## Sliding Dynamic Behavior of a Nanobubble on a Surface

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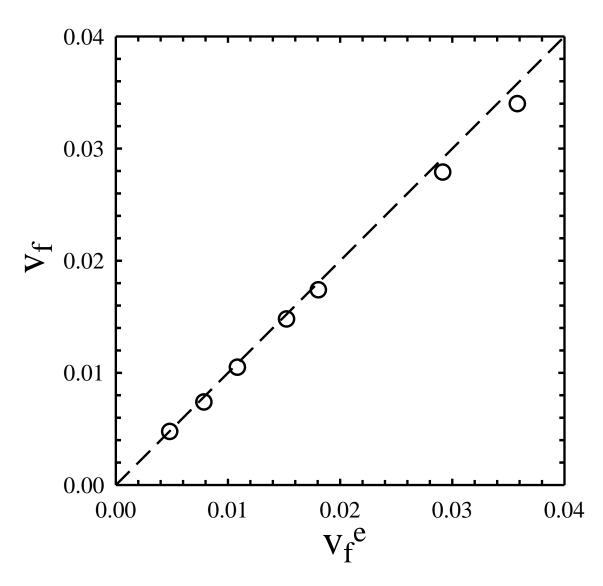


Figure S1. The comparison of sliding velocity obtained from our simulation and theoretical calculation.

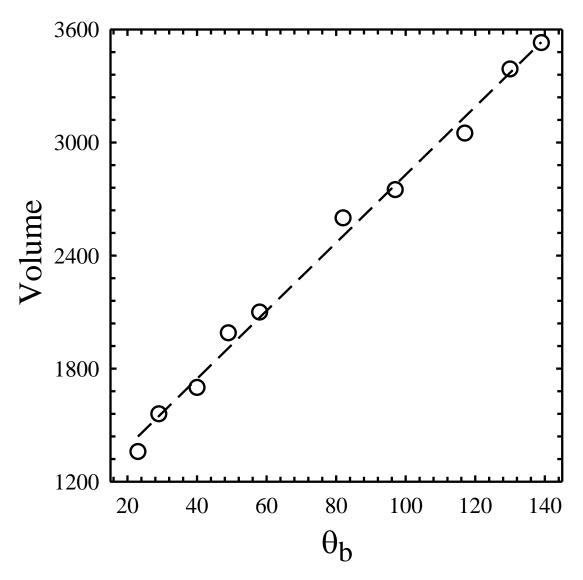


Figure S2. The variation of the bubble volume with bubble angles.

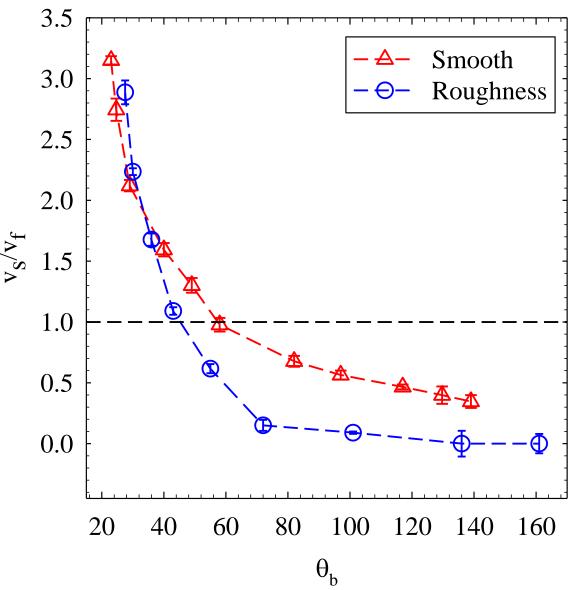


Figure S3. The variation of the terminal velocity of sliding bubbles  $(v_s)$  scaled by  $v_f$  with the bubble contact angle  $(\theta_b)$  along smooth and rough surfaces, respectively.

## Calculation of the viscosity

For a single-phase fluid flow, the Hagen-Poiseuille law is obeyed. In standard fluid-dynamics notation, the velocity profile is parabolic:

$$\mathbf{U} = \frac{\Delta P R^2}{4\mu L} \left[ 1 - \left( \frac{r}{R} \right)^2 \right] = V_{\text{max}} \left[ 1 - \left( \frac{r}{R} \right)^2 \right]$$

where

 $\Delta P$  is the pressure difference between the two ends,

L is the length of pipe,

μ is the dynamic viscosity,

R is the pipe radius,

 $V_{max}$  is the centerline velocity.

As a consequence, we can perform a simulation of a simple fluid flow in a tube and determine the velocity distribution U(r) corresponding to the body force driven flow. Then, the dimensionless viscosity is determined according to the above equation with given  $\Delta P/L$  and R. The fluid viscosity is not affected by  $a_{SL}$ . The detail of the calculation of viscosity can be found in our previous study in Reference 50.

## Sensitivity analysis of the box size on bubble mobility

We have performed more simulations to study the effect of box size on the mobility of the free rising bubbles. The results are listed as follows. As we can see, the mobility of the bubbles remain essentially the same as the box size varies from  $33\times33\times100$  to  $50\times50\times100$ . Similar results were found for sliding bubbles. This is because the system is dilute enough (bubble volume fraction is about 1-2%) for the system size to be ineffective.

## Free rising bubbles

box size	33×33×100	40×40×100	50×50×100
mobility	$1.58 \times 10^{-3} \pm 8 \times 10^{-5}$	$1.57 \times 10^{-3} \pm 9 \times 10^{-5}$	$1.59 \times 10^{-3} \pm 6 \times 10^{-5}$