A Novel Solution Approach to a Priority-Slot-Based Continuous-Time Mixed Integer Nonlinear Programming Formulation for a Crude-Oil Scheduling Problem

Table S1. Objective Values for Solutions Obtained by Different Methods for Example 3

	MILP solution	NLP solution	Optimality gap	Composition discrepancy
The MILP/NLP method	61.667	No solution	N/A	Yes
The proposed method	59.583	59.583	0%	No

Table S2. Detailed Operations of the Obtained Solution by the Proposed Method for Example 3

Priority-slot	Operation	Device	Oil types and amounts in a device	Oil types and amounts delivered from a device
1	c1→d1	c1	C(50)	C(50)
2	s1→c1	s1	A(41.167)	A(41.167)
3	s1→c2	s1	A(20.833)	A(20.833)
4	v1→s1	v1	A(100)	A(100)
5	s2→c2	s2	B(29.167)	B(29.167)
6	s1→c1	s1	A(45.833)	A(45.833)
7	c2→d1	c2	A(20.833) B(29.167) D(50)	A(20.833) B(29.167) D(50)
8	s1→c2	s1	A(32.738)	A(32.738)
9	s2→c2	s2	B(45.833)	B(45.833)
10	v2→s2	v2	B(100)	B(100)
11	c1→d1	c1	A(50)	A(50)

Table S3. Solution Details for Example 4

Solution method	The number of slots	The number of binary variables	Iteration	Objective obtained by MILP	Objective obtained by NLP	Optimality gap	CPU time (s)
MILP/NLP method	10	80		79.750	79.750	0%	4.063
The proposed method	10	80	1	79.750		0%	1.762
		8	2	79.750			
MILP/NLP method	13	104		79.750	79.750	0%	7.459
The proposed method	13	104	1	79.750		0%	5.157
		8	2	79.750			

Table S4. Comparison on Performance between Different Methods with the Same Number of Slots for Examples 5, 6, and 7

		The number of slots	Iteration	Objective by MILP	Objective by NLP	Optimality gap	CPU time(s)
Example 5	MILP/NLP method	21		101.175	101.175	0%	187.375
	The proposed method	21	1	101.175		0%	184.294
Example 6	MILP/NLP method	13		87.400	84.500	3.32%	16.275
	The proposed method	13	1	87.400		1.79%	13.391
			2	86.329			0.505
			3	84.329			0.132
			4	85.836			0.045.
	MILP/NLP method	21		87.400	84.500	3.32%	196.357
	The proposed method	21	1	87.400		1.79%	192.643
			2	86.329			0.538
			3	84.329			0.141
			4	85.836			0.048
Example 7	MILP/NLP method	19		132.548	132.548	0%	887.651
	The proposed method	19	1	132.548		0%	884.482

Table S5. Solution Details for Example 8

Method	The number of slots	The number of binary variables	Iteration number	*t**	Objective by MILP	Objective by NLP	Optimality gap	CPU time (s)
MILP/NLP method	18	252			91.575	No solution		127.358
Proposed one	18	252	1	0	91.575		0.6877%	127.358
		126	2	1.1067	90.945			5.157

Table S6. Solution Details for Example 9

Method	The number of slots	The number of binary variables	Iteration number	t*	Objective by MILP	Objective by NLP	Optimality gap	CPU time(s)
MILP/NLP method	13	182			8.569	No solution		13.487
The proposed one	13	182	1	0	83.569		2.5045%	13.487
		78	2	2.3748	81.476			2.325

Table S7. Data for Example 10

	Scheduling horizon		15 days
Vessel	Arrival time	Composition	Amount of crude oil (Mbbl)
Vessel 1	0	100% A	500
Vessel 2	5	100% B	500
Vessel 2	10	100% C	500
Storage tank	Capacity (Mbbl)	Initial composition	Initial amount of crude oil (Mbbl)
Tank 1	[100,900]	100% D	200
Tank 2	[100,1100]	100% D	200
Tank 3	[100,1100]	100% E	200
Tank 4	[100,1100]	100% F	200
Tank 5	[100,900]	100% H	300
Tank 6	[100,900]	100% H	600
charging tank	Capacity (Mbbl)	Initial composition	Initial amount of crude oil (Mbbl)
Tank 1 (mix X)	[0,800]	100% G	300
Tank 2 (mix Y)	[0,800]	100% E	500
Tank 3 (mix Z)	[0,800]	100% F	300
Tank 4 (mix W)	[0,800]	100% H	300
Crude oil type	Sulfur concentration	Gross margin($/bbl)	
Crude oil A	0.035	3	
Crude oil B	0.085	5	
Crude oil C	0.06	6.5	
Crude oil D	0.02	3.1	
Crude oil E	0.05	7.5	
Crude oil F	0.08	3.17	
Crude oil G	0.03	4.83	
Crude oil H	0.095	6.33	
Crude mixture	Sulfur concentration	Demand (Mbbl)	
Crude oil mix X	[0.025,0.035]	[600,600]	
Crude oil mix Y	[0.045,0.065]	[600,600]	
Crude oil mix X	[0.075,0.085]	[600,600]	
Crude oil mix Y	[0.09,0.11]	[600,600]	
Unloading flow rate	[0,500]	transportation flow rate	[0,500]
Distillation flow rate	[20,500]	Number of distillations	7

Table S8. Solution Details for Example 10

Method	Priority-slots number	Number of binary variables	Iteration number	t*	Objective by MILP	Objective by NLP	Optimality gap	CPU time(s)
MILP/NLP method	17	340			143.415	No solution		746.381
The proposed one	17	340	1	0	143.415		7.65%	746.381
		240	2	1.0	140.1721			129.462
		120	3	6.0	138.4638			4.653
		60	4	11.0	136.9415			1.947
		40	5	11.32	132.4433			0.592

■ APPENDIX

A. SOS mathematical model based on priority-slot for crude oil operations scheduling problems

Maximize

$$\sum_{i \in T} \sum_{r \in R_D} \sum_{v \in I_r} \sum_{c \in C} G_c \cdot V_{ivc} \tag{7}$$

Subject to

Constraints of time for unloading oil from a vessel: Constraints (8)-(9) are used to enforce that only after the arrival of crude oil vessels to the dock, a vessel can be unloaded.

$$S_{iv} \geq \underline{S_v} \cdot Z_{iv} \quad i \in T,\ v \in W_U \tag{8}$$

$$\underset{v \in O_r}{S_{iv}} \leq S_r \quad r \in R_V,\ i \in T \tag{9}$$

Time constraints: Constraints (10) and (11) restrict the beginning time, duration, and the ending time of operation *v*.

$$E_{iv} \leq H \cdot Z_{iv} \quad i \in T,\ v \in W \tag{10}$$

$$E_{iv} = S_{iv} + D_{iv} \quad i \in T,\ v \in W \tag{11}$$

Cardinality constraints for unloading and distillation operations: Constraint (12) ensures that each vessel must be unloaded its cargo exactly once. In order to decrease the changeover cost of CDU switches, the total number of distillation operations is bounded by Constraint (13) using lower bound $\underline{N_D}$ and upper bound $\overline{N_D}$.

$$\sum_{i \in T} \sum_{v \in O_r} Z_{iv} = 1 \quad r \in R_V \tag{12}$$

$$\underline{N_D} \leq \sum_{i \in T} \sum_{v \in W_D} Z_{iv} \leq \overline{N_D} \tag{13}$$

Unloading sequence constraints: Constraints (14)-(15) define the unloading sequence of crude oil vessels that must be unloaded in order of their arrival time to the dock.

$$\sum_{i \in T} \sum_{v \in O_{r_1}} E_{iv} \leq \sum_{i \in T} \sum_{v \in O_{r_2}} S_{iv} \quad r_1, r_2 \in R_V,\ r_1 < r_2 \tag{14}$$

$$\sum_{j \in T, j < i} \sum_{v \in O_{r_1}} Z_{jv} \leq \sum_{j \in T, j \leq i} \sum_{v \in O_{r_2}} S_{iv} \quad i \in T,\ r_1, r_2 \in R_V,\ r_1 < r_2 \tag{15}$$

Continuous distillation constraint: The continuousness of CDU distillation is ensured by Constraint (16). Since each CDU can be charged by only one charging tank, continuous distillation can be defined by the total distillation time equating the whole scheduling horizon *H*.

$$\sum_{i \in T} \sum_{v \in I_r} D_{iv} = H \quad r \in R_D \tag{16}$$

Assignment constraint: Constraint (17) enforces that at most one operation must be assigned to each priority-slot.

$$\sum_{v\in W} Z_{iv} \le 1 \quad i \in T \tag{17}$$

Symmetry breaking constraint: Constraint (18) is used to eliminate non-occupancy of a priority-slot for avoiding slot redundancy.

$$\sum_{v\in W} Z_{iv} \ge 1, \quad i \in T \tag{18}$$

Non-overlapping constraints: Constraints (19)-(21) ensure that two operations v_1 and v_2 must not be simultaneously fulfilled.

$$\begin{gathered} E_{i_1 v_1} + E_{i_1 v_2} \le S_{i_2 v_1} + S_{i_2 v_2} + H \cdot (1 - Z_{i_2 v_1} - Z_{i_2 v_2}) \\ i_1, i_2 \in T, i_1 < i_2, v_1, v_2 \in W, NO_{v_1 v_2} = 1 \end{gathered} \tag{19}$$

$$E_{i_1 v_1} \le S_{i_2 v_2} + H \cdot (1 - Z_{i_2 v_2}) \; i_1, i_2 \in T, i_1 < i_2, v_1, v_2 \in W, NO_{v_1 v_2} = 1 \tag{20}$$

$$E_{i_1 v_2} \le S_{i_2 v_1} + H \cdot (1 - Z_{i_2 v_1}) \; i_1, i_2 \in T, i_1 < i_2, v_1, v_2 \in W, NO_{v_1 v_2} = 1 \tag{21}$$

Constraints (22)-(23) bound crude oil volume transferred by operation v using lower bound $\underline{V_v^t}$ and upper one $\overline{V_v^t}$.

$$V_{iv}^t \le \overline{V_v^t} \cdot Z_{iv} \quad i \in T, \; v \in W \tag{22}$$

$$V_{iv}^t \ge \underline{V_v^t} \cdot Z_{iv} \quad i \in T, \; v \in W \tag{23}$$

Constraints (24)-(26) enforce material balance for transferring operation.

$$V_{iv}^t = \sum_{c\in C} V_{ivc} \quad i \in T, \; v \in W \tag{24}$$

$$L_{irc} = L_{0rc} + \sum_{j\in T, j<i} \sum_{v\in I_r} V_{ivc} - \sum_{j\in T, j<i} \sum_{v\in O_r} V_{ivc} \; i \in T, r \in R, c \in C \tag{25}$$

$$L_{ir}^t = \sum_{c\in C} L_{irc} \quad i \in T, \; v \in W \tag{26}$$

Constraints (27) bound the flowrate by $\underline{FR_v}$ and $\overline{FR_v}$.

$$\underline{FR_v} \cdot D_{iv} \le V_{iv}^t \le \overline{FR_v} \cdot D_{iv} \quad i \in T, \; v \in W \tag{27}$$

Property constraint: Constraint (28) bounds property k of the blender transferred by operation v, and calculates property k of the blender from property x_{ck} of crude oil c by the assumption that the mixing procedure is linear.

$$\underline{x_{vk}} \cdot V_{iv}^{t} \leq \sum_{c \in C} x_{ck} V_{ivc} \leq \underline{x_{vk}} \cdot V_{iv}^{t} \quad i \in T, v \in W, k \in K \tag{28}$$

Constraints (29)-(32) ensure material balance for inventory of tanks.

$$\underline{L_r^t} \leq L_{ir}^t \leq \overline{L_r^t} \quad i \in T,\ r \in R_S \cup R_C \tag{29}$$

$$0 \leq L_{irc} \leq \overline{L_r^t} \quad i \in T, c \in C, r \in R_S \cup R_C \tag{30}$$

$$\underline{L_r^t} = L_{0r}^t + \sum_{i \in T} \sum_{v \in I_r} V_{iv}^t - + \sum_{i \in T} \sum_{v \in O_r} V_{iv}^t \leq \overline{L_r^t} \quad r \in R_S \cup R_C \tag{31}$$

$$0 \leq L_{0rc} + \sum_{i \in T} \sum_{v \in I_r} V_{ivc} - + \sum_{i \in T} \sum_{v \in O_r} V_{ivc} \leq \overline{L_r^t} \quad r \in R_S \cup R_C, c \in C \tag{32}$$

Constraint (33) is a demand constraint, which define lower and upper bounds, $\underline{D_r}$ and $\overline{D_r}$, to restrict the total volume of feedstock charged by each charging tank during the whole scheduling horizon *H*.

$$\underline{D_r} \leq \sum_{i \in T} \sum_{v \in O_r} V_{iv}^t \leq \overline{D_r} \quad r \in R_C \tag{33}$$