Proceedings of DYCOPS-2007, Cancun, Mexico, 6-8 June, 2007

CONSTRAINED RECURSIVE PARAMETER ESTIMATION FOR ADAPTIVE CONTROL

Angadh Singh, Abhijit S. Badwe, Sachin C. Patwardhan*

Department of Chemical Engineering, Indian Institute of Technology, Bombay, Powai, Mumbai, 400076, India.
**Email:sachinp@che.iitb.ac.in*

Abstract: In adaptive control of systems with poles close to the unit circle, application of recursive estimation techniques can lead to excursions of poles outside the unit circle even when the process has no unstable poles. In this work, we propose a novel constrained formulation for recursive estimation of parameters using pseudo linear regression approach. The Jury's stability criterion is used to impose constraints in the parameter space. The proposed strategy has considerable computational advantage when compared to moving window constrained estimation . We demonstrate the efficacy of the proposed method using experimental data collected from a laboratory heater-mixer setup.
Copyright © 2007 IFAC

Keywords: Recursive parameter estimation, Constrained Estimation, Dynamic Data Reconciliation, Jury's stability criterion, Pseudo linear regression

1. INTRODUCTION

The field of adaptive control with on-line recursive parameter estimation has received considerable attention in the past three decades. The ease of implementation of on-line parameter estimation algorithms developed over the years has made adaptive model based control a competent alternative to nonlinear process control. Parameter estimation is of paramount importance in context of an adaptive control system and the need to generate consistent parameter estimates cannot be ignored. The recursive least square (RLS) estimation of linear in parameter models (ARX/FIR) and its extensions to nonlinear in parameter models (pseudo-linear regression or PLS) have been extensively employed for this purpose and the convergence results of such schemes have been discussed by Ljung (1999). These methods belong to the class of prediction error methods, which fail to generate meaningful estimates when the predictors are unstable.

Parameter estimation scheme based on recursive least squares can be regarded as a form of the Kalman filter (Astrom and Wittenmark, 2001). It is a well acknowledged fact that the Kalman filter or its extensions do not handle bounds or constraints that may exist on the state or parameter estimates. When identifying stable systems which are close to being unstable, it may happen at any step that the parameter values make the predictor unstable (Forssell and Ljung, 1998). They proposed an alternate structure for the OE and Box-Jenkins model structures to overcome this problem when identifying stable systems close to instability. Our objective would be to address the former issue and propose an algorithm which takes into account the bounds on the stability of models recursively identified for such systems in the context of adaptive control.

An alternative to the Kalman filter based methods is the moving-horizon estimation technique. The Moving-Horizon estimation (MHE) techniques were introduced to include the knowledge of inequality and equality constraints on states, parameters and disturbances. In the case of linear time-invariant systems, the resulting optimization problem reduces to quadratic programming. To extend this approach to processes described by nonlinear dynamics, Liebman et al., (1992) proposed non-linear dynamic data reconciliation (NDDR) scheme, which can also account for nonlinear algebraic constraints and bounds on the estimates. In the case of nonlinear constraints (nonlinear process dynamics) or a non-quadratic objective function, the solution is obtained by solving a constrained non-linear optimization problem (NLP) over a moving window. However, the computational burden of numerically solving a nonlinear optimization problem over a window of data at every sampling instant renders it unsuitable for real time applications like adaptive control. Recently, Vachhani et al. (2005) have proposed a Recursive NDDR technique which combines the computational benefits of the extended Kalman filter and the constraint handling properties of NDDR. The *a priori* knowledge of states and parameters is included in a nonlinear optimization problem to be solved numerically at the current instant. The computation time for RNDDR is similar to that of the Kalman filter based methods and hence provides the motivation for the inclusion of constraints in online recursive parameter identification.

In the context of adaptive control of systems with poles close to the unit circle, application of recursive estimation techniques can lead to excursions of poles outside the unit circle due to variance errors even when the process has no unstable poles. These excursions can have detrimental effects on the resulting closed loop behavior. Taking motivation from the RNDDR formulation, we propose a constrained formulation for recursive estimation of parameters. The proposed constrained recursive parameter algorithm consists of recursively identifying the model parameters using the recursive output error (ROE) and extended least squares (ELS) for OE and ARMAX models, respectively. A check on the model stability is done by investigating the zeros of the denominator polynomial of the predictor and the constrained optimization is invoked only when the parameters violate the stability criteria. The Jury's stability criterion is used to formulate the constraints in the nonlinear optimization problem. We then proceed to demonstrate the efficacy of the proposed method on experimental data collected from a laboratory scale heater-mixer setup.

The paper is organized in 3 sections. We begin by discussing about the motivation for constrained parameter estimation in Section 2 and present the proposed constrained recursive estimation method. The results of constrained and unconstrained parameter estimation are presented in Section 3. Finally, we present the conclusions based on the analysis of the obtained results.

2. CONSTRAINED RECURSIVE PARAMETER ESTIMATION

All RPLR algorithms belong to the class of prediction error methods (PEM) in which one step-ahead errors are minimized sequentially. An important requirement of the PEM is that the one step ahead predictor should be stable. (Ljung, 1999). Consider the popularly used ARMAX model structure,

$$A(q^{-1})y(t) = B(q^{-1})u(t) + C(q^{-1})e(t) \tag{1}$$

The one step ahead predictor for ARMAX model is given as follows

$$\widehat{y}(t) = \frac{B(q^{-1})}{C(q^{-1})}u(t) + \left(\frac{C(q^{-1}) - A(q^{-1})}{C(q^{-1})}\right)y(t) \tag{2}$$

A sufficient condition for the above predictor to be stable is that the $C(q^{-1})$ polynomial is stable, which does not impose any stability constraint on the deterministic dynamics. The ARMAX model structure can thus be used to identify a system provided the stability of the noise model polynomial can be guaranteed. In the case of output error (OE) structure

$$y(t) = \frac{B(q^{-1})}{A(q^{-1})}u(t) + e(t) \tag{3}$$

the stability of the predictor is ensured if the roots of the $A(q^{-1})$ polynomial lie inside the unit circle. In the following sub-sections, we first briefly review moving window constrained parameter estimation and pseudo linear regression scheme with reference to OE structure. We then proceed to formulate the constrained recursive parameter estimation algorithm.

2.1 Constrained Parameter Estimation

The coefficients of polynomial $A(q^{-1})$, $B(q^{-1})$ in equation (3) can be estimated by using prediction error method, which minimizes variance of residual signal $\widehat{v}(t)$ defined as

$$\widehat{v}(t) = y(t) - \boldsymbol{\varphi}(t)^T\boldsymbol{\theta} \tag{4}$$

$$\begin{aligned}\boldsymbol{\varphi}(t) = [&-\widehat{y}(t-1)..-\widehat{y}(t-n_a)\\ &u(t-1)..u(t-n_b)]^T\end{aligned} \tag{5}$$

$$\boldsymbol{\theta} = \left[\, a_1 \;...\; a_{na} \; b_1 \;...\; b_{nb} \,\right] \tag{6}$$

In the present work we assume that the dynamics of the process under consideration can be adequately represented by a set of local linear models with poles strictly inside the unit circle. Thus, while performing on-line parameter estimation, it is necessary to ensure that local models do not become unstable due to variance errors caused by low signal to noise ratios. Following Ydstie (1997), the problem of on-line parameter estimation can be posed as a constrained optimization problem over a time window $[t-N, t]$ as follows

$$\widehat{\boldsymbol{\theta}}(t) = \begin{matrix} \min \\ \boldsymbol{\theta}(t) \end{matrix} \left[\Omega + \sum_{i=t-N}^{t} \alpha(i) \left(\widehat{v}(i)\right)^2 \right]$$

$$\Omega = \left(\Delta\boldsymbol{\theta}(t)\right)^T \mathbf{P}(t-N)^{-1} \left(\Delta\boldsymbol{\theta}(t)\right)$$

$$\Delta\boldsymbol{\theta}(t) = \boldsymbol{\theta}(t) - \widehat{\boldsymbol{\theta}}(t-N)$$

$$\text{Subject to } \boldsymbol{\theta}(t) \in \Theta$$

where Θ represents constraint set for parameter estimates, $\alpha(i)$ represents discount factor and $\mathbf{P}_d(t-N) \geq \varepsilon I > 0$ represents a positive definite covariance matrix, which defines a trade-off between emphasis placed on new measurements and old estimate. The constraints on the estimated parameters can be derived using the Jury's stability criteria. It may be noted that the resulting formulation is qualitatively similar to nonlinear dynamic data reconciliation (NDDR) task (Liebman et al., 1992), which is employed on-line for improving the quality of operating data. The main difficulty with this formulation is that it results in a highly nonlinear constrained optimization problem and solving it on-line in real-time may not be feasible.

2.2 Unconstrained Pseudo-Linear Regression

In adaptive control, recursive solution to the unconstrained version of the above optimization problem is popularly employed for on-line computations mainly due to its computational simplicity. Even the unconstrained problem is a nonlinear optimization problem, which can be solved on-line using a *recursive pseudo-linear regression* (RPLS) method called *recursive output error* (ROE) as follows (Ljung, 1999)

$$\widehat{\boldsymbol{\theta}}(t+1) = \widehat{\boldsymbol{\theta}}(t) + \mathbf{K}(t) \left[y(t+1) - \boldsymbol{\varphi}(t+1)^T \widehat{\boldsymbol{\theta}}(t) \right]$$

where gain $\mathbf{K}(t)$ is computed as follows

$$\mathbf{K}(t) = \frac{\mathbf{P}(t-1)\boldsymbol{\varphi}(t+1)}{\lambda + \boldsymbol{\varphi}(t+1)^T \mathbf{P}(t-1) \boldsymbol{\varphi}(t+1)} \tag{7}$$

$$\mathbf{P}(t) = \frac{1}{\lambda} \left[\mathbf{P}(t-1) - \mathbf{K}(t) \boldsymbol{\varphi}(t+1)^T \mathbf{P}(t-1) \right] \tag{8}$$

Here $\mathbf{P}(t-1)$ represents the covariance matrix of model parameters and λ is an exponential forgetting factor.

Though the recursive estimation procedure outlined above is computationally attractive, the parameter estimates are always subject to uncertainty due to various reasons. As a result, it could so happen that the parameters identified from the data provide us with an unstable model despite the fact that the process is open loop stable. This is not an uncommon occurrence during the identification of stable systems which are close to being unstable (Forssell and Ljung, 1998) or if the signal to noise ratio is not sufficiently high (Soderstrom and Stoica, 1981). Also, when the forgetting factor is used, the window of data in the past also known as the asymptotic sample length (ASL), that influences the current parameter estimates is defined as

$$ASL = \frac{1}{1-\lambda}$$

where λ is the forgetting factor. For small values of ASL, variance errors are expected to be large, which can again cause excursions of poles into the unstable region.

2.3 Constrained Recursive Parameter Estimation

Taking motivation from RNDDR formulation (Vachhani et al, 2005), we propose a novel constrained recursive formulation for on-line parameter estimation. By this approach, let us assume that parameter estimates $\widehat{\boldsymbol{\theta}}(t) \in \Theta$ are available at instant $(t+1)$. We first compute $\widetilde{\boldsymbol{\theta}}(t+1)$ using unconstrained recursive estimation scheme discussed above. If estimated $\widetilde{\boldsymbol{\theta}}(t+1) \in \Theta$ (i.e. roots of $A(q^{-1})$ polynomial are inside unit circle), then we set $\widehat{\boldsymbol{\theta}}(t+1) = \widetilde{\boldsymbol{\theta}}(t+1)$ and proceed with the control law calculation. However, if $\widetilde{\boldsymbol{\theta}}(t+1) \notin \Theta$, then we formulate a constrained optimization problem over one sampling period as follows

$$\widehat{\boldsymbol{\theta}}(t+1) = \begin{matrix} \min \\ \boldsymbol{\theta}(t+1) \end{matrix} \Omega(t+1) + \left[\widehat{v}(t+1)\right]^2$$

$$\Omega(t+1) = \left(\delta\boldsymbol{\theta}(t+1)\right)^T \mathbf{P}(t)^{-1} \left(\delta\boldsymbol{\theta}(t+1)\right)$$

$$\delta\boldsymbol{\theta}(t+1) = \boldsymbol{\theta}(t+1) - \widehat{\boldsymbol{\theta}}(t)$$

$$\text{Subject to } \boldsymbol{\theta}(t+1) \in \Theta$$

In particular, when polynomial $A(q^{-1})$ is of first or second order the above constrained optimization problem can be reduced to a QP problem. For example, for second order OE model given by

$$y(t) = \left[\frac{b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1} + a_2 q^{-2}} \right] u(t) + v(t) \tag{9}$$

the constrained optimization problem can be stated as follows

$$\widehat{\boldsymbol{\theta}}(t+1) = \begin{matrix} \min \\ \boldsymbol{\theta}(t+1) \end{matrix} \begin{bmatrix} \boldsymbol{\theta}(t+1)^T \mathbf{H}(t+1) \boldsymbol{\theta}(t+1) \\ +\mathbf{f}(t+1)^T \boldsymbol{\theta}(t+1) \end{bmatrix}$$

$$\mathbf{H}(t+1) = \left[\mathbf{P}(t)^{-1} + \boldsymbol{\varphi}(t+1) \boldsymbol{\varphi}(t+1)^T \right]$$

$$\mathbf{f}(t+1) = -2 \left[\left(\mathbf{P}(t)^{-1}\right)^T \widehat{\boldsymbol{\theta}}(t) + y(k+1) \boldsymbol{\varphi}(t+1) \right]$$

$$\boldsymbol{\theta}(t) = \left[\, a_1(t) \ a_2(t) \ b_1(t) \ b_2(t) \,\right]$$

Subject to

$$\begin{bmatrix} -1 & -1 & 0 & 0 \\ 1 & -1 & 0 & 0 \end{bmatrix} \boldsymbol{\theta}(t+1) \leq \begin{bmatrix} 1-\varepsilon \\ 1-\varepsilon \end{bmatrix}$$

A small number ε has been introduced in the above constraint equation to convert the inequality constraints arising from Jury's criteria into equality constraints. It may be noted that, for higher order models, the objective function remains identical but the constraints become nonlinear functions of model parameters. This formulation ensures that the poles of $A(q^{-1})$ remain within the unit circle at significantly less computational cost when compared to the moving window formulation.

Though the proposed constrained optimization approach ensures the stability of the estimated model, the procedure is computationally cumbersome when compared to the recursive estimation techniques. Thus, we propose to combine the two approaches to make the parameter estimation computationally efficient.

A check for stability is performed every time the parameters are estimated using the unconstrained recursive estimation procedures outlined above. We define the spectral radius of $A(q^{-1})$ as

$$\boldsymbol{\rho} = \max |p_i| \tag{10}$$

where $p_i's$ are zeros of $A(q^{-1})$. The constrained optimization procedure is invoked only when

$$\boldsymbol{\rho} \geq \mathbf{1} \tag{11}$$

Similar to RNDDR approach, the covariance matrix $\mathbf{P}(t+1)$ is updated using equations (7)-(8) irrespective of the method being used to estimate the parameters.

While developing ARMAX model by extended least square (ELS) algorithm, it is required that the poles of polynomial $C(q^{-1})$ should remain within unit circle to meet the requirement of stable predictor. If poles of $C(q^{-1})$ are found to move outside unit circle, then a constrained optimization problem can be formulated and solved in a similar manner.

3. ILLUSTRATIVE EXAMPLE

In this section, the stability of the model obtained at every sampling instant using the ordinary recursive estimation techniques is compared to the proposed scheme by open-loop identification from experimental data. The problem of poles of the estimated model at a particular instant violating stability requirements due to parameter estimation is discussed using the Experimental Two-Tank Heater mixer setup.

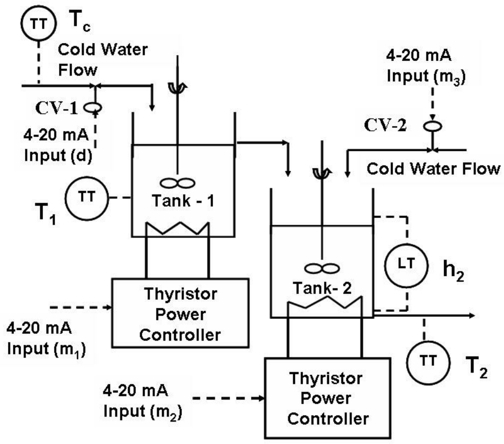

Fig. 1. Schematic of heater-mixer experimental setup

3.1 Heater Mixer Setup

A schematic of the setup is shown in Figure (1). The heater-mixer setup consists of two tanks. The flow to Tank-1 is maintained constant and heating is provided in this tank. The constant overflow from this tank is fed to Tank-2. Cold water can be fed to Tank-2 through control valve CV-2. Temperature measurements are available in both tanks and level in Tank-2 can also be measured. The heat supplied to both the tanks can be varied by the thyristor power controllers. The input to the thyristor power controller is in the range 4 - 20 mA i.e. 4 mA corresponds to zero heat input while 20 mA corresponds to maximum heat input. Also, the flow to Tank-2 can be varied using control valve CV-2.

Identification of this system is carried out by considering three output variables and three input variables. The output variables of interest are the temperatures in both the tanks (T_1 and T_2) and the level in Tank-2 (L_2). The inlet flow to Tank 2 (F_2) and inputs to the thyristor control units (U_1 and U_2) are treated as the manipulated inputs. Fluctuations in the inlet cold water flow to Tank-1 is treated as an unmeasured disturbance in the identification exercise. A sampling time of 1 second is used while collecting data. The inputs to the setup were designed using the *idinput* function in System Identification toolbox in MATLAB. The manipulated inputs are random binary signals in the frequency range of $\begin{bmatrix} 0 & 0.01\pi \end{bmatrix}$. Also, while generating data for identification, a random Gaussian signal (RGS) in the frequency range of $\begin{bmatrix} 0 & 0.005\pi \end{bmatrix}$ is introduced in the inlet cold water flow. The output and input data used for the recursive identification exercise is presented in Figures (2) and (3), respectively. We recursively identify following models using this data

- second order OE model given by 9
- second order ARMAX model of the form

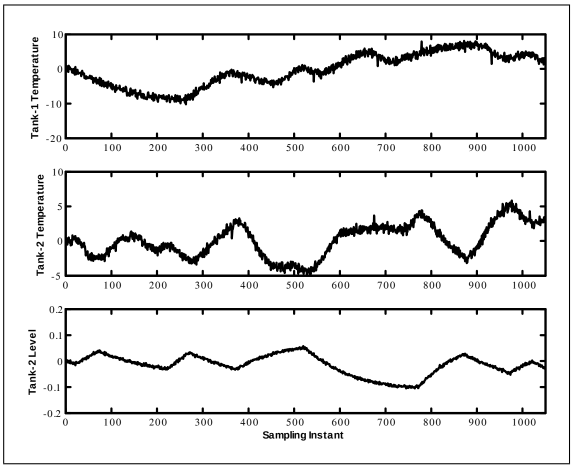

Fig. 2. Output data

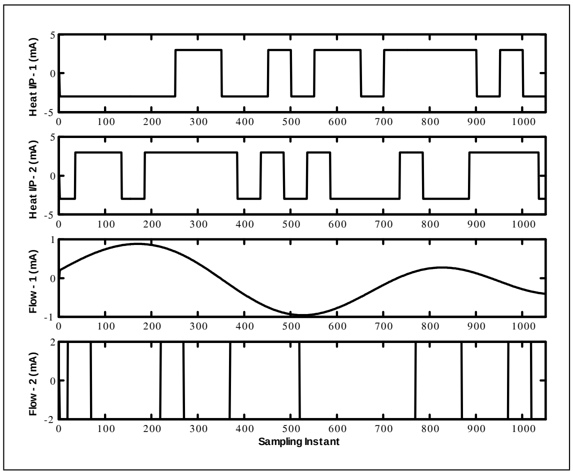

Fig. 3. Input data

Table 1. Input-Ouput selection for model development

Output	**Inputs**	**Model Type**
T_1	U_1	SISO
T_2	U_1, U_2, F_2	MISO
L_2	F_2	SISO

$$y(t) = \frac{b_1 q^{-1} + b_2 q^{-2}}{1 + a_1 q^{-1} + a_2 q^{-2}} u(t) + \frac{1 + c_1 q^{-1} + c_2 q^{-2}}{1 + a_1 q^{-1} + a_2 q^{-2}} e(t) \quad (12)$$

The inputs influencing a particular output are mentioned in Table 1. The recursive identification for both OE and ARMAX models is carried out using (a) unconstrained RPLR methods (b) proposed constrained RPLR method. The forgetting factors were chosen to be constant at 0.99 and 0.98 for OE and ARMAX models, respectively.

It can be argued based on the underlying physics of the system that the poles of the OE model should be inside the unit circle. Thus, the model identified at every sampling instant using the ROE method is expected to be stable. Also the roots of the $C(q^{-1})$ should lie inside unit circle to insure the stability of the predictor. The open loop process time constants in the neighborhood of the chosen operating point are of the order of 150 sec. As a consequence, the poles of discrete OE/ ARMAX model are expected to be close to unit circle when sampling time is chosen as 1 sec. Figure (4) summarizes the variation of the ARMAX model parameters with time and compares the estimates obtained from the proposed method to that of the unconstrained ELS. As can be seen from this Figure, the constrained estimates follow the trend in the unconstrained estimates except at some instants. This is when the unconstrained estimates result in poles of $C(q^{-1})$ outside the unit circle and the proposed estimation procedure projects them inside the unit circle so as to ensure stability of the predictor.

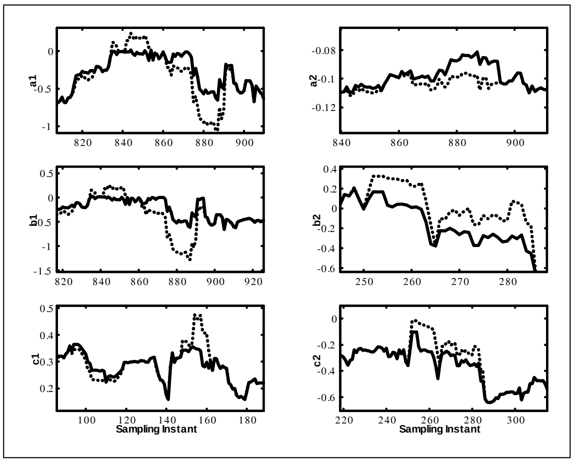

Fig. 4. Comparison of variation in identified ARMAX model parameters with unconstrained (....) and constrained (—) estimation

The spectral radii of the A polynomial for OE model and C polynomial for ARMAX model are shown in Figures (5) and (6), respectively. It is observed that the spectral radii frequently exceed 1, verifying the excursions of the poles of estimated models outside unit circle. However, the proposed constrained estimation scheme ensures that the spectral radii are less than 1, thus guaranteeing the stability of the identified model.

The effect of decreasing the forgetting factor was also studied and has been demonstrated in Figure (7). Decrease in the forgetting factor results in smaller ASL and higher variance errors in parameter estimation. As can be expected, there are more frequent excursions of the poles outside the unit circle when forgetting factor is reduced.

4. SUMMARY AND CONCLUSIONS

In adaptive control of systems with poles close to the unit circle, application of recursive estimation techniques can lead to excursions of poles outside the unit circle even when the process has no unstable poles. Such excursion of poles of the

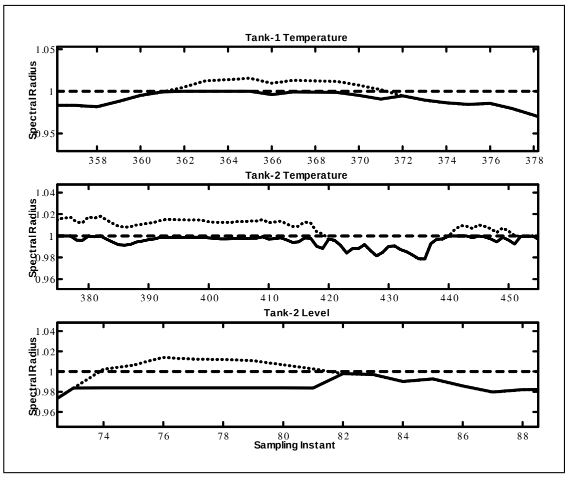

Fig. 5. Comparison of variation in spectral radii for identified OE models - Unconstrained estimate (....), Constrained estimate (—), Unit circle (- - -)

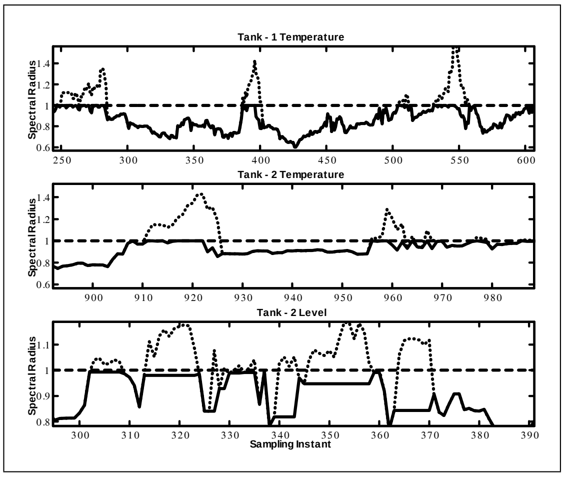

Fig. 6. Comparison of variation in spectral radii of C polynomial for identified ARMAX models - Unconstrained estimate (....), constrained estimate (—), unit circle (- - -)

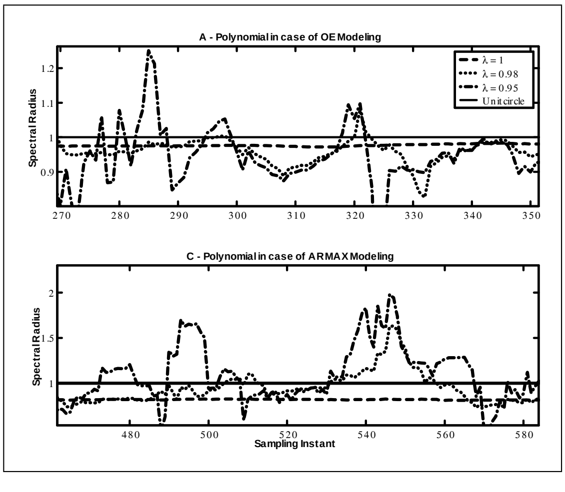

Fig. 7. Effect of forgetting factor on the spectral radii of identified models

estimated model into the unstable region may degrade the adaptive controller performance. In this work, we propose a novel constrained formulation for recursive estimation of parameters using pseudo linear regression approach. The Jury's stability criterion is used to impose constraints in the parameter space. The proposed strategy has considerable computational advantage when compared to moving window constrained estimation . We demonstrate the efficacy of the proposed method using experimental data collected from a laboratory heater-mixer setup.

The analysis of the experimental results reveals that the excursions of the poles outside the stable region in unconstrained RPLR can be mainly attributed to the variance errors. The proposed combination of RPLR and constrained estimation is able to guarantee stability of the predictor even when the ASL is small. Thus, the proposed constrained parameter estimation scheme provides a systematic and computationally attractive approach to impose estimator stability constraints in on-line parameter estimation .

5. REFERENCES

Astrom, K. J. and B. Wittenmark. (2001). *Adaptive Control.* Second Edition, Pearson Education Asia.

Forssell, U. and L. Ljung (1998). Identification of Unstable systems using Output Error and Box-jenkins Model Structures. Proceedings of the 37th IEEE conference on decisions & Control, Tampa, Florida, USA.

Liebman, M. J., T. F. Edgar and L. S. Lasdon (1992). Efficient Data reconciliation and Estimation for Dynamic Processes using Nonlinear Programming techniques. *Computers Chem. Engng.* **16**(10/11), 963-986.

Ljung, L. (1999). *System Identification: Theory for the User.* Second Edition, Prentice-Hall, Inc., Eaglewood Cliffs, New Jersey.

Soderstrom, T. and P. Stoica (1981). On the stability of Dynamic Models Obtained by Least-Squares Identification. *IEEE Trans Auto.Cont.* **AC-26**(2), 575-577

Vachhani, P., R. Rengaswamy, V. Gangwal and S. Narasimhan (2005). Recursive Estimation in Constrained Nonlinear Dynamical Systems. *AIChE J.* **51**(3), 946-959.

Ydstie, B. E. (1997). Certainty Equivalence Adaptive Control: What's New in the Gap. *Chemical Process Control - V.,* Tahoe City, California, J. C. Kantor, C. E. Garcia and B. Carnaham (Eds.), *CACHE-AIChE*, 9-23.