Current-phase Relationship, Thermal and Quantum Phase Slips in Superconducting Nanowires made on Scaffold Created using Adhesive Tape

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SUPPORTING INFORMATION

When one attaches two adhesive tapes into their glue sides and peels off from each other, bunches of glue strands are suspended across two tapes (Figs. S1a and b). The glue strands of the acrylate adhesive tape is generally composed of hard (high T_g), soft (low T_g), and functional monomers, where T_g is a glass transition temperature [S1]. Fig. S1c shows a representative example of chain composition of an acrylic adhesive [S1]. The internal strength is provided by the hard monomers such as ethyl and methyl acrylate with high T_g. The adhesion property originates from the soft monomers with low glass temperatures such as 2-ethylhyxyl, *n*-butyl, and *n*-octyl acrylate. The acrylic acid and acrylic amide play a role of functional monomers for the specific adhesion to desired object.

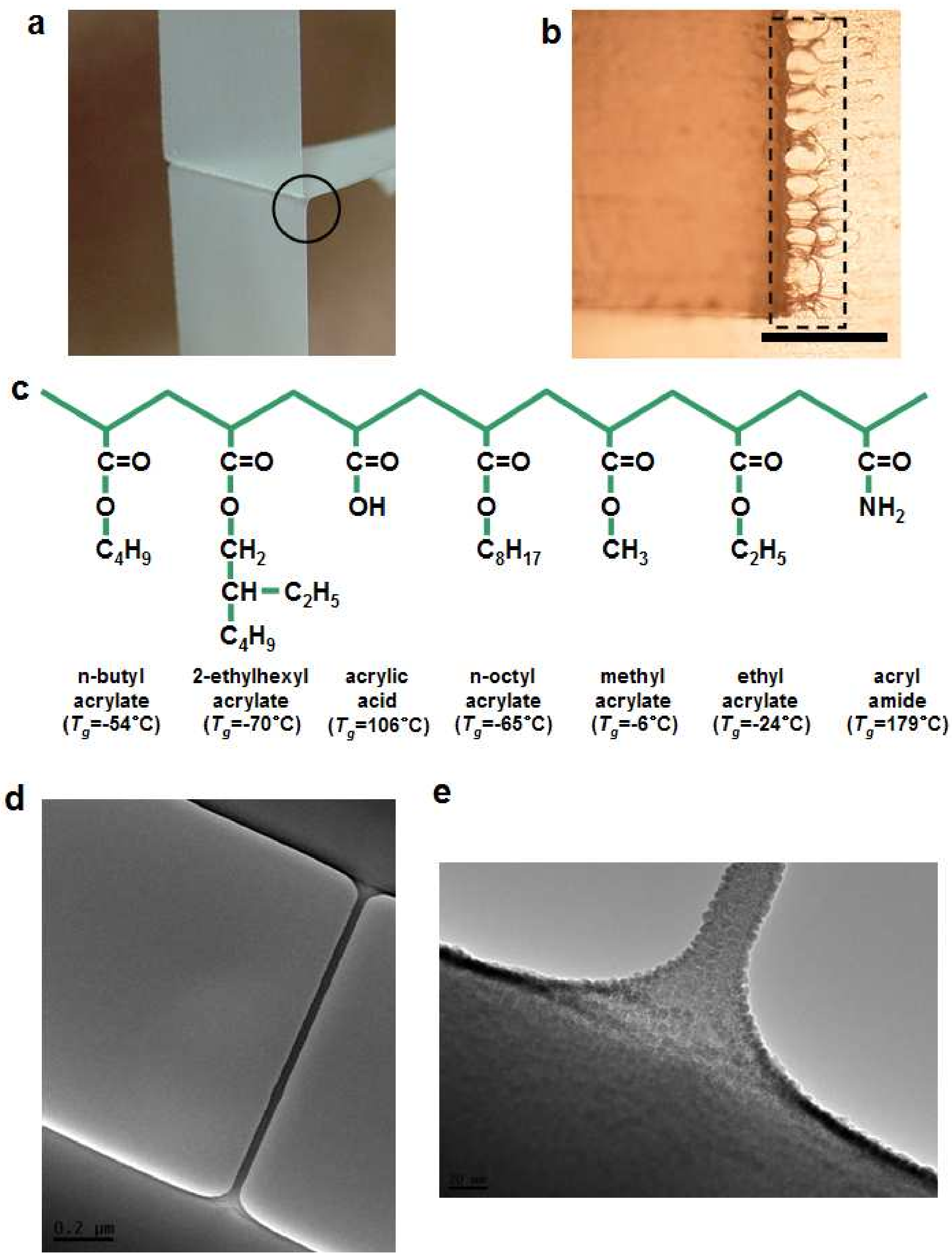

Figure S1 (a) 3M scotch tape was folded and attached each other. When one tries to separate from each other, one would find polymer strings at an interface such as a black circled area between two tapes. These strings are shown in (b) at the interfaces between two tapes as indicated by a dashed box. Scale bar is 1 mm. (c) Schematic of the representative chain composition of an acrylic adhesive, where T_g is a melting temperature of the monomers from Ref. [S1]. This is not for a 3M Scotch tape. (d) TEM total image of the 12 nm-thick-$Mo_{76}Ge_{24}$ wire in Fig. 1c. (e) TEM image of the junction part between the thin film and the nanowire from another 12 nm-thick-$Mo_{76}Ge_{24}$wire.

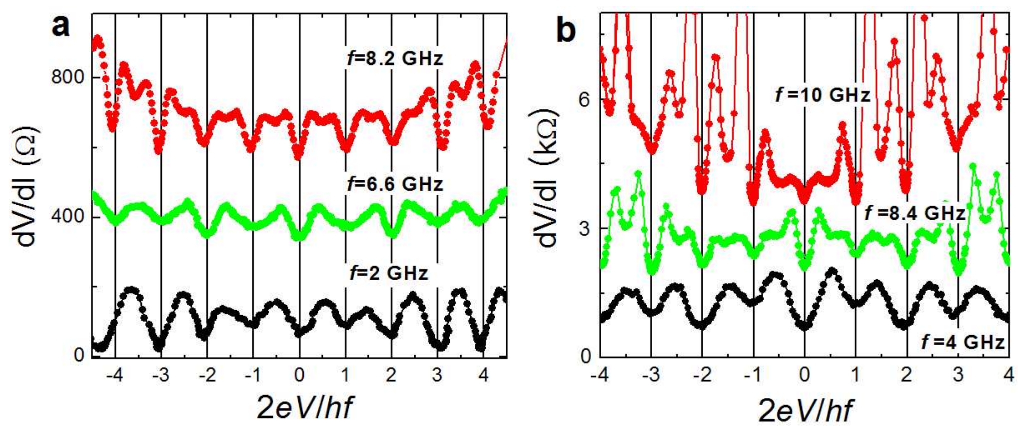

Figure S2. (a) d*V*/d*I*-2*eV*/*hf* curves with various frequencies of MoGe2, where 8.2 GHz-one is shifted vertically as 300 Ω for the clarity. (b) d*V*/d*I*-2*eV*/*hf* curves with various frequencies of Al wire, where data are shifted vertically as 1.8 kΩ for the clarity

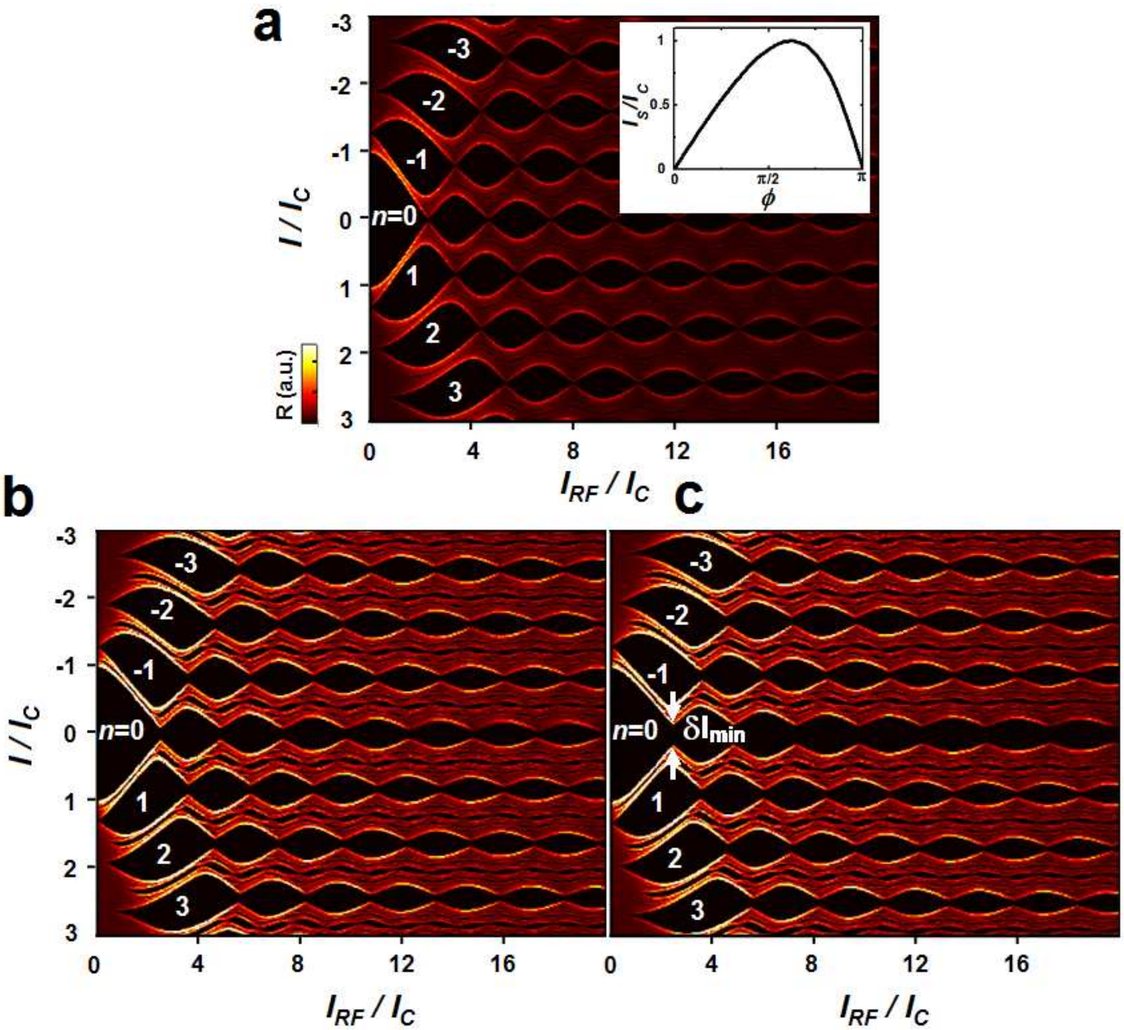

Figure S3. dV/dI (I/I_C, I_{RF}/I_C) plot numerically calculated by the McCumber-Stewart model based on (a) single-valued nonsinusoidal CPR as shown in the inset and multivalued nonsinusoidal CPR with (b) $L/\xi(0)$=5.6 (c) $L/\xi(0)$=5.9 at Ω=0.8. The integer numbers are index numbers of integer Shapiro steps. Inset of (a): An example of the CPR in the case of a short wire limit. To produce this, we used Kulik and Omelyanchuk model for a diffusive wire [S2]: $I_S(\phi) = \frac{4\pi T}{eR_N}\sum_{\omega>0}\frac{\Delta\cos(\phi/2)}{\delta}\arctan\frac{\Delta\sin(\phi/2)}{\delta}$, where $\delta = \sqrt{\Delta^2 \cos^2(\phi/2) + \omega^2}$, Δ is the superconducting gap energy, $\omega = \pi k_B T(2m+1)$ are the Matsubara frequency, and m is integer number.

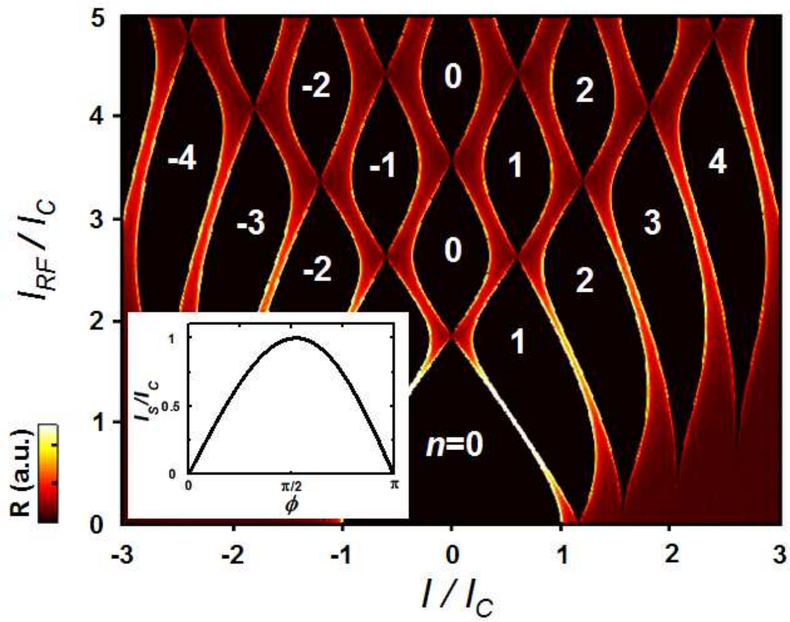

Figure S4. dV/dI (I/I_C, I_{RF}/I_C) plot numerically calculated based on the CPR in the case of L[=80 nm]~$\xi(T)$ [=73 nm] as shown in the inset at Ω=0.6 (see main text).

Current-Phase relation from Mathieu equation in a QPS-dominated superconducting wire

In this note, the phase difference of the wire is ϕ, and the charge transported through the wire is θ. The Hamiltonian [S3] is

$$H = -K/2(\partial^2/\partial\theta^2) - \alpha\cos\theta$$

and the energies are given by the Schrödinger equation $H\Psi = E\Psi$.

Make a change of variable $\theta = \pi+2z$. Then, the Schrödinger equation becomes of the standard Mathieu form:

$$(\partial^2/\partial z^2)\Psi + (a - 2q\cos 2z)\Psi = 0\,.$$

Here, $a=8E/K$ and $q=4\alpha/K$. Mathieu functions are of the form

$$F_\nu(z) = e^{i\nu z}P(z)\,.$$

Here, $P(z)$ is periodic with period π. The characteristic exponent ν is related to the phase difference ϕ on the wire by $\nu = \phi/\pi$. So, $\phi = \pi$ corresponds to $\nu=1$.

A good superconductor corresponds to $q << 1$. In this case, the critical current corresponds to $\nu \approx 1$ and is approximately $I_C \approx K/4\pi$ (in units of $2e$). Expansion of a in q is [Abramowitz and Stegun, Eq. 20.3.15]

$$a = \nu^2 + q^2/2(\nu^2 - 1) + O(q^4)\,.$$

This is useful as long as ν is not too close to 1, i.e., the current is not too close to critical current. This gives energy as a function of ϕ:

$$E = (K/8)(\phi/\pi)^2 + \alpha^2/\{K[(\phi/\pi)^2 - 1]\} + O(\alpha^4/K^3)\,.$$

Differentiating with respect to ϕ, we obtain the current in units of $2e$

$$I \approx (K\phi/4\pi^2) - (2\alpha^2/\pi^2 K)\{\phi/[1-(\phi/\pi)^2]^2\}\,.$$

Supporting Information References

[S1] Czech Z.; Milker R. *Mater. Scien. Pol.* **2005**, *23*, 1015.

[S2] Kulik, I. O.; A. N. Omelyanchuk, *Pis'ma Zh. Eksp. Teor. Fiz.* **1975**, *21*, *216* [*JETP Lett.* **1975**, *21*, *96*].

[S3]Khlebnikov, S. *Phys. Rev. B* **2008**, *78*, *014512*.