

Supporting Information Section

Origin of the α -, β -, ($\alpha\beta$)-, and “slow” dielectric processes in poly(ethyl methacrylate).

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A. Derivation of the expressions for the ratio \mathfrak{R} of the constant volume and constant pressure apparent activation energies

Case 1. $W = Y, X \neq Z$

e.g. VP/VT , Eq.(6) of the text, is obtained as follows.

We start by writing $\tau=\tau(T, V)$, $\tau=\tau(T, P)$ with $V=V(T, P)$ giving

$$(\partial \ln \tau / \partial V)_P = (\partial \ln \tau / \partial T)_V (\partial T / \partial V)_P + (\partial \ln \tau / \partial V)_T \quad (\text{S.1})$$

$$(\partial \ln \tau / \partial T)_P = (\partial \ln \tau / \partial T)_V + (\partial \ln \tau / \partial V)_T (\partial V / \partial T)_P \quad (\text{S.2})$$

hence

$$\begin{aligned} VP/VT &= (\partial \ln \tau / \partial V)_P / (\partial \ln \tau / \partial V)_T = 1 + ((\partial \ln \tau / \partial T)_V (\partial T / \partial V)_P / (\partial \ln \tau / \partial V)_T) \\ &= 1 - (\partial V / \partial T)_T / (\partial V / \partial T)_P \end{aligned} \quad (\text{S.3})$$

Using Eq.(S.2) to replace $(\partial \ln \tau / \partial V)_T$ on the right hand side of Eq.(S.3) gives

$$VP/VT = (1 - \mathfrak{R})^{-1} \quad (\text{S.4})$$

Eqs.(S3) and (S4) correspond to Eq.(6) of the text. Eq.(7) follows from the definition of \mathfrak{R} while Eq.(8) is obtained by a procedure similar to that used to derive Eq.(6).

Case 2. $W = Z, X = Y$

e.g. PT/TP Eq.(11) is obtained by writing $\tau = \tau(T, P)$

$$(\partial \ln \tau / \partial T)_V = (\partial \ln \tau / \partial T)_P + (\partial \ln \tau / \partial P)_T (\partial P / \partial T)_V \quad (\text{S.5})$$

$$\begin{aligned}
PT/TP &= [(\partial \ln \tau / \partial T)_V - (\partial \ln \tau / \partial T)_P] / ((\partial \ln \tau / \partial T)_P (\partial P / \partial T)_V) \\
&= -(1 - \mathfrak{R}) / (\partial P / \partial T)_V = -(\partial T / \partial P)_\tau
\end{aligned} \tag{S.6}$$

Eq.(9) and (10) of the text are obtained by a similar procedure, writing $\tau = \tau(T, V)$ and $\tau = \tau(P, V)$ respectively.

Case 3. $W = Z, X \neq Y$

e.g. TP/VT , Eq.(12) is obtained from Eq.(7) and (9) as

$$(TV/VT) / (TV/TP) = (\partial V / \partial T)_P (1 - \mathfrak{R})^{-1} \tag{S.7}$$

In a similar manner Eq.(7) and Eq.(11) give TV/PT (Eq.13); Eq.(8) and (10) give PT/VP , Eq.(14); Eq.(8) and (11) give PV/TP Eq.(15); Eq.(6) and (9) give VP/TV (Eq.(16) while Eq.(6) and (10) give VT/PV (Eq.17).

Case 4. $W \neq X, Y \neq Z$

e.g. Eq.(18)-(20) may be confirmed as follows; e.g. for Eq.(20) using Eq.(8) and (17)

$$(PT/VT) = (PT/PV)(PV/VT) = (\partial V / \partial P)_T \tag{S.8}$$

A further set of equations of the form $\mathfrak{R} = (1 + \lambda)^{-1}$ may be obtained from the generalized Arrhenius equation $\tau = \tau_0 \exp[Q(T, P) / RT]$. It is easy to show that

$$\begin{aligned}
TP &= -\frac{Q}{RT^2} [1 - T(\partial \ln Q / \partial T)_P] & ; & \quad TV = -\frac{Q}{RT^2} [1 - T(\partial \ln Q / \partial T)_V] \\
PT &= \frac{1}{RT} (\partial Q / \partial P)_T & ; & \quad VT = \frac{1}{RT} (\partial Q / \partial V)_T
\end{aligned} \tag{S.9a-d}$$

From Eq.7 we write

$$\Re = TV/TP = \frac{1 - T(\partial \ln Q / \partial T)_V}{1 - T(\partial \ln Q / \partial T)_P} = \frac{A}{1 + \lambda_a} \quad (\text{S.10})$$

where $A = 1 - T(\partial \ln Q / \partial T)_V$ and $\lambda_a = -T(\partial \ln Q / \partial T)_P$. Two further relations for \Re are obtained by a similar method, giving

$$\Re = [1 + \lambda_b / A]^{-1} \quad ; \quad \Re = [1 + \lambda_c / A]^{-1} \quad (\text{S.11a,b})$$

where $\lambda_b = T(\partial \ln Q / \partial P)_T(\partial P / \partial T)_V$ and $\lambda_c = -T(\partial \ln Q / \partial V)_T(\partial V / \partial T)_P$.

For the case $Q = Q(V)$ then $A = 1$. Roland and coworkers proposed that $Q = Q(V) = C/V^\gamma$ may be applied to the α -process in glass-forming materials.^{34,35} For this case $(d \ln Q / d \ln V) = -\gamma$ giving $\lambda_a = \lambda_b = \lambda_c = T\alpha_p\gamma$, so $\Re = [1 + T\alpha_p\gamma]^{-1}$.

B. Further Dielectric Data

Additional dielectric loss data are provided for PEMA ($M_w = 2.0 \times 10^3$ g/mol) under “isothermal” conditions at 383.15 K ($T \gg T_g$) (in Figure S1). The spectra show the separation of the $\alpha\beta$ -process to α - and β -processes that are well resolved at elevated pressures. Figure S2, comprises the temperature normalized dielectric strengths ($T\Delta\epsilon$) for the three processes at all temperatures investigated as a function of applied pressure. The complementarity in the dielectric strengths of the α - and β -processes can be seen. In Figure S3, the experimentally obtained ratio of activation energies, obtained from the $\tau(\rho)$ representation using Eq. (6), is shown for each of the relaxation processes. This is made by coupling the relaxation times measured under “isobaric” $\tau(T)$ and “isothermal” $\tau(P)$ conditions with the equation of state $V(T, P)$.

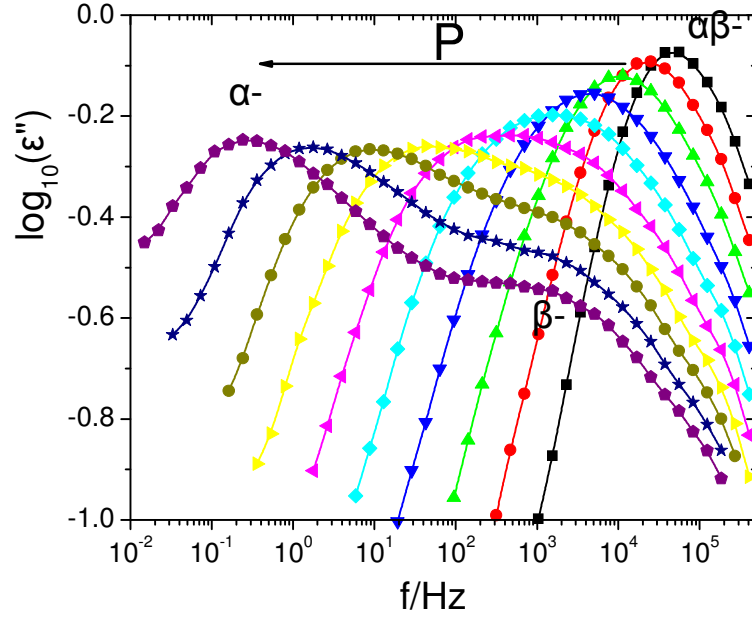


Figure S1. Dielectric loss curves for PEMA ($M_w=2.0 \times 10^3$ g/mol) under “isothermal” conditions at $T=383.15$ K. The curves are at (squares): 0.1, (circles): 30, (up triangles): 60, (down triangles): 90, (rhombus): 120, (left triangles): 150, (right triangles): 180, (polygons): 210, (stars): 240 and (pentagons): 270 MPa.

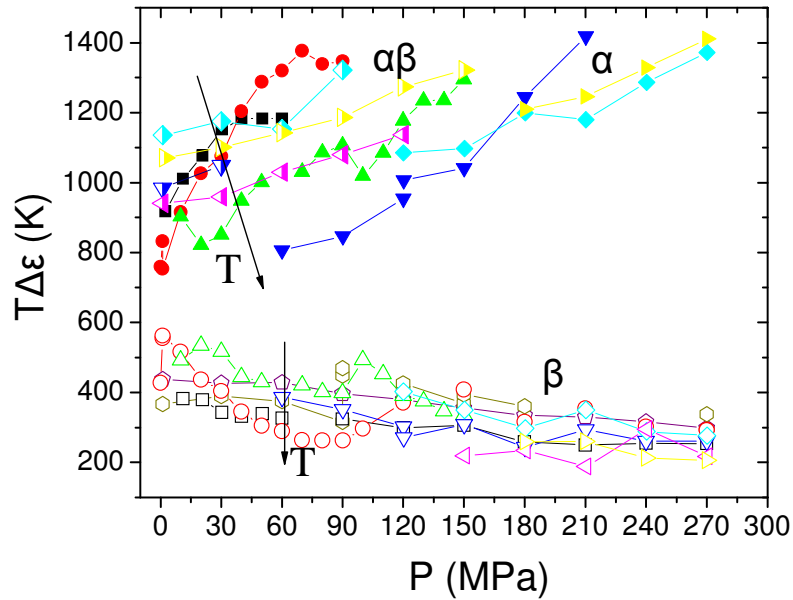


Figure S2. Pressure dependence of the dielectric strength $T\Delta\epsilon$ of PEMA ($M_w=2.0 \times 10^3$ g/mol), corresponding to the three processes as follows: β - (open symbols), α - (filled symbols) and $\alpha\beta$ -

process (half-filled symbols) at the different temperatures investigated: (pentagons): 323.15, (polygons): 333.15, (squares): 343.15, (circles): 353.15, (up triangles): 363.15, (down triangles): 373.15, (rhombus): 383.15, (left triangles): 393.15 and (right triangles): 403.15 K.

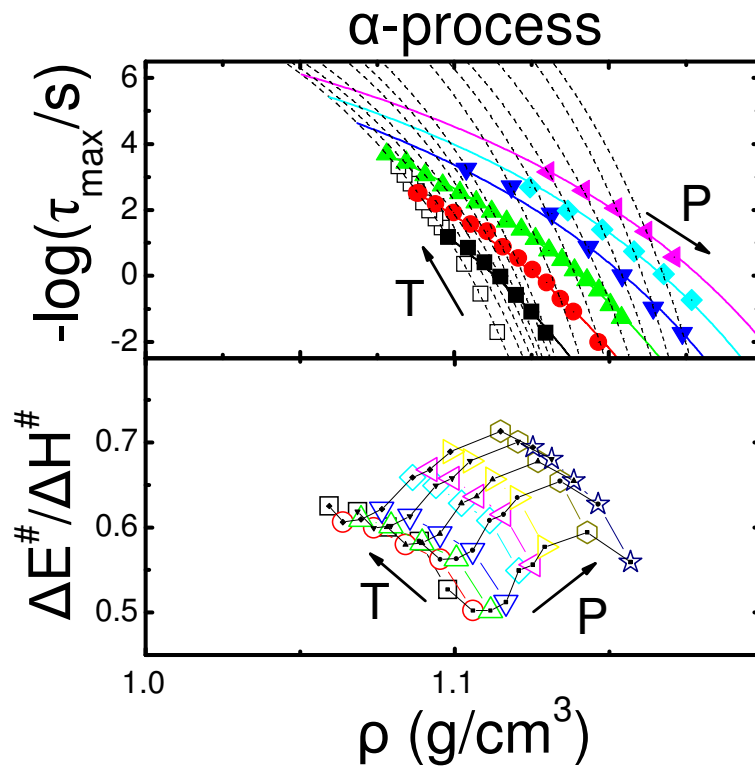


Figure S3a. (Top): Dependence of the “isothermal” (solid lines) and “isobaric” (dashed lines) relaxation times of PEMA ($M_w=2.0 \times 10^3$ g/mol) on density. The “isothermal” and “isobaric” lines, are the result of the fits to the modified VFT equation (Eq. 28) to the α -process. The different “isotherms” are: (squares): 343.15, (circles): 353.15, (up triangles): 363.15, (down triangles): 373.15, (rhombus): 383.15 and (left triangles): 393.15 K. In all cases the “isobaric” data at $P=0.1$ MPa are shown with open squares. **(Bottom):** Ratio \mathfrak{R} , of the constant-volume activation energy ($\Delta E^{\#}$) to the enthalpy of activation ($\Delta H^{\#}$) for the α -process plotted against density. The “isobars” are now shown with open symbols: (squares): 0.1, (circles): 10, (up triangles): 20, (down triangles): 30, (rhombus): 40, (left triangles): 50, (right triangles): 60, (polygons): 90 and (stars): 120 MPa.

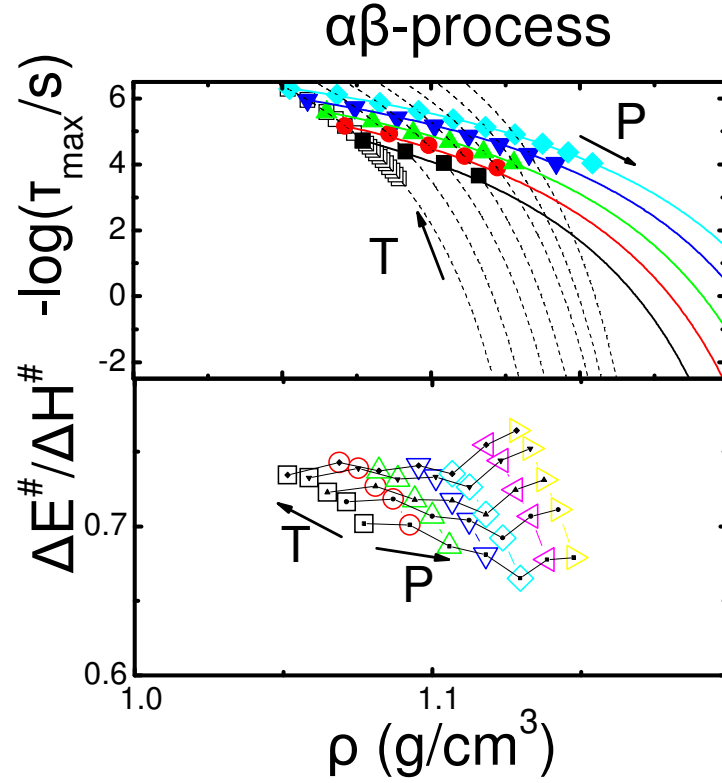


Figure S3b. (Top): Dependence of the “isothermal” (solid lines) and “isobaric” (dashed lines) relaxation times of PEMA ($M_w=2.0 \times 10^3$ g/mol) on density corresponding to the ($\alpha\beta$)-process. The “isothermal” and “isobaric” lines, are the result of the fits to the modified VFT equation (Eq. 28). The different “isotherms” are: (squares): 383.15, (circles): 393.15, (up triangles): 403.15, (down triangles): 413.15 and (rhombus): 423.15 K. In all cases the “isobaric” data at $P=0.1$ MPa are shown with open squares. **(Bottom):** Ratio \mathfrak{R} , of the constant-volume activation energy ($\Delta E^{\#}$) to the enthalpy of activation ($\Delta H^{\#}$) for the ($\alpha\beta$)-process plotted against density. The “isobars” are now shown with open symbols that correspond to: (squares): 0.1, (circles): 30, (up triangles): 60, (down triangles): 90, (rhombus): 120, (left triangles): 150 and (right triangles): 180 MPa.

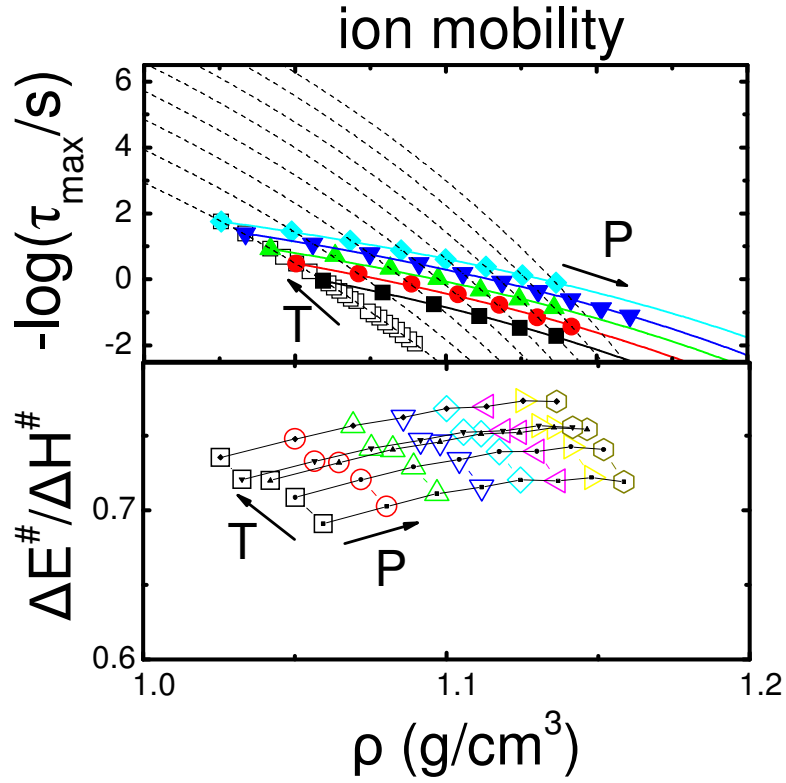


Figure S3c. (Top): Dependence of the “isothermal” (solid lines) and “isobaric” (dashed lines) relaxation times of PEMA ($M_w=2.0 \times 10^3$ g/mol) on density. The “isothermal” and “isobaric” lines, are the result of the fits to the modified VFT equation (Eq. 28) to ion mobility process. The different “isotherms” are: (squares): 383.15, (circles): 393.15, (up triangles): 403.15, (down triangles): 413.15 and (rhombus): 423.15 K. In all cases the “isobaric” data at $P=0.1$ MPa are shown with open squares. **(Bottom):** Ratio \mathfrak{R} , of the constant-volume activation energy ($\Delta E^{\#}$) to the enthalpy of activation ($\Delta H^{\#}$) for the ion mobility process plotted against density. The “isobars” are now shown with open symbols: (squares): 0.1, (circles): 30, (up triangles): 60, (down triangles): 90, (rhombus): 120, (left triangles): 150, (right triangles): 180 and (polygons): 210 MPa.