SUPPORTING INFORMATION: Overlap criterion for cubes

The overlap criterion for cubes was found by using the oriented bounding box (OBB) approach used in the field of computer graphics for collision detection. The algorithm allows the exact and efficient interference detection between complex models undergoing rigid motion. An OBB is a rectangular bounding box at an arbitrary orientation in 3D space. This method involves hierarchical representation of models using tightfitting oriented bounding box trees (OBBTrees). The algorithm traverses two such trees and tests for overlaps between OBBs based on a separating axis theorem. The overlap test checks all edges of one box for intersection with any of the faces of the other box and vice versa.

The intersection testing is based on the following theorem: Two non–intersecting convex polyhedra can be separated by a plane that is either parallel to a face of one of the polyhedra or that contains an edge from each of the polyhedra. It is necessary and sufficient to determine whether or not two convex polyhedra intersect by examining the intersections of the projections of the polyhedra on lines that are perpendicular to the planes described in the theorem. If the minimal intervals containing the projections of the polyhedra onto one of these lines do not intersect, then the polyhedra themselves do not intersect. These lines are called separating axes.

An oriented bounding box is defined by a center $\boldsymbol{C}$, a set of right–handed orthonormal axes $\boldsymbol{A}_0$, $\boldsymbol{A}_1$, and $\boldsymbol{A}_2$, and a set of extents $a_0 > 0$, $a_1 > 0$, and $a_2 > 0$. The solid box is represented by $\vec{C} + \sum_{i=0}^{2} x_i \vec{A}_i : |x_i| \le |a_i|$ for all i. The eight vertices of the box are given by $\vec{C} + \sum_{i=0}^{2} \sigma_i a_i \vec{A}_i$ where $|\sigma_i| = 1$ for all i. Projection of a point $\boldsymbol{P}$ onto line $\boldsymbol{C}_0 + s\boldsymbol{L}$

relative to the line origin $\boldsymbol{C}_0$ is given by $\frac{\vec{L}.(\vec{P}-\vec{C}_0)}{\vec{L}.\vec{L}}\vec{L}$. The distance of the projection from the line origin is: ProjDist($\vec{P}$) = $\frac{\vec{L}.(\vec{P}-\vec{C}_0)}{\vec{L}.\vec{L}}$. The projection distances of the first OBB's vertices relative to the line origin $\boldsymbol{C}_0$ are given by ProjDist $\left(\vec{C}_0+\sum_{i=0}^{2}\sigma_i a_i \vec{A}_i\right)=\sum_{i=0}^{2}\sigma_i a_i \frac{\vec{L}.\vec{A}_i}{\vec{L}.\vec{L}}$.

The minimum length interval containing all eight projection distances has center $K_0 = 0$ and radius $r_A = \frac{\sum_{i=0}^{2} a_i Sign(\vec{L}.\vec{A}_i)\vec{L}.\vec{A}_i}{\vec{L}.\vec{L}}$ as seen in the Figure. The projection distances of the second OBB's vertices relative to the line origin $\boldsymbol{C}_0$ are ProjDist $\left(\vec{C}_0+\sum_{i=0}^{2}\sigma_i a_i \vec{A}_i\right)=\frac{\vec{L}.\vec{T}}{\vec{L}.\vec{L}}+\sum_{i=0}^{2}\sigma_i b_i \frac{\vec{L}.\vec{B}_i}{\vec{L}.\vec{L}}$ where $\boldsymbol{T} = \boldsymbol{C}_1 - \boldsymbol{C}_0$. The minimum length interval containing all eight projection distances has center $K_1 = \vec{L}.\vec{T}$ and radius $r_B = \frac{\sum_{i=0}^{2} b_i Sign(\vec{L}.\vec{B}_i)\vec{L}.\vec{B}_i}{\vec{L}.\vec{L}}$. The condition for non-overlap is $|\boldsymbol{K}_1 - \boldsymbol{K}_0| > r_A + r_B$. This implies $\left|\vec{L}.\vec{T}\right| > \sum_{i=0}^{2} a_i Sign(\vec{L}.\vec{A}_i)\vec{L}.\vec{A}_i + \sum_{i=0}^{2} b_i Sign(\vec{L}.\vec{B}_i)\vec{L}.\vec{B}_i$.

Eberly[1] has tabulated the different possible choices of $\vec{L}$ and the corresponding non-overlap check conditions. It has to be noted that it is better to use the basic formula for overlap detection and not the simplified formulae given in the tables for the case $\vec{L} = \vec{A}_i \times \vec{B}_j$. It was observed that the overlap condition was not commutative in these cases. The results obtained when the overlap of x with y was checked was different from the overlap of y with x. Using the basic criterion in this case $\left|\vec{L}.\vec{T}\right| > \sum_{i=0}^{2} a_i Sign(\vec{L}.\vec{A}_i)\vec{L}.\vec{A}_i + \sum_{i=0}^{2} b_i Sign(\vec{L}.\vec{B}_i)\vec{L}.\vec{B}_i$ is the right approach.

References:

(1) Eberly, D. Dynamic Collision Detection using Oriented Bounding Boxes, www.geometrictools.com , 1999.

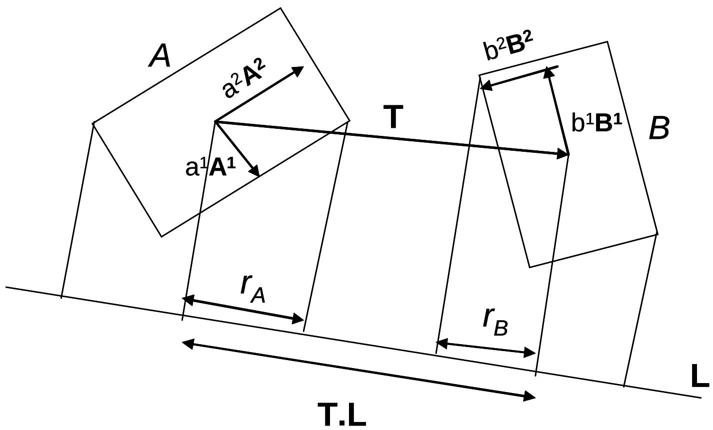

Figure: Schematic of the overlap between the two oriented bounding boxes A and B.