

Spectral Properties and Energy Transfer Between $\text{Ce}^{3+} \rightarrow \text{Yb}^{3+}$ in the $\text{Ca}_3\text{Sc}_2\text{Si}_3\text{O}_{12}$ Host: Is It an Electron Transfer Mechanism?

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Experimental Methods and Measurements

Materials and Synthesis. The phosphors were synthesized by a sol-gel combustion method. Firstly, the metal nitrate solutions were obtained in stoichiometric proportions by dissolving rare earth oxides in nitric acid. Anhydrous ethanol was added to the resulting solution with heating at 65 °C to evaporate superfluous water until a transparent sol was obtained. The sol was dried at 95 °C to form a dry gel which was then fired in a muffle furnace at 700 °C in air for 3 hours to obtain a precursor. After grinding, the precursor was sintered at 1400 °C for 6 h in a CO reducing atmosphere created by burning activated carbon. The product was reground, washed twice by warm dilute nitric acid and water, and finally oven-dried. The XANES (Figure S1) shows that ytterbium is present in the samples in the trivalent state even though a reducing atmosphere was employed in synthesis. In fact the PL and PLE spectra are unchanged whether an ambient air or reducing atmosphere is employed in the synthesis (Figure S2).

Instrumental Measurements. The final products were examined on a Bruker D8 Advance type powder X-ray diffractometer, using Cu Ka radiation ($\lambda = 1.5404 \text{ \AA}$) and operating at 40 kV and 40 mA. The UV-excitation and UV-visible emission spectra at room temperature, as well as the luminescence decay curves, were measured by a FLS920-combined Fluorescence Lifetime and Steady State Spectrometer (Edinburgh Instruments) equipped with a 450 W Xe lamp, a TM300 excitation monochromator and two TM300 emission monochromators, equipped with a red sensitive PMT for visible spectral measurements and a R5509-72 NIR-PMT in a liquid nitrogen cooled housing (Hamamatsu Photonics) for NIR spectral measurements. A 150 W nF900 flash lamp was employed for measurement of nanosecond lifetime decay curves. For lifetimes in the microsecond and millisecond range a 60W uF900 flash lamp with a pulse width of 1.5–3.0 μs and a pulse repetition rate of 50 Hz was employed. The Yb X-ray absorption near-edge structure (XANES) analysis (Fig. S1) was carried out in the fluorescence mode at room temperature on the beamline 1W2B of the Beijing Synchrotron Radiation Facility (BSRF). The oxidation state detected in all samples is +3. The excitation and emission spectra of samples prepared in an ambient or reduced atmosphere are similar (Fig. S2).

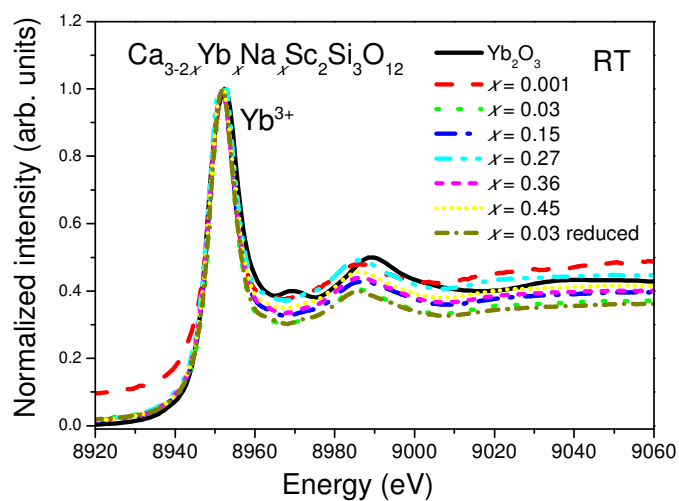


Fig. S1. Yb-L3 X-ray Absorption Near-edge Structure in $\text{Ca}_{3-2x}\text{Yb}_x\text{Na}_{0.03}\text{Sc}_2\text{Si}_3\text{O}_{12}$ Samples.

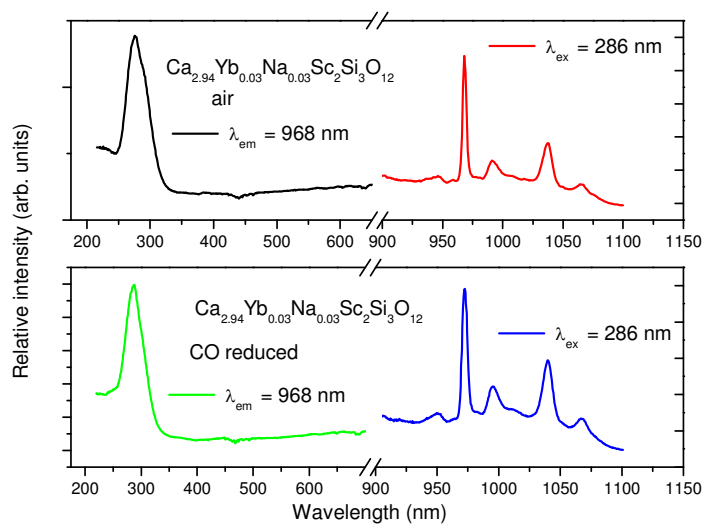


Fig. S2. The Excitation and Emission Spectra of $\text{Ca}_{2.94}\text{Yb}_{0.03}\text{Na}_{0.03}\text{Sc}_2\text{Si}_3\text{O}_{12}$ Synthesized Under Air and CO Reduced Atmosphere.

Evidence That Yb³⁺ Substitutes at the Ca²⁺ and Not the Sc³⁺ Site. The electronic spectra of several lanthanide ions diluted into the Ca₃Sc₂Si₃O₁₂ host have been reported and it has been demonstrated that Ln³⁺ substitutes at the Ca²⁺ site of D₂ point group symmetry.^{S1,S2} The Yb³⁺ ion is slightly smaller than the Eu³⁺ or Tb³⁺ ions so that some additional proof may be required to show that it does not substitute at the Sc³⁺ site. Therefore, we synthesized Ca₃Sc_{2-x}Yb_xSi₃O₁₂ (x = 0.03) and compared the X-Ray diffractogram as in Figure S3 below. It is evident that Sc₂O₃ and SiO₂ are produced in this case, but not for Ca_{3-2x}Sc₂Yb_xNa_xSi₃O₁₂ (x = 0.03).

Figure S4 shows the Yb³⁺ emission spectra of samples prepared with starting stoichiometries for Yb³⁺ substituting at the Ca²⁺ or Sc³⁺ sites. The spectra are very similar.

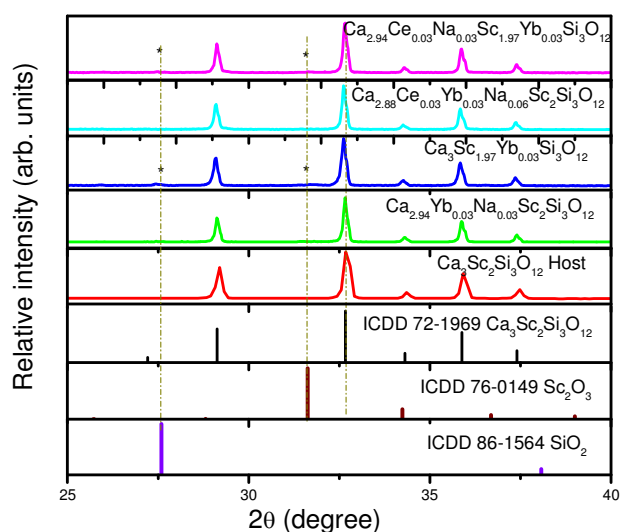


Fig. S3. X-ray Diffractograms of Ce, Yb, and Na-substituted Ca₃Sc₂Si₃O₁₂ Host.

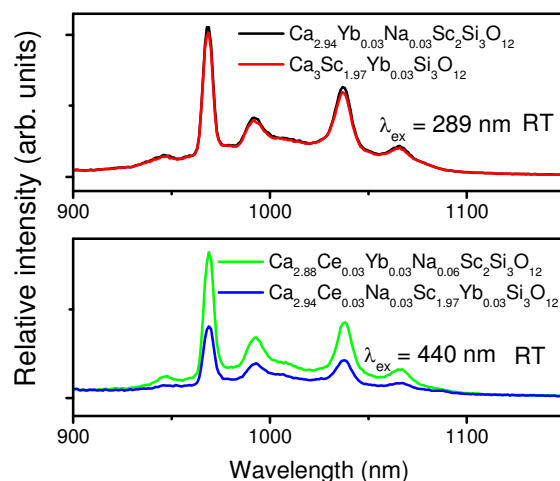


Fig. S4. Yb³⁺ Emission Spectra at RT of Yb- and Ce, Yb-doped Ca₃Sc₂Si₃O₁₂.

Artefact in Lifetime Measurements. For 289 nm excitation of CSS:Yb_xNa_x (and CSS:Ce_{0.1}Yb_xNa_{x+0.1}), in addition to excitation at 440 nm and 896 nm, we observe a biexponential decay of Yb³⁺ emission with a rise lifetime of ~9.7 μ s. Since this phenomenon also occurs during direct excitation at 896 nm, it is attributed to the instrumental response using NIR detection. The decay curves of the ²F_{5/2} *J*-multiplet of CSS:Yb_{0.27}Na_{0.27} at 3 K are shown in Figure S5. On taking into account this artifact, the lifetime decay at RT is monoexponential but a slight decrease from 0.97 \pm 0.04 ms to 0.86 \pm 0.03 ms is observed for the (different) direct ²F_{5/2} population mechanism when the excitation wavelength changes from 289 nm to 884 or 896 nm.

Ce³⁺ Lifetime at 77 K. At 77 K, with excitation at 440 nm and emission at 550 nm, the lifetime of Ce³⁺ in CSS:Ce_{0.1}Yb_{0.03}Na_{0.13} can be fitted by a monoexponential function ($\tau = 58.1 \pm 0.2$ ns $R^2_{\text{adj}} = 0.9964$), or a biexponential function with similar results to that in Table 1 for this sample ($\tau_1 = 66.9 \pm 1.3$ ns, $\tau_2 = 26.8 \pm 2.5$ ns, $R^2_{\text{adj}} = 0.9970$).

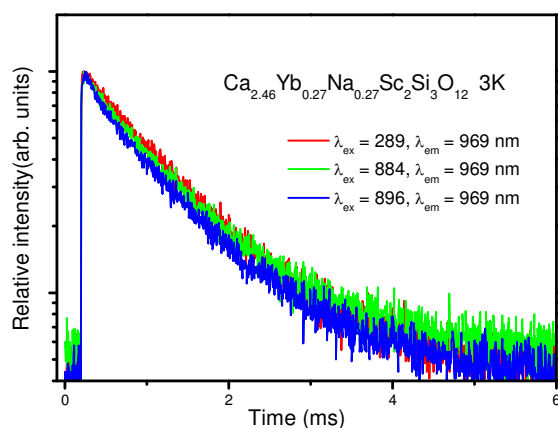


Fig. S5. The Emission Decay Curves of CSS:Yb_{0.27}Na_{0.27} at 3 K.

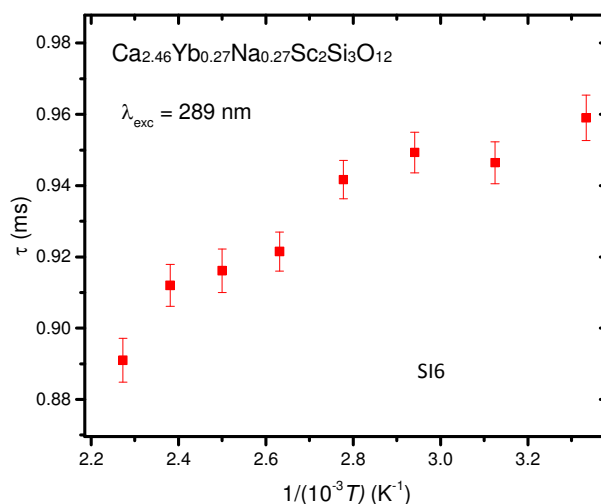


Fig. S6. Temperature Dependence of the Yb^{3+} Lifetime of $\text{CSS:Yb}_{0.27}\text{Na}_{0.27}$ Upon CT Excitation.

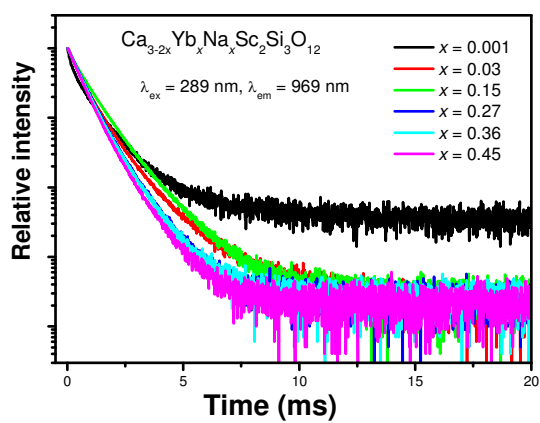


Fig. S7. Room Temperature Decay Curves of the $^2\text{F}_{5/2}$ State as a Function of Yb^{3+} Concentration in $\text{CSS:Yb}_x\text{Na}_x$.

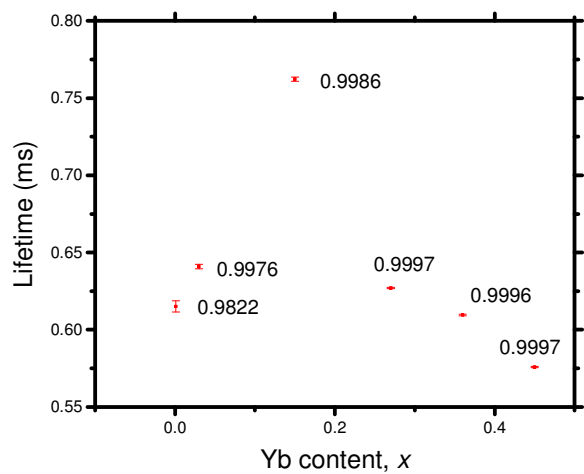
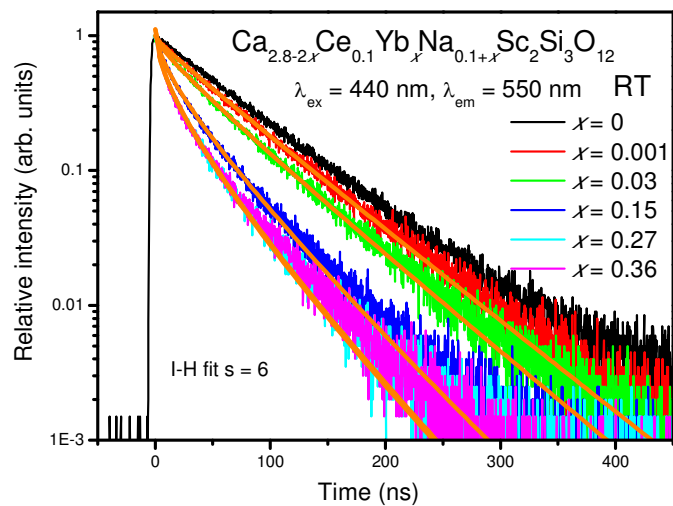


Fig. S8. Room Temperature Monoexponential Lifetimes of the $\text{Yb}^{3+} {}^2\text{F}_{5/2}$ J -multiplet as a Function of Yb Concentration in $\text{CSS:Yb}_x\text{Na}_x$. The Numbers are the Adjusted Coefficients of Determination, R^2_{adj} , For the Fits. ($\lambda_{exc} = 289 \text{ nm}$, $\lambda_{em} = 969 \text{ nm}$)



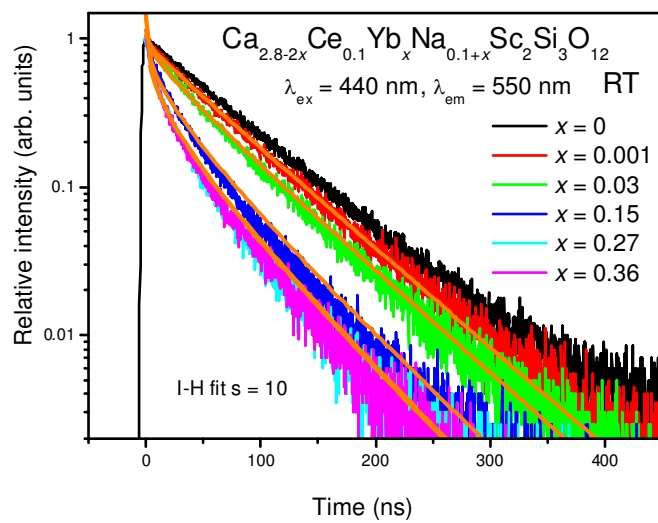
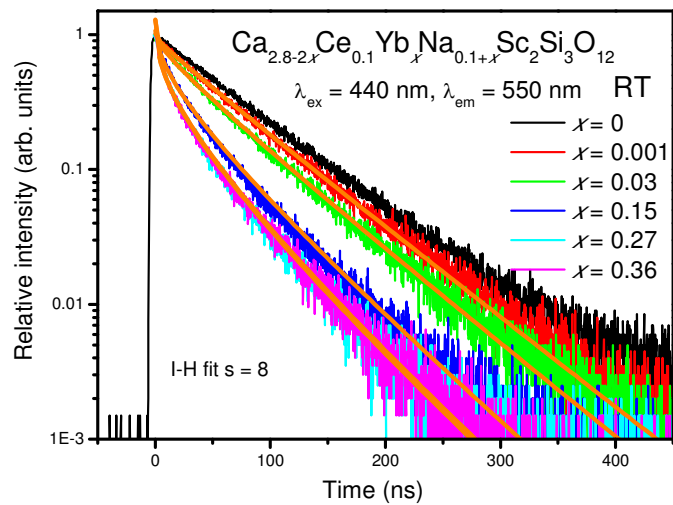


Fig. S9. Inokuti-Hirayama Fits to the RT Ce^{3+} Emission in $\text{CSS}:\text{Ce}_{0.1}\text{Yb}_x\text{Na}_{0.1+x}$ for $s = 6, 8$ and 10

$\text{Ca}_{2.8-2x}\text{Ce}_{0.1}\text{Yb}_x\text{Na}_{0.1+x}\text{Eu}_x\text{Sc}_2\text{Si}_3\text{O}_{12}$		
$x\text{Yb}^{3+}$	R_{adj}^2	C_A/C_0
0	0.9984	-
0.001	0.9984	0.14
0.03	0.9979	0.33
0.15	0.9949	0.95
0.27	0.8437	1.24
0.36	0.9932	1.25

Table S1. Inokuti-Hirayama Fitting Parameters for $s = 8$.

Analysis of Energy Transfer in CSS:Ce_{0.1}Yb_xNa_{x+1} Using the Dornauf-Heber Model^{S3}

The acceptor Ca²⁺ lattice points are arranged in spherical coordination shells around the Yb_{Ca}[•] donor, where N , the number of acceptors per unit volume is approximated by its mean value Zx , where, as previously, x is the molar concentration and Z is the sum over k shells where shell l has lZ sites. The donor-acceptor distance in the l th shell is lR . Then for multipole-multipole interactions (index s), the change of intensity with time is given by:

$$\phi_{EX}(t) = \exp\left\{-\frac{t}{\tau_0}\right\} \cdot \exp\left\{-\frac{t \cdot 4\pi c_A \cdot R_0^s}{\tau_0 (S-3)^{k+1} R^{s-3}}\right\} \\ \times \left[\sum_{l=1}^k \frac{{}^lZ}{Z} \cdot \exp\left\{-\frac{t}{\tau_0} \cdot \left(\frac{R_0}{{}^lR}\right)^s\right\} \right]^{Z \cdot x}$$

where $\tau_0 = 66.8$ ns; c_A is the acceptor (Yb³⁺) concentration = $(x \times 8/1837.8)$; R_0 is the critical transfer radius (Å); and the particulars for the shells are listed in Table S2. The fitted values of R_0 are listed in Table S3.

Shell	Coordination number	Distance (Å)
1	4x	3.7508
2	8x	5.7294
3	2x	6.1250
4	8x	6.8480
5	4x	7.1822
6	8x	8.3870
7	4x	8.6621
8	12x	9.4393
9	8x	10.3854
10	8x	10.6088
11	8x	11.0420
12	16x	11.2523
13	16x	12.0571
14	6x	12.2500
15	8x	12.8114
16	8x	13.5236
17	8x	13.6959
18	16x	14.0341
19	20x	14.2002
20	16x	14.8460
21	8x	15.0031
22	8x	15.4649
23	24x	16.0599
24	24x	16.4921
25	12x	16.6336
26	24x	17.1882
27	12x	17.3241
28	28x	17.7255
29	8x	18.2470
30	10x	18.3750
31	8x	18.6285
32	20x	18.7539
33	16x	19.2475
34	8x	19.3689
35	20x	19.7288

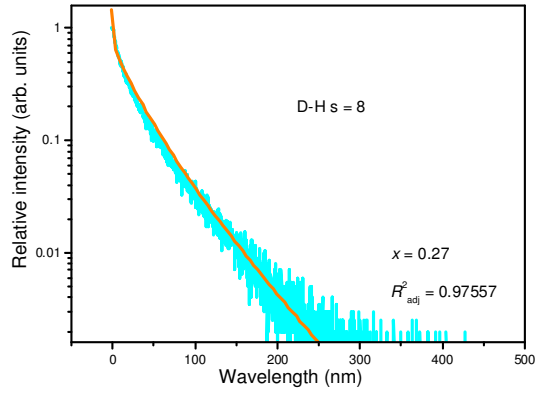
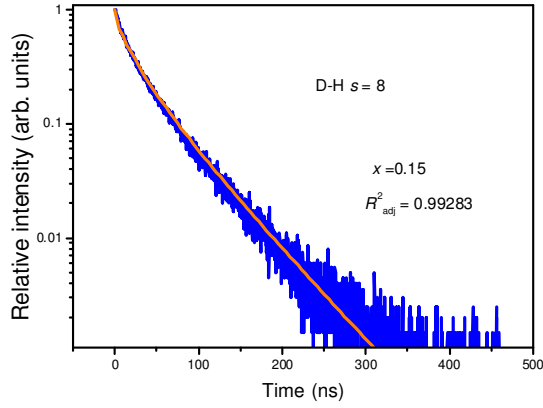
Table S2. Ce³⁺ Donor - Yb³⁺ Acceptor Shell Distances and Coordination Numbers in CSS.

In the case of exchange interaction:

$$\phi_{EX}(t) = \exp\left\{-\frac{t}{\tau_0}\right\} \cdot \exp\left\{-\frac{t \cdot 4\pi c_A}{\tau_0}\right\} \\ \times \left(\frac{R^{k+1}}{\gamma} + 2\frac{R^{k+1}}{\gamma^2} + \frac{2}{\gamma^3}\right) \cdot \exp\left\{\gamma(R_0 - R)\right\} \\ \times \left[\sum_{l=1}^k \frac{l}{Z} \cdot \exp\left\{-\frac{t}{\tau_0} \cdot \exp\left\{\gamma(R_0 - lR)\right\}\right\}\right]^{Z \cdot x}$$

where γ has the dimension L^{-1} .

There is only one variable parameter, R_0 . In the case of CSS:Ce_{0.1}Yb_xNa_{0.1+x}, the fittings for multipole interactions are superior for $s = 8$ and are displayed below for 15 shells:



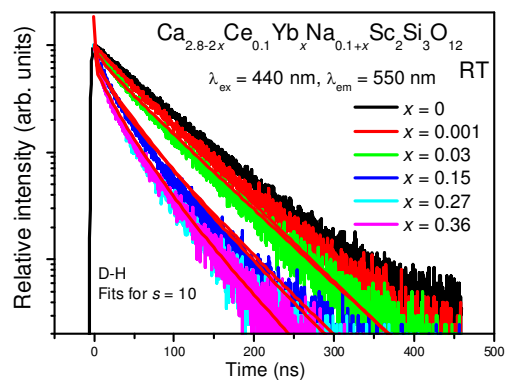
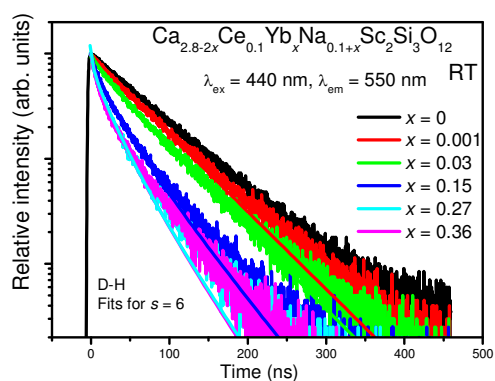
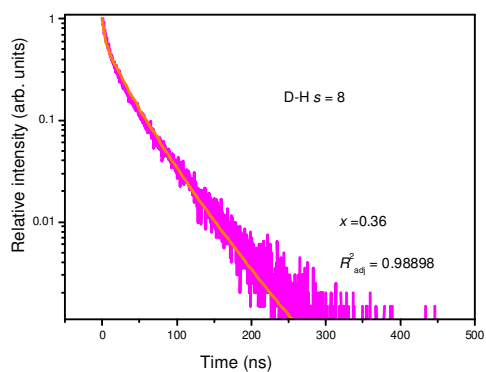


Fig. S10. Energy Transfer in $\text{CSS}:\text{Ce}_{0.1}\text{Yb}_x\text{Na}_{x+1}$ Using the Dornauf-Heber Model:^{S3} Individual Fits for $s = 8$ For Different Yb^{3+} Concentrations and Fits For $s = 6$ and 10.

x	R_0 (Å)
0.03	7.55 ± 0.01
0.15	6.38 ± 0.01
0.27	5.88 ± 0.01
0.36	5.56 ± 0.01

Table S3. Dornauf-Heber Parameter R_0 for Fits With $s = 8$.

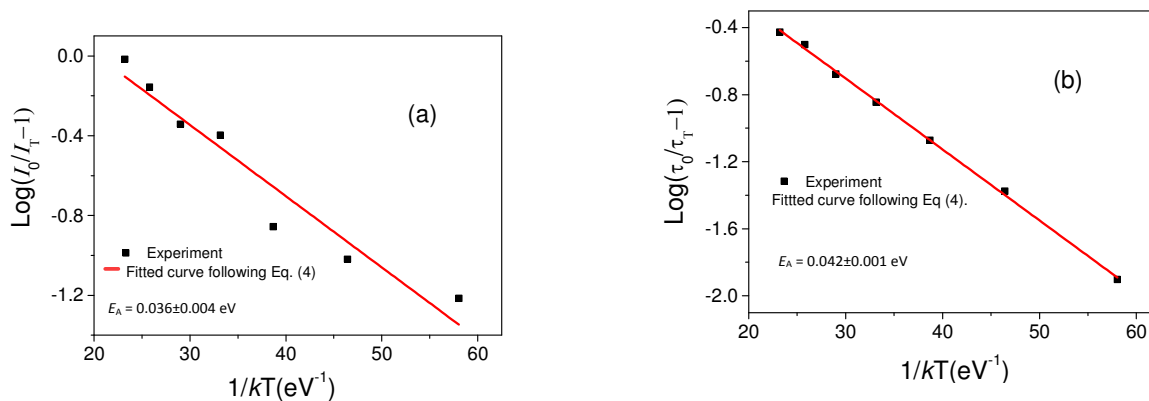


Fig. S11. The Fits Using Eq. (5) of Emission Intensities (a) and Lifetimes (b) as a Function of Temperature Dependence For Ce^{3+} in $\text{CSS:Ce}_{0.1}\text{Yb}_{0.03}\text{Na}_{0.13}$. ($\lambda_{\text{exc}} = 440 \text{ nm}$, $\lambda_{\text{em}} = 550 \text{ nm}$).

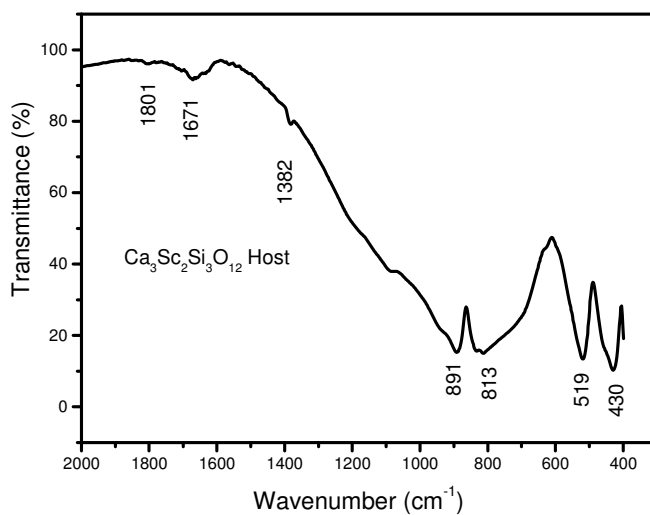


Fig. S12. FT-IR Spectrum of CSS Host at Room Temperature.

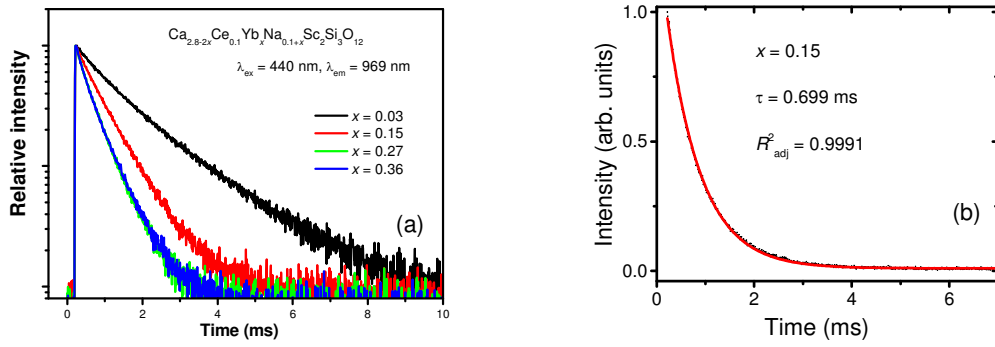


Fig. S13. (a) The Decay Curves of Yb³⁺ Emission at 969 nm Under Excitation of Ce³⁺ at 440 nm For Different Yb³⁺ Concentration Samples at RT; (b) Monoexponential Fit For the Decay of $x = 0.15$.

Decay of Yb³⁺ Emission in CSS:Ce_{0.1}Yb_xNa_{0.1+x}. Figure S13(a) presents the decay curves of Yb³⁺ $^2F_{5/2} \rightarrow ^2F_{7/2}$ emission of CSS: Ce_{0.1}Yb_xNa_{0.1+x} under excitation of 440 nm at RT. The fitting with a monoexponential function for $x = 0.15$ is shown in Figure S13(b). The fit of $x = 0.03$ gives the decay time of 1.36 ms and this decreases to 0.455 ms when $x = 0.36$. Under 440 nm excitation at 77 K, the lifetime of Yb³⁺ for $x = 0.03$ (i.e., Ca_{2.74}Ce_{0.1}Yb_{0.03}Na_{0.13}Sc₂Si₃O₁₂) emission measured at 969 nm is similar 1.267 ± 0.004 ms ($R^2_{\text{adj}} = 0.9967$) (Figure S14). This figure shows that direct excitation at 896 nm gives a fast component superimposed upon this lifetime, which could be due to Yb³⁺ defect site emission. However, the important result observed from this figure is that excitation at 289 nm into the CT band gives Yb³⁺ emission at the same wavelength of 969 nm with a lifetime of 0.630 ± 0.005 ms ($R^2_{\text{adj}} = 0.9711$). This is one-half of the lifetime under 440 nm excitation. It is also shorter than the RT lifetime of CSS:Yb_{0.03}Na_{0.03} where Ce³⁺ is absent: 1.005 ± 0.003 ms ($R^2_{\text{adj}} = 0.9982$). It is therefore clear from the different lifetimes of the decay curves of Ca_{2.74}Ce_{0.1}Yb_{0.03}Na_{0.13}Sc₂Si₃O₁₂ at 77 K under 289 nm and 440 nm excitation that different excitation mechanisms are operative in these cases. In fact, the former excitation wavelength has been associated with Ce³⁺ ions at minority sites^{S4} so that a different energy transfer scenario is operative in this case and it requires further investigation.

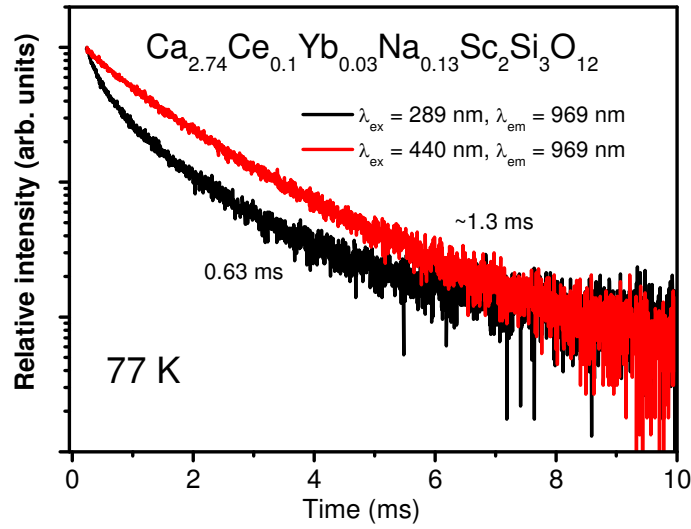


Fig. S14. The Decay Curves of Yb³⁺ Emission in CSS:Ce_{0.1}Yb_{0.03}Na_{0.13} at 77 K.

Contributions of Dipole-dipole and Dipole-quadrupole Energy Transfer Mechanisms to the Total Energy Transfer Rate

Kushida^{S5} generalized the theory of energy transfer (ET) to calculate the average rate \overline{P}_{AB} of ET between the manifolds J_a and J_b by summing over all the possible terminal crystal field (CF) states $|a'b'\rangle$ and taking the Boltzmann-weighted average of the initially occupied CF states, $|ab\rangle$:

$$\overline{P}_{AB} = \frac{2\pi}{\hbar} \overline{J^2 S} \quad (1)$$

and:

$$\overline{J^2} = \frac{1}{[J_a][J_b]_{a,b}^{a,b}} \sum | \langle ab | H_{AB} | a'b' \rangle |^2 \quad (2)$$

is the 'squared electronic transition element', averaged over the initial crystal field (CF) states but summed over the terminal CF states, for ET between the J_a'

J'_b manifolds and

$$\overline{S} = \frac{1}{[J'_a][J'_b]} \sum_{a,b} Y_a Y_b \int g_{aa'}(E) g_{bb'}(E) dE \quad (3)$$

is the averaged overlap integral between individual CF emission and individual CF absorption transitions, in which $[J] = 2J + 1$, Y is a normalized Boltzmann factor, and g is a line shape function. Here, $a = \text{Ce}^{3+}$, $b = \text{Yb}^{3+}$. The typical value of \overline{S} at room temperature is $1 \times 10^{-3}/\text{cm}^{-1}$.^{S5} In the present case, for multiphonon \overline{S} assisted ET, is given by an expression from Wassam and Fong^{S6} and it is not necessary to include it here since we are taking a ratio.

Since the $\text{Ce}^{3+} 5d^1 \rightarrow 4f^1$ transition is electric dipole allowed, the possible multipolar mechanisms for $\text{Ce}^{3+} \rightarrow \text{Yb}^{3+}$ ET are dipole – quadrupole (dq) or dipole – (forced) electric dipole (dd). The $\overline{J^2}$ can be calculated for these two mechanisms by the formulae as follows:

$$\overline{J^2} = \frac{1}{[J'_a][J'_b]} \left(\frac{2}{3} \right) \left(\frac{e^4}{R^6} \right) \left[\langle J'_a | D^{(1)} | J'_a \rangle^2 \right] \left[\sum_{\lambda} \Omega_{B\lambda} \langle J'_b || U^{(\lambda)} || J'_b \rangle^2 \right]$$

$$\overline{J^2} = K \left(\frac{2}{3} \right) \left(\frac{1}{R^6} \right) \left[\sum_{\lambda} \Omega_{B\lambda} \langle J'_b || U^{(\lambda)} || J'_b \rangle^2 \right]$$

or

$$\overline{J^2} = K \left(\frac{2}{3} \right) \left(\frac{1}{R^6} \right) 2 \times 10^{-20} \quad (4)$$

for (dd) interaction between the donor and acceptor with separation R ; is the reduced $\langle J'_b || U^{(\lambda)} || J'_b \rangle$ matrix element of the unit tensor $U^{(\lambda)}$; and Ω_{λ} ($\lambda = 2, 4, 6$) is the Judd parameter obtained by fitting manifold–manifold ED transition intensities. The reduced matrix elements are

$$\langle {}^2F_{5/2} \| U^{(2)} \|^2 F_{7/2} \rangle^2 = 6/49, \quad \langle {}^2F_{5/2} \| U^{(4)} \|^2 F_{7/2} \rangle^2 = 20/49 \quad \text{and}$$

$$\langle {}^2F_{5/2} \| U^{(6)} \|^2 F_{7/2} \rangle^2 = 6/7. \text{S5}$$

The values for Ω_λ parameters for Yb^{3+} are scarce. Ω_2 was given between $0.93\text{-}36.9 \times 10^{-20} \text{ cm}^2$, Ω_4 as $1.6 \times 10^{-20} \text{ cm}^2$; Ω_6 as $1.6 \times 10^{-20} \text{ cm}^2$.^{S7} This gives a value of $\sum_\lambda \Omega_\lambda \langle {}^2F_{7/2} \| U^\lambda \|^2 F_{5/2} \rangle^2$ between $(2.1\text{-}6.5) \times 10^{-20} \text{ cm}^2$.

Kushida gives the value for $\text{Yb}^{3+} = 2 \times 10^{-20} \text{ cm}^2$ which we adopt here.^{S5}

The corresponding formula for dq interaction is:

$$\overline{J^2} = \frac{1}{[J_a][J_b]} \left(\frac{e^4}{R^8} \right) \left[\langle J_a \| D^{(1)} \| J_a \rangle^2 \right] \langle 4f \| r_{\text{Yb}^{3+}}^2 \| 4f \rangle^2 \langle f \| C^{(2)} \| f \rangle^2 \langle J_b \| U^{(2)} \| J_b \rangle^2$$

$$\begin{aligned} \overline{J^2} &= K \left(\frac{1}{R^8} \right) \langle 4f \| r_{\text{Yb}^{3+}}^2 \| 4f \rangle^2 \langle f \| C^{(2)} \| f \rangle^2 \langle J_b \| U^{(2)} \| J_b \rangle^2 \\ &= K \left(\frac{1}{R^8} \right) 3.739 \times 10^{-34} \times 1.867 \times 0.1224 \\ &= K \left(\frac{1}{R^8} \right) 8.547 \times 10^{-35} \end{aligned}$$

(5)

where $\langle 4f \| r_{\text{Yb}^{3+}}^2 \| 4f \rangle = 0.691 \text{ a.u.} = (0.691 \times 5.29 \times 10^{-9})^2 = 1.9337 \times 10^{-17} \text{ cm}^2$;

$$\langle f \| C^{(2)} \| f \rangle = (-7) \begin{pmatrix} 3 & 2 & 3 \\ 0 & 0 & 0 \end{pmatrix} = (-7) 0.195180014 = -1.366260098.$$

The total ET rate is:

$$\overline{P}_{\text{total}} = \sum_{\text{shells}} \overline{P}_{AB}(R) N C_{\text{Yb}} P \quad (6)$$

where $\overline{P}_{AB}(R)$ is the ET rate for Ce^{3+} - Yb^{3+} transfer at the separation R , N is the number of acceptor ions in the shell, C_{Yb} is the concentration of Yb doping, and P is the probability that the acceptor ion is in the initial state at the relevant ET process.

From the crystal structure, $R_1 = 3.7508 \text{ \AA} = 3.7508 \times 10^{-8} \text{ cm}$, $N_1 = 4$, etc., so

$$\bar{P}_{total} = \sum_{shells} \bar{P}_{AB}(R) N C_{Yb} P = [4 \left(\frac{1}{3.7508 \times 10^{-8}} \right)^6 + 8 \left(\frac{1}{5.7294 \times 10^{-8}} \right)^6 + \dots] C_{Yb} P = 0.00191 \times 10^{48} \bar{P}_{AB} C_{Yb} P \quad (7)$$

(cm^{-6} units for $1/R^6$)

For dd transfer, up to 35 shells.

$$\bar{P}_{total} = \sum_{shells} \bar{P}_{AB}(R) N C_{Yb} P = [4 \left(\frac{1}{3.7508 \times 10^{-8}} \right)^8 + 8 \left(\frac{1}{5.7294 \times 10^{-8}} \right)^8 + \dots] C_{Yb} P = 0.000113212 \times 10^{64} \bar{P}_{AB} C_{Yb} P \quad (8)$$

(cm^{-8} units for $1/R^8$) for dq transfer up to 35 shells.

$$\text{So for up to 35 shells: } \frac{\bar{P}_{total(dd)}}{\bar{P}_{total(dq)}} = \frac{1.91 \times 10^{45}}{1.13212 \times 10^{60}} \times \frac{1.333 \times 10^{-20}}{8.547 \times 10^{-35}} = 0.263$$

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