Supporting Information to

Controlled introduction of diameter modulations in arrayed magnetic iron oxide nanotubes

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Additional figures

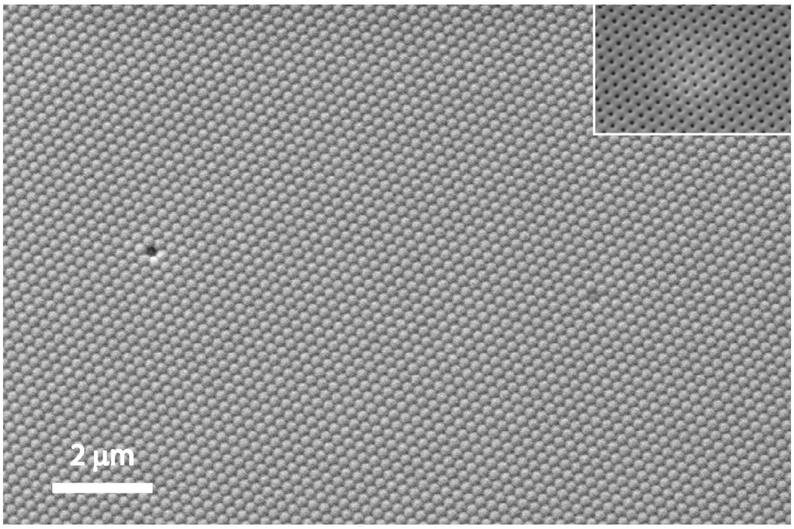

Figure S1. Scanning electron micrographs of a nanoimprinted and anodized membrane of approximately 20 μm thickness. The main panel shows the bottom side of it, to be compared with the top (imprinted) side, in the inset. The prestructuring is maintained throughout the anodization.

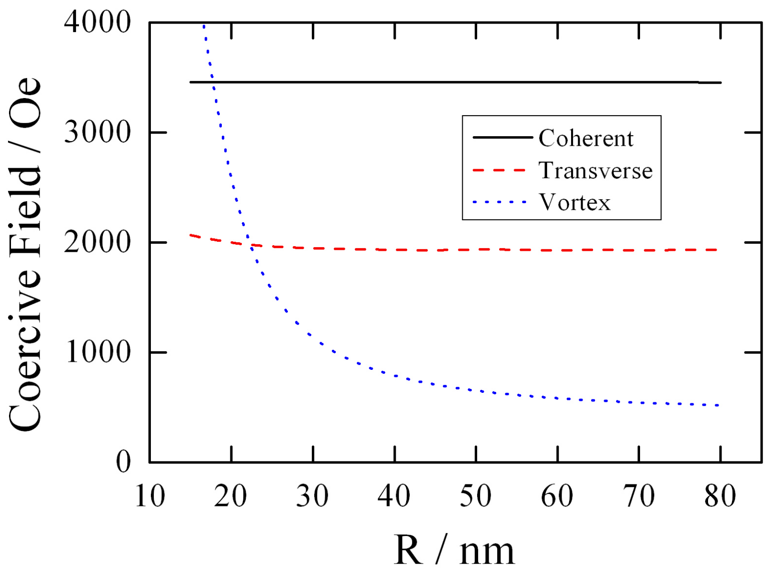

Figure S2. Coercive fields obtained from Eqs. (1), (2) and (3) (for the **C**, **T** and **V** modes, respectively shown in black, red and blue) for tube radii varying between 15 and 80 nm, and a tube wall thickness of d_w = 10 nm.

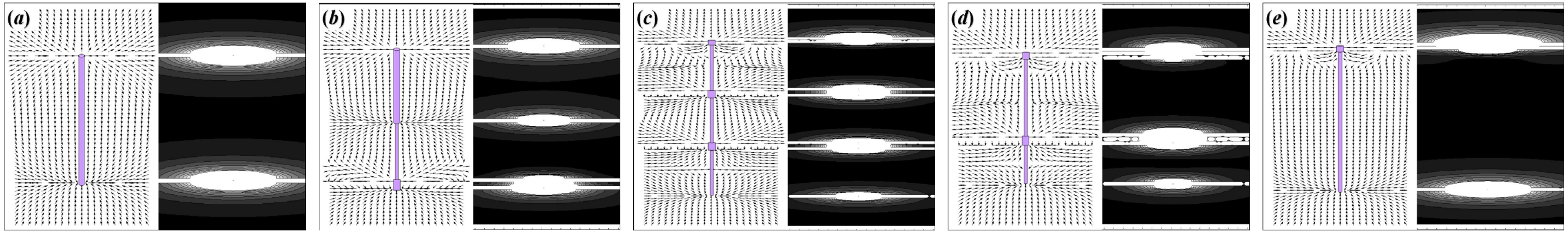

Figure S3. Magnetostatic field profiles and intensities originating from tubes uniformly magnetized in the +*z* direction and with the geometries A-E as defined as defined in Figure 3. The color scale is chosen such that higher absolute values of the field are represented by lighter shades.

Theoretical treatment of the magnetostatic energy for various tube geometries

We can quantitatively compare the strength of this dipolar interaction between neighbors for various types of modulations by adding contributions for each pair of segments. The magnetostatic interaction of segment j with the stray field H_i generated by segment i in the neighboring tube of the tubes can be calculated from Eq. (4) mentioned in the main text:[i]

$$E_{\text{int}} = -\mu_0 \int \vec{M}_j(\vec{r}) \cdot \vec{H}_i(\vec{r}) dv,$$

The expression for the magnetostatic field has been previously reported,[ii,iii] and is given by

$$\vec{H}_i(r,z) = H_{ir}(r,z)\hat{r} + H_{iz}(r,z)\hat{z},$$

where

$$H_{ir}(r,z) = \frac{M_0}{2}\int_0^\infty dk\left(R_i J_1(kR_i) - a_i J_1(ka_i)\right) J_1(kr)\left(e^{-k\left|\frac{L_i}{2}-z\right|} - e^{-k\left|-\frac{L_i}{2}-z\right|}\right),$$

and

$$H_{iz}(r,z) = \frac{M_0}{2}\int_0^\infty dk\left(R_i J_1(kR_i) - a_i J_1(ka_i)\right) J_0(kr)\left(\operatorname{sgn}\left(\frac{L_i}{2} - z\right) e^{-k\left|\frac{L_i}{2}-z\right|} - \operatorname{sgn}\left(-\frac{L_i}{2} - z\right) e^{-k\left|-\frac{L_i}{2}-z\right|}\right).$$

This field corresponds to the magnetostatic field due to one single segment of our modulated tube. Thus, if we want to calculate the total magnetostatic field for the whole modulated tube, we have to sum the contribution to the field of each segment. **Figure S3** illustrates the results obtained for our various types

[i] A. Aharoni, *Introduction to the Theory of Ferromagnetism;* Clarendon, 1996.

[ii] Escrig, J; Allende, S; Altbir, D; Bahiana, M. *Appl. Phys. Lett.* **2008**, *93*, 023101.

[iii] Escrig, J; Allende, S; Altbir, D; Bahiana, M; Torrejón, J; Badini, G; Vázquez, M. *J. Appl. Phys.* **2009**, *105*, 023907.

of modulated nanotubes. Now, if we write the separation between the segments i and j in terms of the interaxial distance, d, and the vertical separation, s, as depicted in **Figure S4**, then the magnetostatic interaction between two arbitrary segments is given by

$$E^{ij}{}_{\mathrm{int}}(d,s) = -\pi\mu_0 M_0^2 \int_0^\infty \frac{dk}{k^2} J_0(kd)\left(R_i J_1(kR_i) - a_i J_1(ka_i)\right)\times$$
$$\times\left(R_j J_1(kR_j) - a_j J_1(ka_j)\right) e^{-\frac{k}{2}(L_i+L_j+2s)} \begin{cases} \left(1-e^{kL_i}\right)\left(1-e^{kL_j}\right) & s \geq \dfrac{L_i+L_j}{2} \\ \left(1-e^{kL_i}-e^{kL_j}+e^{2ks}\right) & s \geq \dfrac{L_i+L_j}{2} \end{cases}$$

This equation has been previously obtained for two identical nanotubes interacting.[ii] The overall interaction energy E_{int} between two modulated tubes is then the sum of the $E^{ij}{}_{int}$. The results are presented for geometries A-E in **Table 1** in the main text.

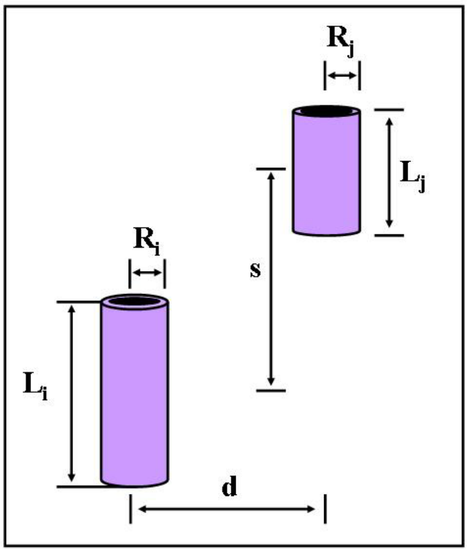

Figure S4. Relative position of interacting segments i and j of our modulated nanotubes: d is the interaxial distance and s is the vertical separation.