\text {a, d }}$ |  |
|  | P $\overline{\mathrm{BE}}$ |  |  |  |
| $\varepsilon_{\text {LUMO }}$ | -5.48 | $-5.22$ | -5.57 | -5.21 |
| $\varepsilon_{\text {номо }}$ | -7.29 | -7.03 | -7.17 | -6.80 |
| Band gap ${ }^{\text {e }}$ | 1.81 | 1.82 | 1.61 | 1.59 |
|  | B3LYP |  |  |  |
| $\varepsilon_{\text {LUMO }}$ | -4.83 | -4.57 | -4.94 | -4.58 |
| $\varepsilon_{\text {номо }}$ | -8.32 | -8.08 | -8.30 | -7.94 |
| Band gap ${ }^{\text {e }}$ | 3.49 | 3.51 | 3.35 | 3.36 |
|  | distorted | $\mathrm{AlPO}_{4}{ }^{\text {a }}$ | distorted | $\mathrm{PO}_{4}-\mathrm{L}^{\mathrm{a}, \mathrm{d}}$ |
|  | P $\overline{\mathrm{BE}}$ |  |  |  |
| $\varepsilon_{\text {LUMO }}{ }^{\text {f }}$ | -8.35 | -7.45 | -8.47 | -7.29 |
| $\varepsilon_{\text {номо }}{ }^{\text {f }}$ | $-8.98^{\text {g }}$ | $-8.08^{\mathrm{g}}$ | $-9.06^{\text {g }}$ | $-7.89^{\text {h }}$ |
| Band gap ${ }^{\text {e }}$ | $0.63{ }^{\text {i }}$ | $0.63{ }^{\text {i }}$ | $0.59{ }^{\text {i }}$ | $0.60^{\text {j }}$ |
|  | B3LYP |  |  |  |
| $\varepsilon_{\text {LUMO }}{ }^{\text {f }}$ | -7.84 | -6.97 | -7.97 | -6.84 |
| $\varepsilon_{\text {номо }}{ }^{\text {f }}$ | $-10.17^{\text {g }}$ | $-9.29{ }^{\text {g }}$ | $-10.24^{\mathrm{g}}$ | $-9.13^{\text {h }}$ |
| Band gap ${ }^{\text {e }}$ | $2.33{ }^{\text {i }}$ | $2.32{ }^{\text {i }}$ | $2.27^{\text {i }}$ | $2.29{ }^{\text {j }}$ |

${ }^{\mathrm{b}}$ These geometries were taken to be the same as the corresponding moiety of $\mathrm{Rh}_{2} / \mathrm{Al}_{2} \mathrm{O}_{3}$ and $\mathrm{Rh}_{2} / \mathrm{AlPO}_{4}$ optimized by the slab calculations.
${ }^{\mathrm{b}}$ A number of point charges is $1451940\left(920 \times 920 \times 15 \AA^{3}\right.$ ) for the $\mathrm{Al}_{2} \mathrm{O}_{3}$ embedded models with very large number of point charges (VLP), and that is $1016310\left(970 \times 920 \times 15 \AA^{3}\right)$ for the $\mathrm{AlPO}_{4}$ ones with VLP.
${ }^{c}$ PE indicates periodic electrostatic potential.
${ }^{\mathrm{d}} \mathrm{Rh}_{2} / \mathrm{Al}_{2} \mathrm{O}_{3}$-L and $\mathrm{Rh}_{2} / \mathrm{AlPO}_{4}$ - L mean $\mathrm{Rh}_{2} /\left(\mathrm{Al}_{2} \mathrm{O}_{3}\right)_{18}$ and $\mathrm{Rh}_{2} /\left(\mathrm{AlPO}_{4}\right)_{19}$ cluster models, respectiveky.
${ }^{\text {e }}$ Band gap is $\varepsilon_{\text {LUMO }}-\varepsilon_{\text {HOMO }}$ unless caution is presented as superscript.
${ }^{\mathrm{f}}$ Frontier orbitals similar to HO and LU bands of the slab model.
${ }^{\mathrm{g}} \mathrm{HOMO}-2$.
${ }^{h} \mathrm{HOMO}-4$.
${ }^{\mathrm{i}} \varepsilon_{\text {LUMO }}-\varepsilon_{\text {HOMO-2 }}$.
${ }^{\mathrm{j}} \varepsilon_{\text {LUMO }}-\varepsilon_{\text {HOMO-4 }}$.
$\left(\mathrm{Al}_{2} \mathrm{O}_{3}\right)_{12}$ model

$E_{\text {int }}-5.99$
$\left(\mathrm{Al}_{2} \mathrm{O}_{3}\right)_{18}$ model
$\left(\mathrm{AlPO}_{4}\right)_{15}$ model

$E_{\text {int }}-5.55$
$\left(\mathrm{AlPO}_{4}\right)_{19}$ model

$-5.61$

Figure S2: Cluster size effects on interaction energies $\left(E_{\text {int }} ; \mathrm{eV}\right)^{\mathrm{a}}$ of $\mathrm{Rh}_{2}$ with $\mathrm{Al}_{2} \mathrm{O}_{3}$ and $\mathrm{AlPO}_{4}$.
${ }^{\text {a }}$ PBE functional was used. The embedded cluster model with VLP charges was employed.

VLP
HOMO


PE
HOMO



Figure S3: HOMO and HOMO-1 of the $\mathrm{Rh}_{2} / \mathrm{AlPO}_{4}$ cluster models with VLP and PE.

These HOMO and HOMO-1 are localized on the edge, which correspond to the artificial dangling bond. These orbitals cannot be compared with the HO band of the slab model.

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gamma-Al203(001) Rh2/Al24036

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7.1509298772701921
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| 0 | 10.690894289380708 | 11.858411032826920 | 7.3347978088918501 |
| :--- | :--- | :--- | :--- | :--- |
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| 0 | 8.863277623169965 | 16.092356678187890 | 4.9749583881887631 |
| 0 | 8.819170879904139 | 13.309447449789429 | 4.9980151643780741 |
| 0 | 7.414631251417997 | 11.961198267000304 | 6.8726889881758551 |
| 0 | 7.252285536362678 | 8.980971487913052 | 6.8951317675229301 |
| 0 | 6.226791450154687 | 16.163373108438648 | 5.0802144527358161 |
| 0 | 6.161678402477171 | 13.287211478933864 | 5.0382120288740671 |
| 0 | 4.702592262651906 | 6.285471743784201 | 7.0600785271757301 |
| 0 | 12.883948750419218 | 6.202673494476930 | 7.1986398940379481 |
| 0 | 14.451896442425268 | 7.665419789650470 | 4.9678788789372561 |
| 0 | 14.451230230049461 | 4.897948128585901 | 4.9759009193422851 |
| 0 | 11.840181550924667 | 7.723979013035787 | 5.1488564456212781 |
| 0 | 11.816430551457355 | 4.850491199039271 | 5.0833847235304681 |
| 0 | 4.697196271256240 | 14.695013378889559 | 7.0608282289528681 |
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92
tridymite-AlPO4(110) Rh2/Al15P15060

Al $\quad 7.195031995533290$
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