Calculation of the Spin Correlation Functions:

In expanding the summation over j, j-1 in eq. 3 and in using eq. 6 of the main manuscript the intensity $I_j(T)$ is written:

$$I_j(T) = \left(\Pi^b_{j,j}\right)^2 \begin{bmatrix} \frac{1}{6}\langle \mathbf{s}_j \cdot \mathbf{s}_j \rangle + \frac{i}{4S_j}\langle \mathbf{S}_j \cdot [\mathbf{s}_j \times \mathbf{s}_j] \rangle - \frac{1}{2S_j(2S_j-1)}\langle \mathbf{s}_j \cdot \mathbf{Q}_j \cdot \mathbf{s}_j \rangle \\ -\frac{1}{6}\langle \mathbf{s}_j \cdot \mathbf{s}_{j-1} \rangle - \frac{i}{4S_j}\langle \mathbf{S}_j \cdot [\mathbf{s}_j \times \mathbf{s}_{j-1}] \rangle + \frac{1}{2S_j(2S_j-1)}\langle \mathbf{s}_j \cdot \mathbf{Q}_j \cdot \mathbf{s}_{j-1} \rangle \end{bmatrix} \tag{S1}$$

$$+ \text{same expression where } \mathbf{s}_j \text{ and } \mathbf{s}_{j-1} \text{ are permuted.}$$

Some of the spin correlation functions appearing in eq. S1 have been determined by Seiden.[S1] Here, we restrict ourselves to recall their expression, taking into account the fact that $\mathbf{S}_{Mn}$ is treated as a vector of length $\sqrt{S_{Mn}(S_{Mn}+1)}$.

$$\langle \mathbf{s}_j \cdot \mathbf{s}_j \rangle = s_{Cu}(s_{Cu}+1) \tag{S2}$$

$\mathbf{s}_j \times \mathbf{s}_j = i\,\mathbf{s}_j$, therefore

$$\langle \mathbf{S}_j \cdot [\mathbf{s}_j \times \mathbf{s}_j] \rangle = i\langle \mathbf{S}_j \cdot \mathbf{s}_j \rangle = -i\Lambda s_{Cu}\sqrt{S_{Mn}(S_{Mn}+1)} \tag{S3}$$

Using the commutation rules of the spin operators $S_{j_x}, S_{j_y}, S_{j_z}$ and the definition of the operator $\mathbf{Q}_j$ in eq. 4 of the main manuscript, we can write:

$$\langle \mathbf{s}_j \cdot \mathbf{Q}_j \cdot \mathbf{s}_j \rangle = \langle (\mathbf{s}_j \cdot \mathbf{S}_j)(\mathbf{S}_j \cdot \mathbf{s}_j) \rangle - \frac{1}{3} S_{Mn}(S_{Mn}+1) s_{Cu}(s_{Cu}+1) + \frac{1}{2}\langle \mathbf{S}_j \cdot \mathbf{s}_j \rangle \tag{S4}$$

The specific properties of the spin operator $\mathbf{s}_j$ allow to write:[S2]

$$\left\langle \left(\mathbf{s}_j \cdot \mathbf{S}_j\right)\left(\mathbf{S}_j \cdot \mathbf{s}_j\right)\right\rangle = s_{Cu}^2 S_{Mn}\left(S_{Mn}+1\right) - s_{Cu}\left\langle \mathbf{S}_j \cdot \mathbf{s}_j\right\rangle \tag{S5}$$

Replacing s_{Cu} by its value 1/2 results in:

$$\left\langle \mathbf{s}_j \cdot \mathbf{Q}_j \cdot \mathbf{s}_j\right\rangle = 0 \tag{S6}$$

$$\left\langle \mathbf{s}_j \cdot \mathbf{s}_{j\text{-}1}\right\rangle = \Lambda^2 s_{Cu}^2 \tag{S7}$$

In eq. S1, the second term obtained when $\mathbf{s}_{j\text{-}1}$ and $\mathbf{s}_j$ are permuted is strictly identical to the first one because the manganese ions are located on an inversion center. The two- and four-body correlation functions are identical. It can be demonstrated that:

$$\left\langle \mathbf{S}_j \cdot \left[\mathbf{s}_j \times \mathbf{s}_{j\text{-}1}\right]\right\rangle = \left\langle \mathbf{S}_j \cdot \left[\mathbf{s}_{j\text{-}1} \times \mathbf{s}_j\right]\right\rangle = -\left\langle \mathbf{S}_j \cdot \left[\mathbf{s}_j \times \mathbf{s}_{j\text{-}1}\right]\right\rangle = 0 \tag{S8}$$

Using the same procedure than in eq. S5, we can write:

$$\left\langle \mathbf{s}_j \cdot \mathbf{Q}_j \cdot \mathbf{s}_{j\text{-}1}\right\rangle = \left\langle \left(\mathbf{s}_j \cdot \mathbf{S}_j\right)\left(\mathbf{S}_j \cdot \mathbf{s}_{j\text{-}1}\right)\right\rangle - \frac{1}{3} S_{Mn}\left(S_{Mn}+1\right)\left\langle \mathbf{s}_j \cdot \mathbf{s}_{j\text{-}1}\right\rangle - \frac{i}{2}\left\langle \mathbf{S}_j \cdot \left[\mathbf{s}_j \times \mathbf{s}_{j\text{-}1}\right]\right\rangle \tag{S9}$$

Making the following approximation:

$$\left\langle \left(\mathbf{s}_j \cdot \mathbf{S}_j\right)\left(\mathbf{S}_j \cdot \mathbf{s}_{j\text{-}1}\right)\right\rangle \approx \left\langle \mathbf{s}_j \cdot \mathbf{S}_j\right\rangle\left\langle \mathbf{S}_j \cdot \mathbf{s}_{j\text{-}1}\right\rangle = \Lambda^2 s_{Cu}^2 S_{Mn}\left(S_{Mn}+1\right) \tag{S10}$$

results in:

$$\left\langle \mathbf{s}_j \cdot \mathbf{Q}_j \cdot \mathbf{s}_{j\text{-}1} \right\rangle = \frac{S_{Mn}\left(2S_{Mn}-1\right)}{3}\Lambda^2 s_{Cu}^2 \qquad \text{(S11)}$$

Introducing eq. S2, S3, S6, S7, S8, S11 in eq. S1 leads to eq. 7 of the main manuscript.

(S1) Seiden, J. *J. Phys. Lett.* **1983**, *44*, L947.

(S2) Cohen-Tanoudji, C.; Diu, B.; Laloë, F. *Mécanique Quantique*; Hermann: Paris, 1977.