Optimal Parameters for Hyperthermia Treatment Using Biomineralized Magnetite Nanoparticles: a Theoretical and Experimental Approach

Alicia Muela[1,5], David Muñoz[1], Rosa Martín-Rodríguez[2], Iñaki Orue[4], Eneko Garaio[3], Ana Abad Díaz de Cerio[1], Javier Alonso[5], José Ángel García[3,5], Mª Luisa Fdez-Gubieda[2,5,]*

Depto. de Inmunología, Microbiología, y Parasitología[1], Depto. de Electricidad y Electrónica[2], Física Aplicada II[3], Universidad del País Vasco (UPV/EHU), Leioa, Spain

SGIker Medidas Magnéticas, Universidad del País Vasco (UPV/EHU), Leioa, Spain[4]

BCMaterials, Edificio No. 500, Parque Tecnológico de Zamudio, Derio, Spain[5]

***corresponding Author:** *malu@we.lc.ehu.es*

DC Hysteresis Loop

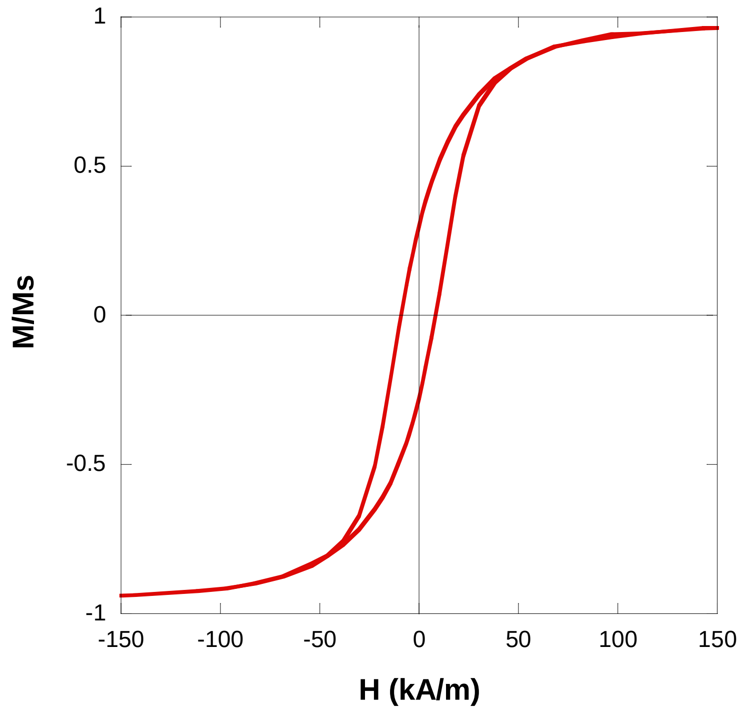

Figure S1. DC hysteresis loops measured by VSM at room temperature for the magnetosomes embedded in agar.

In Figure S1 we present the DC M-H loop as obtained by VSM for the magnetosomes embedded in agar. As can be observed, the coercive field is close to 9 kA/m.

AC Hysteresis Loops

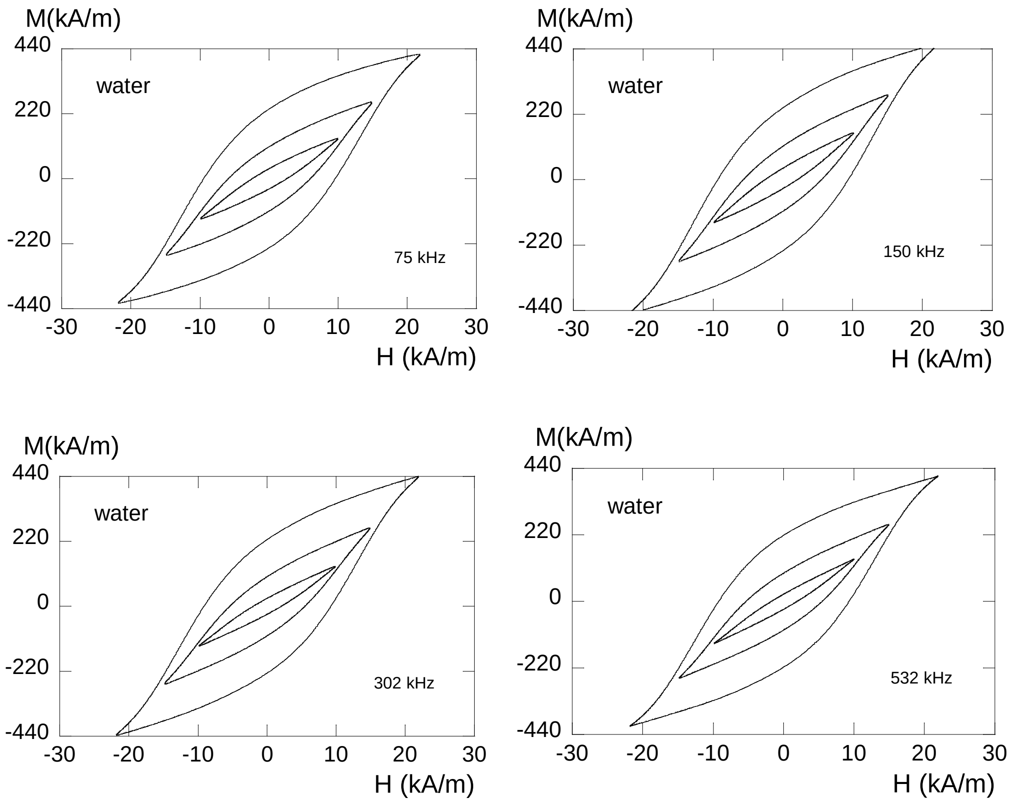

Figure S2. AC hysteresis loops measured at different frequencies, between 75 and 532 kHz

In Figure S2 the AC hysteresis loops measured at different frequencies have been represented. As observed, the shape and area of the hysteresis loops doesn't change remains nearly the same in all the cases, independently of the frequency of the AC field. This explains why the SAR/f vs H curves (Figure 4 in the main text) practically overlap.

Dynamic hysteresis loops simulation

The high energy barrier approach followed in this work is a Stoner-Wohlfarth based model (SWBM). In this model, magnetization is allowed to point only to discrete orientations by assuming that thermal energy, k_BT, is much smaller than the energy barrier between minimum energy states ($K_{eff}v$, being K_{eff} the anisotropy energy density and v, the particle volume). The dynamical problem is therefore reduced to the calculation of the probabilities $p_i(t)$ of finding the magnetization in any of the minimum energy states i at a given time t, as determined from the energy landscape of the system, $E(\theta, \varphi, t)$. This method was developed explicitly by Carrey et al[1] for the case of uniaxial single domain magnetic particles, where magnetization depends only on the polar angle (a one dimensional problem). The approach can be generalized for more complex 2-dimensional problems (magnetization depending on both polar and azimuthal angles), as those involving the cubic, mixed or multiaxial anisotropies[2].

The instantaneous magnetization of each particle is given by:

$$M_H(t) = M \sum_i p_i(t) \hat{u}_i(t) \cdot \hat{u}_H(t) \qquad \text{(S–1)}$$

where the unit vectors $\hat{u}_i(t)$ define the directions of these minima that depend on the sinusoidal magnetic field given by $H(t) = H_0 sin\omega t \, \hat{u}_H$ where $f = 2\pi/\omega$ is the frequency of AC field and unit vector $\hat{u}_H$ defines its direction. The time evolution of the probabilities $p_i(t)$ can be calculated by solving a set of ordinary differential equations as:

$$\frac{\partial p_i}{\partial t} = \sum_{j \neq i} w_{ji} p_j - \left(\sum_{j \neq i} w_{ij} \right) p_i \qquad \text{(S–2)}$$

where index i runs through the total number of minima. This equation is a way of saying that the change of the population in minimum i is the result of all the incoming jumps (first term) from the available neighbor states minus those departing from i (second term), with the condition $\sum p_i = 1$, which states that magnetization M is constant. The coefficients w_{ij} denote the rate of jumps (in units of frequency) from state i to state j, which depend on the instantaneous energy barrier E_{ij}, as $w_{ij}(t) = c_{ij} \exp(-vE_{ij}/k_B T)$, being v the volume of the single domain and pre-factor c_{ij} being the maximum jumps rate related to the natural precession frequency of the particle magnetization, which has been considered a constant equal to 10^{-10} s.

Biaxial anisotropy

The dipolar interaction between two magnetic dipoles gives rise to an effective magnetic anisotropy directed along the line joining them, under the assumption that both dipoles rotate coherently. This interaction or extrinsic anisotropy is superimposed to that of each isolated particle, giving rise to a biaxial anisotropy single domain. This line of reasoning can be easily implemented in the high energy barrier approximation by taking an energy density landscape given by:

$$E(\theta,\varphi,t) = K_1[1-(\hat{u}_1\cdot\hat{u}_m)^2] + K_2[1-(\hat{u}_2\cdot\hat{u}_m)^2] - \mu_0 M H_0 sin\omega t(\hat{u}_H\cdot\hat{u}_m) \qquad \text{(S–3)}$$

where θ and φ are the polar and azimuthal angles, respectively, of the magnetization vector $\mathbf{M} = M\hat{u}_m$. As shown in Figure S3 (b), unit vectors of equation (S-3) are defined as follows: $\hat{u}_1 \equiv \hat{z}$, defines the uniaxial anisotropy of isolated particles (referred as intrinsic in the text), fixed along the z axis for convenience, $\hat{u}_2 = sin\alpha cos\lambda\,\hat{x} + sin\alpha sin\lambda\,\hat{y} + cos\alpha\hat{z}$ determines the direction of extrinsic or interaction anisotropy, dependent on spherical angles (α,λ), and $\hat{u}_H = sin\beta cos\delta\,\hat{x} + sin\beta sin\delta\,\hat{y} + cos\beta\hat{z}$, determines the orientation of the external magnetic field, given by spherical angles (β,δ).

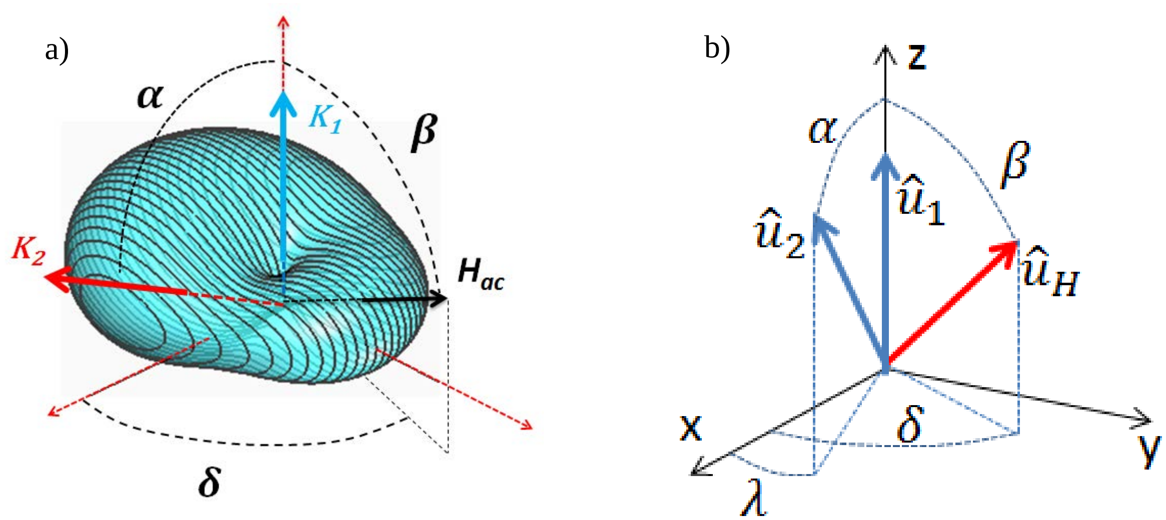

Figure S3. (a) Surface energy of the biaxial system at a given external magnetic field. In general, it has two easy axes above and below the plane z=0. (b) Angle definitions used in equation (S-3) for the three unit vectors defining the direction of the two uniaxial axes ($\hat{u}_1$, $\hat{u}_2$) and the external magnetic field $\hat{u}_H$.

The energy surface shown in Figure S3 (a) corresponds to an arbitrary external magnetic field and has two minima located in the north (z > 0) and south (z < 0) hemispheres. Critical points of the surface (minima, maxima and saddle points) can be obtained by numerically finding the zeroes of the energy density gradient function $\nabla E = (\partial E/\partial\theta)\hat{u}_\theta + 1/sin\theta\,(\partial E/\partial\varphi)\hat{u}_\varphi$, and equation (S-2) reduces in this case to the two states problem, as:

$$\frac{\partial p_{1(2)}}{\partial t} = w_{21(12)}p_{2(1)} - w_{12(21)}p_{1(2)} \qquad \text{(S–4)}$$

By making $K_2 = 0$ in equation (S-3), the energy density is the corresponding one to the well-known uniaxial problem:

$$E(\theta, t) = K_1 sin^2\theta - \mu_0 M H_0 sin\omega t cos(\theta - \beta) \qquad \text{(S–5)}$$

When the easy axis is oriented at random respect to the external applied field, equation (S-4) leads to the Carrey´s approach for a set of isolated or non-interacting uniaxial particles. In the case of the biaxial case, magnetization is averaged over all the possible orientations between the two easy axes and the external magnetic field, as:

$$M_{random} = \frac{\int_{\delta=0}^{\delta=\pi}\int_{\beta=0}^{\beta=\pi/2} M_H(t) sin\beta d\beta d\delta}{\iint sin\beta d\beta d\delta} \qquad \text{(S–6)}$$

Figure S4 illustrates the hysteresis loops calculated in this way with increasing external field amplitude. Each loop, as shown in Figure S4, results from averaging around 1500 single simulations corresponding to given directions $\hat{u}_1$. $\hat{u}_2$ and $\hat{u}_H$.

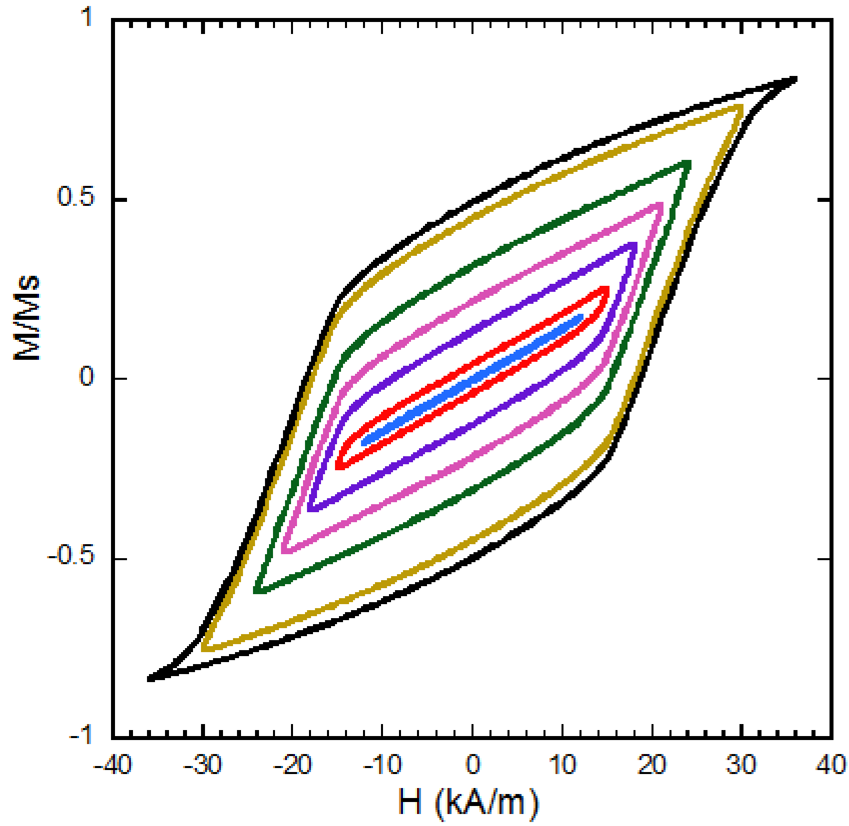

Figure S4. Simulated hysteresis loops as calculated using equation (S-6)

Cubic anisotropy

For the more complex cubic or mixed (cubic plus uniaxial) anisotropy problems, the calculation of critical points and easy axes as a function of time becomes much more laborious. In the case of cubic anisotropy, the instantaneous energy density $E(\theta,\varphi,t)$ depends on both the polar (θ) and azimuthal (φ) angles of the particle magnetization as follows:

$$E(\theta,\varphi,t) = \frac{K_c(sin^4\theta sin^2 2\varphi + sin^2 2\theta)}{4} - \mu_0 M H_0 sin\omega t[sin\theta sin\beta \cos(\varphi - \delta) + cos\theta cos\beta] \qquad \text{(S–7)}$$

where β and δ are the polar and azimuthal angles which determine the orientation of the external field relative the principal axis 100, 010 and 001 of the cubic crystal. For the case $K_c < 0$ the problem consists in the calculation of eight energy minima as a function of time, $\hat{u}_i(t)$ and the 8×8 jumps matrix $(w_{ij}(t))$ whose main diagonal entries are all zeroes.

In this work, uniaxial and biaxial models have been directly used for discussion of the heating performance of magnetosomes, because they are able to explain to a greater or lesser degree the experimental findings. For the uniaxial models, we have also analyzed the effect of implementing a Gaussian size distribution (see Figure S5) and we have observed that a slight improvement is obtained, but still the results are not as good as those obtained with the biaxial model. In addition, in order to put these results into context we also used the cubic model for the sake of comparison. For this, we considered a first cubic anisotropy constant of 11 kJ/m3, equal to the magnetocrystalline anisotropy of magnetite. As seen in Figure S5, SAR becomes very small and saturates at low fields (for H > 10 kA/m).

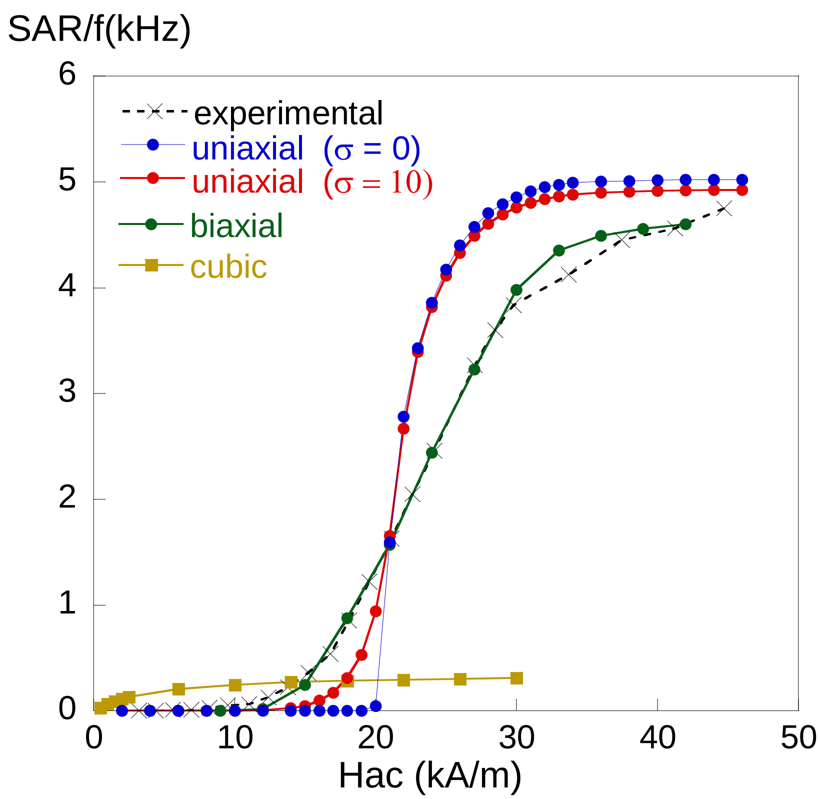

Figure S5. Comparison of the experimental evolution of SAR vs field with the calculated Stoner-Wohlfarth model assuming uniaxial (with different standard deviations), biaxial, and cubic anisotropy for the magnetosomes.

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