Supporting Information for:

Tyndall windows: tunable scattering of disordered solid-liquid matching mixtures

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Supplementary Note 1 - Theoretical framework

Tyndall scattering is scattering of electromagnetic radiation from particle with any shape. In the specific case of spherical particles Tyndall scattering reduces to Mie scattering. The scattering efficiency and scattering anisotropy based on Mie theory is obtained by solving Maxwell's equations in spherical coordinates by fulfilling the boundary conditions at the particle-liquid interface.

The scattering cross section σ_s and the scattering anisotropy g can be obtained for spherical particles in the Mie scattering regime as a combination of first kind spherical Bessel functions and spherical Hankel functions[1]. The scattering efficiency Q_s, $Q_s = \frac{\sigma_s}{\pi a^2}$ with a being the particle radius and the scattering anisotropy g can be expressed as following with $x = ka$:

$$Q_s = \frac{2}{x^2}\sum_{l=1}^{\infty} (2l+1)\ (|a_l|^2 + |b_l|^2) \qquad (1)$$

$$g = \frac{4}{Q_s\, x^2}\sum_{l=1}^{\infty}\left[\frac{l(l+2)}{l+1} Re(\, a_l\, a_{l+1} + \, b_l\, b_{l+1}{}^*) + \frac{2l+1}{l\,(l+1)}\, Re(\, a_l\, b_l{}^*)\right] \qquad (2)$$

where a_l and b_l coefficients are expressed in terms of x and $y = m\,x$ with $m = \frac{n_P}{n_m}$ by:

$$a_l = \frac{\psi_l'(y)\,\psi_l(x) - m\ \psi_l(y)\,\psi_l'(x)}{\psi_l'(y)\,\xi_l(x) - m\ \psi_l(y)\,\xi_l'(x)} \qquad (3)$$

$$\mathrm{b_l} = \frac{\mathrm{m}\,\psi_l'(\mathrm{y})\,\psi_l(\mathrm{x}) - \ \psi_l(\mathrm{y})\,\psi_l'(\mathrm{x})}{\mathrm{m}\,\psi_l'(\mathrm{y})\,\xi_l(\mathrm{x}) - \ \psi_l(\mathrm{y})\,\xi_l'(\mathrm{x})} \qquad (4)$$

and

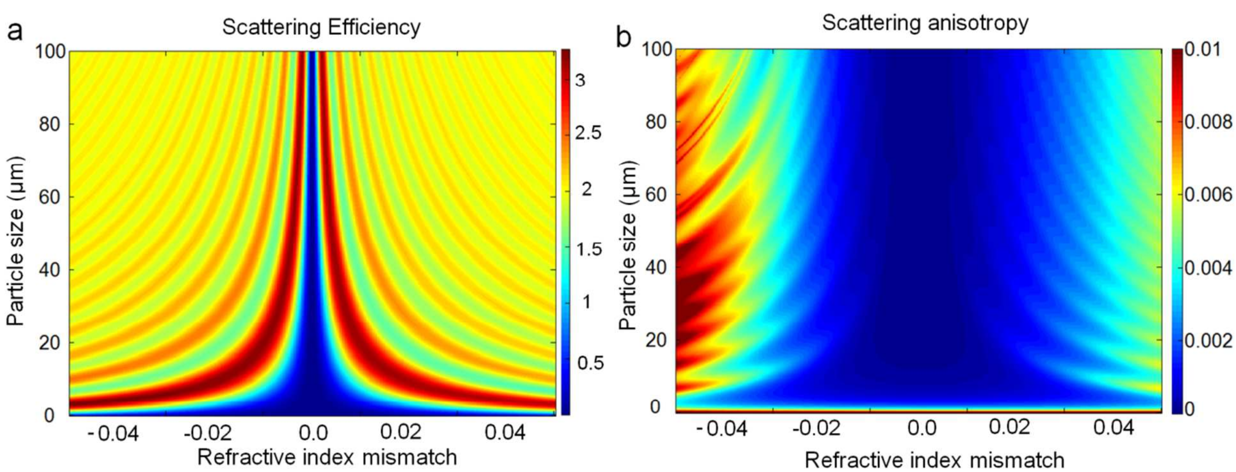
$$\psi_l(z) = \ z\, j_l(z) = \left(\frac{\pi\, z}{2}\right)^{0.5} J_{l+1}(z) \qquad (5)$$

$$\chi_l(z) = -\, z\, y_l(z) = -\left(\frac{\pi\, z}{2}\right)^{0.5} \; Y_{l\,+\,0.5}(z) \quad (6)$$

$$\xi_l(z) = \;\; \psi_l(z) + \; i \;\; \chi_l(z) = z\, h_l{}^{(2)}(z) \qquad (7)$$

Here, *l* and *l+0.5* indicate the orders, $j_l(z)$ and $y_l(z)$ are the spherical Bessel function of the first kind and second kind respectively, $J_{l+1}(z)$and $Y_{l+0.5}(z)$ denote the Bessel function of the first and second kind respectively, $h_l{}^{(2)}(z)$ represent the spherical Hankel function of second kind[2].

Supplementary figure 1 shows the scattering efficiency and the scattering anisotropy based on Mie theory as a function of particle radius and refractive index mismatch. The scattering efficiency shown in figure 1a reaches zero at the refractive index match and is symmetric around the refractive index match. The ripples come from the Bessel and Hankel function. The scattering anisotropy (*1-g*) shown in supplementary figure 1b is not symmetric around the refractive index match however it does reach a minimum at the refractive index match.



Supplementary figure 1 | Mie theory predicting a window of transparency. (a) Scattering efficiency vs. particle size and refractive index mismatch (b) Scattering anisotropy vs. particle size and refractive index mismatch.

Derivation of equation 1:

The volume fraction *f* can be expressed as:

$$f = \frac{V_P}{V_{total}} = \frac{n\,\frac{4}{3}\pi\, a^3}{V_{total}} = N\;\frac{4}{3}\pi\, a^3 \;(8)$$

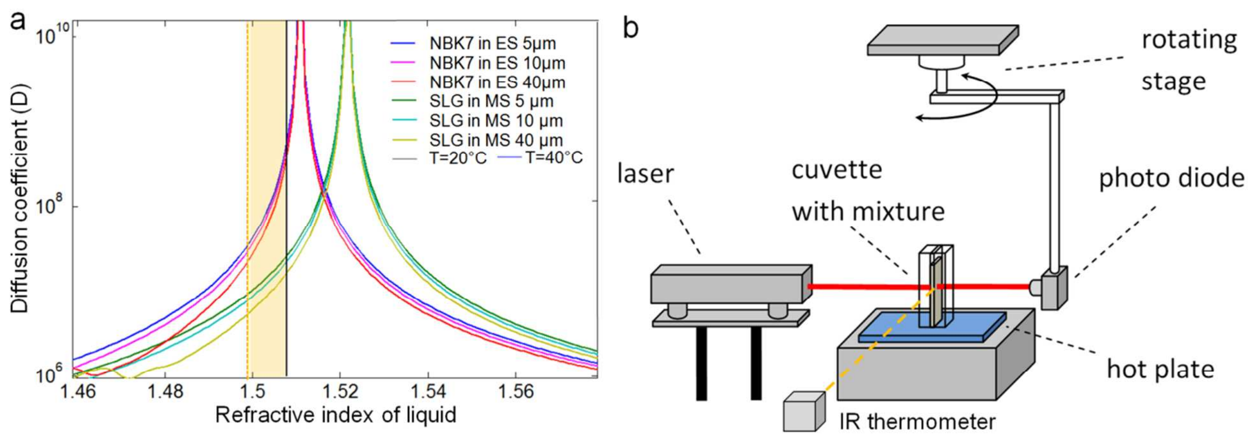
This can be substituted in the decay exponent:

$$\mu_s' \;= N\,\sigma_s \;\;(1-g) = \frac{3\,f\,Q_s}{4\,a}\;(1-g)\;\;(9)$$

with $\sigma_s = \; Q_s\,\pi\, a^2$ to derive the Eq. 1 in the manuscript.

Supplementary Note 2 - Optical diffusion dynamics

Based on Mie theory the optical diffusion ($D = 1/3\mu'_s$) of the mixture goes toward infinity as seen in supplementary figure 2. This translates to transparency of the mixture. In practice the presence of impurities in the particle caps this transparency and therefore there is always some minor absorption in the mixture. The peak will be shifted slightly to right and left on this graph based on the type of mixture used. For example a mixture of sodalime glass in Methyl salicylate will have a peak at ~1.52 which is the refractive index of the liquid at room temperature. As noted the change in particle size does not notably affect these curves.



Supplementary figure 2 | Diffusion dynamic and measurements. (a) Diffusion coefficient curves predicted for two types of mixtures, (b) setup schematics for measuring diffusion angle, the photodiode is rotated around the sample to measure the angular profile.

To measure the diffusion angle we used the setup in supplementary figure 2 b where the photodiode is rotated around the cuvette to record the angular profile of diffused light.

Supplementary References:

1. Wang, L. V. & Wu, H. Biomedical Optics: Principles and Imaging (Wiley, 2007).
2. A. J. Cox, Alan J. DeWeerd, and Jennifer Linden, "An experiment to measure Mie and Rayleigh total scattering cross sections", Am. J. Phys. 70, 620 (2002)