Supporting Information

Theoretical Understanding of Absorption-Based SPR Sensor Based on Kretchmann's Theory

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Derivation of Eq 23

The Approximation of Eq 23 is described. In the usual SPR experiments, $k_0' > |k_R'|$ is satisfied so that Eq 21 is simplified to

$$n_p \frac{\omega}{c} \sin\theta_{SPR} \cong k_0', \qquad \text{(S1)}$$

where k_0' is given in Eq 19. When $\varepsilon_s' >> \varepsilon_s''$, $|\varepsilon_m'| >> \varepsilon_m''$ and $|\varepsilon_m'| >> \varepsilon_s'$, Eq S1 is transferred to

$$\sin\theta_{SPR} \cong \frac{n_s}{n_p}\sqrt{\frac{|\varepsilon_m'|}{|\varepsilon_m'| - n_s^2}} \cong \frac{n_s}{n_p}, \qquad \text{(23)}$$

where the complex refractive index $n_s + i\kappa_s \equiv \sqrt{\varepsilon_s}$ is used.

Derivation of Eq 25

The approximation of Eq 25 is described. Two approximations of k_0'' and k_R'' are required to estimate η. In the following calculations, we use the relation of $n_s + i\kappa_s \equiv \sqrt{\varepsilon_s}$ and assume that $\varepsilon_s' >> \varepsilon_s''$, $|\varepsilon_m'| >> \varepsilon_m''$ and $|\varepsilon_m'| >> \varepsilon_s'$. Approximations of k_0'' in Eq 19 are given by

$$\begin{aligned} k_0'' &\cong \frac{\omega}{c}\sqrt{\varepsilon_s'}\left(\frac{\varepsilon_s'\varepsilon_m''}{2\varepsilon_m'^2} + \frac{\varepsilon_s''}{2\varepsilon_s'} - \frac{3\varepsilon_s''}{4\varepsilon_m'}\right) \\ &\cong \frac{\omega}{c}\left\{\frac{n_s^3\varepsilon_m''}{2\varepsilon_m'^2} + \left(1 + \frac{3n_s^2}{2|\varepsilon_m'|}\right)\kappa_s\right\} \qquad \text{(S2)} \\ &\cong \frac{\omega}{c}\left(\frac{n_s^3\varepsilon_m''}{2\varepsilon_m'^2} + \kappa_s\right) \end{aligned}$$

On the other hand, approximations of k_R'' in Eq 20 are given by

$$
\begin{aligned}
k_R'' &\cong \frac{\omega}{c}\sin\phi\left(\frac{2}{|\varepsilon_m'|+n_s^2}\right)\left(\frac{|\varepsilon_m'|n_s^2}{|\varepsilon_m'|-n_s^2}\right)^{3/2}\cdot\exp\left[-\frac{4\pi d}{\lambda}\frac{|\varepsilon_m'|}{\left(|\varepsilon_m'|-n_s^2\right)^{1/2}}\right] \\
&\cong \frac{\omega}{c}\sin\phi\frac{2n_s^3}{|\varepsilon_m'|}\left(1+\frac{n_s^2}{2|\varepsilon_m'|}-\frac{3n_s^4}{2\varepsilon_m'^2}\right)\cdot\exp\left[-\frac{4\pi d}{\lambda}\sqrt{|\varepsilon_m'|}\left(1+\frac{n_s^2}{2|\varepsilon_m'|}\right)\right] \qquad \text{(S3)} \\
&\cong \frac{\omega}{c}\sin\phi\frac{2n_s^3}{|\varepsilon_m'|}\cdot\exp\left[-\frac{4\pi d}{\lambda}\sqrt{|\varepsilon_m'|}\left(1+\frac{n_s^2}{2|\varepsilon_m'|}\right)\right]
\end{aligned}
$$

with

$$\exp(i\phi) \equiv \left[r_{pm}\right]_{k_x=k_0} = \frac{A+iB}{A-iB}, \qquad \text{(S4)}$$

$$A = \varepsilon_m\sqrt{\varepsilon_p-\frac{\varepsilon_m\varepsilon_s}{\varepsilon_m+\varepsilon_s}}, \qquad \text{(S5)}$$

and

$$B = -\varepsilon_p\sqrt{-\frac{\varepsilon_m^2}{\varepsilon_m+\varepsilon_s}}. \qquad \text{(S6)}$$

It is noted that Eqs S4 to S6 are equivalent to

$$\tan\frac{\phi}{2} = \frac{B}{A} \cong \frac{\varepsilon_p}{\sqrt{\left\{|\varepsilon_m'|\left(\varepsilon_p-\varepsilon_s'\right)-\varepsilon_p\varepsilon_s'\right\}}}. \qquad \text{(S7)}$$

Eqs. S2 and S3 finally give the approximated value of η as follows

$$\eta = \frac{k_0''}{k_R''} \cong \frac{\varepsilon_m''}{4|\varepsilon_m'|\sin\phi}\left(1+\frac{2\varepsilon_m'^2}{n_s^3\varepsilon_m''}\kappa_s\right)\exp\left[\frac{4\pi d}{\lambda}\sqrt{|\varepsilon_m'|}\left(1+\frac{n_s^2}{2|\varepsilon_m'|}\right)\right]. \quad (25)$$