If we denote as $N_1(t)$ and $N_0(t)$ the population numbers of the $^5D_1$ and $^5D_0$ respectively, the rate equations governing the level population of the two states are:

$$\frac{dN_1(t)}{dt} = -W_{10}N_1(t) - R_1N_0(t) \quad (2)$$

$$\frac{dN_0(t)}{dt} = W_{10}N_1(t) - R_0N_0(t) \quad (3)$$

Where W_{10} is the nanradiation transition rate of $^5D_1$-$^7F_2$, R_1 and R_0 is the radiation rate of $^5D_1$-$^7F_2$ and $^5D_0$-$^7F_2$ respectively. According to eqs 2-3, the time dependence of $N_1(t)$ and $N_0(t)$ can be expressed as:

$$N_1(t) = N_1(0)e^{-(W_{10}+R_1)t} \quad (4)$$

$$N_0(t) = N_0(0)e^{-R_0t} + N_1(0)\frac{W_{10}}{R_1+W_{10}-R_0}(e^{-R_0t} - e^{-(R_1+W_{10})t}) \quad (5)$$

Where $N_1(0)$ and $N_0(0)$ is the population at t = 0. In eq 4-5, it is assumed that $\tau_0 = \frac{1}{R_0}, \tau_1 = \frac{1}{R_1+W_{10}}$ is the lifetime of $^5D_0$ and $^5D_1$ respectively.