## Supporting Information

# New Light-Harvesting Molecular Systems Constructed with a $\mathrm{Ru}(\mathrm{II})$ Complex and a Linear-shaped $\operatorname{Re}(\mathrm{I})$ 

## Oligomer

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Figure S1. ESI-MS spectrum of Ru-Re5 ${ }^{7+}\left(\mathrm{PF}_{6}\right)_{7}$ dissolved in MeCN.


Figure S2. Peaks corresponding to $\left[\mathbf{M}+\left(\mathrm{PF}_{6}{ }^{-}\right)\right]^{6+}$ in the ESI-TOFMS spectrum of $\mathbf{R u} \mathbf{- R e 5}{ }^{7+}\left(\mathrm{PF}_{6}^{-}\right)_{7}(\mathrm{a})$, and the calculated isotope distribution pattern (b).


Figure S3. ESI-MS spectrum of Ru-Re3A ${ }^{5+}\left(\mathrm{PF}_{6}\right)_{5}$ dissolved in MeCN.


Figure S4. UV/vis absorption spectra of Ru-Re3A ${ }^{5+}$ (black line), Ru-Re3B ${ }^{5+}$ (red line) and Ru-Re5 ${ }^{7+}$ (blue line) in MeCN solutions.


Figure S5. Emission spectra from degassed MeCN solutions containing the $\mathrm{Ru}(\mathrm{II})-\mathrm{Re}(\mathrm{I})$ multinuclear complexes by excitation using 350 - and $480-\mathrm{nm}$ light and the corresponding $\operatorname{Re}(\mathrm{I})$ oligomers by excitation using $350-\mathrm{nm}$ light: (a) $\mathbf{R u} \mathbf{R e} \mathbf{~} \mathbf{3} \mathbf{B}^{\mathbf{5 +}}$ and $\mathbf{R e} \mathbf{3}^{\mathbf{3 +}}$; (b) Ru-Re5 ${ }^{7+}$ and $\mathbf{R e 5}{ }^{5+}$. They are standardized by the number of absorbed photons at the excitation wavelength.

## Calculations of the energy transfer rate constants from the terminal $\operatorname{Re}(I)$ unit to the interior

 $\operatorname{Re}(I)$ unit $\left(k_{1}\left(\operatorname{Re} 3^{3+}\right)\right)$ and the radiative and non-radiative rates from each $\operatorname{Re}(I)$ units in $\operatorname{Re} 3^{3+}$.The emission decay of $\mathbf{R e} \mathbf{3}^{3+}$ could be fitted with the following double-exponential functions: with $\tau$ $=15 \mathrm{~ns}$ and 893 ns which can be attributed to the emission from the terminal and the interior $\operatorname{Re}(\mathrm{I})$ units, respectively.

$$
\begin{equation*}
I(\mathrm{t})=\mathrm{A}_{1} \mathrm{e}^{-\mathrm{t} / \tau_{1}}+\mathrm{A}_{2} \mathrm{e}^{-\mathrm{t} / \tau_{2}} \tag{S1}
\end{equation*}
$$

Therefore, the energy transfer rate constant from the excited terminal $\operatorname{Re}(I)$ unit to the interior $\operatorname{Re}(\mathrm{I})$ unit ( $k_{1}\left(\operatorname{Re}^{3+}\right)$ in Scheme 3e) can be calculated using the summed rate constant of the radiative and non-radiative decay from the terminal $\operatorname{Re}(\mathrm{I})$ unit $\left(k_{\mathrm{r}}\left(\operatorname{Re} 3^{3+}\right.\right.$, terminal $)+k_{\mathrm{nr}}\left(\operatorname{Re} 3^{3+}\right.$, terminal $\left.)\right)$ and the following equation:

$$
\begin{equation*}
k_{1}\left(\operatorname{Re} 3^{3+}\right)=\frac{1}{15 \times 10^{-9}}-\left[\left(k_{\mathrm{r}}\left(\operatorname{Re} 3^{3+}, \text { terminal }\right)+k_{\mathrm{nr}}\left(\operatorname{Re} 3^{3+}, \text { terminal }\right)\right]\right. \tag{S2}
\end{equation*}
$$

The emission decay of the dimer $\operatorname{Re}(\mathrm{I})$ complex, $\left[(\mathrm{CO})_{3}(\mathrm{dmb}) \operatorname{Re}-\mathrm{PP}-\operatorname{Re}(\mathrm{dmb})(\mathrm{CO})_{3}\right]^{2+}\left(\boldsymbol{R e}^{2+}\right)$, has a similar structure as the terminal $\operatorname{Re}(\mathrm{I})$ unit in $\mathbf{R e} \mathbf{3}^{3+}$ and could be fitted to a single-exponential with 537 ns. If $\left[k_{\mathrm{r}}\left(\operatorname{Re} 3^{3+}\right.\right.$, terminal $)+k_{\mathrm{nr}}\left(\operatorname{Re} 3^{3+}\right.$, terminal $\left.)\right]$ is the same to that of $\operatorname{Re} 2^{2+}\left(1.9 \times 10^{6} \mathrm{~s}^{-1}\right)$, the $k_{1}\left(\operatorname{Re} 3^{3+}\right)$ can be calculated as $6.5 \times 10^{7} \mathrm{~s}^{-1}$.

## Calculation of the energy transfer efficiency in Ru-Re3A ${ }^{5+}$

The energy transfer efficiencies from the $\operatorname{Re}(\mathrm{I})$ units to the $\mathrm{Ru}(\mathrm{II})$ unit in $\mathbf{R u} \mathbf{- R e} \mathbf{3} \mathbf{A}^{5+}$ were calculated using the following two equations, and all of the estimated energy transfer rate constants are given in the Results and Discussion section.

From the excited terminal $\operatorname{Re}(\mathrm{I})$ unit:

$$
\begin{align*}
& \eta_{\mathrm{ET} 1} \\
& \begin{aligned}
&=\frac{k_{1}\left(\mathrm{Ru}-\operatorname{Re} 3 \mathrm{~A}^{3+}\right)+k_{3}\left(\mathrm{Ru}-\operatorname{Re} 3 \mathrm{~A}^{3+}\right)}{k_{r}\left(\mathrm{Ru}-\operatorname{Re} 3 \mathrm{~A}^{3+}, \text { terminal }\right)+k_{n r}\left(\mathrm{Ru}-\operatorname{Re} 3 \mathrm{~A}^{3+}, \text { terminal }\right)+k_{1}\left(\mathrm{Ru}-\operatorname{Re} 3 \mathrm{~A}^{3+}\right)+k_{3}\left(\mathrm{Ru}-\operatorname{Re} 3 \mathrm{~A}^{3+}\right)} \\
&=\frac{6.5 \times 10^{7}+5.8 \times 10^{7}}{1.1 \times 10^{6}+6.5 \times 10^{7}+5.8 \times 10^{7}} \\
&= 99 \%
\end{aligned} \tag{S3}
\end{align*}
$$

From the interior $\operatorname{Re}(I)$ unit:

$$
\begin{align*}
\eta_{\mathrm{ET} 2}=\frac{k_{2}\left(\mathrm{Ru}-\mathrm{Re} 3 \mathrm{~A}^{3+}\right)}{k_{r}(\mathrm{Ru}}-{\left.\mathrm{Re} 3 \mathrm{~A}^{3+}, \text { interior }\right)+k_{n r}\left(\mathrm{Ru}-\mathrm{Re} 3 \mathrm{~A}^{3+}, \text { interior }\right)+k_{2}\left(\mathrm{Ru}-\mathrm{Re} 3 \mathrm{~A}^{3+}\right)}  \tag{S4}\\
=\frac{6.4 \times 10^{6}}{1.1 \times 10^{6}+6.4 \times 10^{6}} \\
=85 \%
\end{align*}
$$

## Calculation of the emission decay kinetics in Ru-Re5 ${ }^{7+}$

The $\mathbf{R u} \mathbf{u} \mathbf{R e} \mathbf{5}^{7+}$ is constructed with the following four different types of chromophores: the terminal $\operatorname{Re}(\mathrm{I})$ unit ( $\mathbf{R e}-\mathbf{V}$ ), two interior $\operatorname{Re}(\mathrm{I})$ units ( $\mathbf{R e - I I I}$ and $\mathbf{R e - I V}$ ) and the $\mathrm{Ru}(\mathrm{II})$ unit ( $\mathbf{R u}$ ) (Scheme S 1 ).

After UV irradiation, all of the chromophores can absorb photons and become the ${ }^{3}$ MLCT excited state. Each of the excited states is relaxed through the intrinsic decay process (radiative and non-radiative relaxation) and various energy-transfer processes, and these time profiles should follow the set of rate equations (eqs. S5). We define $k_{\text {Ru }}, k_{\mathrm{Re}-\mathrm{V}}, k_{\mathrm{Re}-\mathrm{III}}$, and $k_{\mathrm{Re}-\mathrm{IV}}$ as the intrinsic decay rates of $\mathbf{R u}, \mathbf{R e}-\mathbf{V}$, Re-III and Re-IV excited states, respectively. The energy-transfer rate between the Re-III and Re-IV chromophores is represented by $k_{\mathrm{m}}$. The following relationships are true:

$$
\begin{align*}
& \frac{d \operatorname{Ru}(\mathrm{t})}{d t}=-k_{\mathrm{Ru}} \operatorname{Ru}(\mathrm{t})+k_{2}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right) \operatorname{Re}-\operatorname{III}(\mathrm{t})+k_{3}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right) \operatorname{Re}-\mathrm{IV}(\mathrm{t})+k_{4}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right) \operatorname{Re}-\mathrm{V}(\mathrm{t}) \\
& \frac{d \operatorname{Re}-\mathrm{V}(\mathrm{t})}{d t}=-\left(k_{\mathrm{Re}-\mathrm{V}}+k_{1}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)+k_{4}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)\right) \operatorname{Re}-\mathrm{V}(\mathrm{t})  \tag{S5}\\
& \frac{d \operatorname{Re}-\mathrm{III}(\mathrm{t})}{d t}=-\left(k_{\mathrm{Re}-\mathrm{III}}+k_{m}+k_{2}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)\right) \operatorname{Re}-\operatorname{III}(\mathrm{t})+k_{m} \operatorname{Re}-\mathrm{IV}(\mathrm{t}) \\
& \frac{d \operatorname{Re}-\mathrm{IV}(\mathrm{t})}{d t}=-\left(k_{\mathrm{Re}-\mathrm{V}}+k_{m}+k_{3}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)\right) \operatorname{Re}-\operatorname{IV}(\mathrm{t})+k_{m} \operatorname{Re}-\mathrm{III}(\mathrm{t})+k_{1}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right) \operatorname{Re}-\mathrm{V}(\mathrm{t})
\end{align*}
$$

where $\operatorname{Ru}(\mathrm{t}), \operatorname{Re}-\mathrm{V}(\mathrm{t}), \operatorname{Re}-\mathrm{III}(\mathrm{t})$, and $\mathrm{Re}-\mathrm{IV}$ are the concentration of the ${ }^{3} \mathrm{MLCT}$ excited states of the $\mathbf{R u}$, Re-V, Re-III, and Re-IV chromophores at time t.

The rate equations (eqs. S5) may be rewritten:

$$
\begin{align*}
& \frac{d \operatorname{Ru}(\mathrm{t})}{d t}=-k_{\mathrm{Ru}} \operatorname{Ru}(\mathrm{t})+k_{2}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right) \operatorname{Re}-\mathrm{III}(\mathrm{t})+k_{3}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right) \operatorname{Re}-\mathrm{IV}(\mathrm{t})+k_{4}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right) \operatorname{Re}-\mathrm{V}(\mathrm{t}) \\
& \frac{d \operatorname{Re}-\mathrm{V}(\mathrm{t})}{d t}=-\alpha \operatorname{Re}-\mathrm{V}(\mathrm{t})  \tag{S5'}\\
& \frac{d \operatorname{Re}-\mathrm{III}(\mathrm{t})}{d t}=-(\beta-\delta) \operatorname{Re}-\mathrm{III}(\mathrm{t})+k_{m} \operatorname{Re}-\mathrm{IV}(\mathrm{t}) \\
& \frac{d \operatorname{Re}-\mathrm{IV}(\mathrm{t})}{d t}=-(\beta+\delta) \operatorname{Re}-\mathrm{IV}(\mathrm{t})+k_{m} \operatorname{Re}-\mathrm{III}(\mathrm{t})+k_{1}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right) \operatorname{Re}-\mathrm{V}(\mathrm{t})
\end{align*}
$$

where the parameters $\alpha, \beta, \delta$, and $\mu$ are defined as follows:

$$
\begin{align*}
& \alpha=k_{\mathrm{Re}-\mathrm{V}}+k_{1}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)+k_{4}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right) \\
& \beta=\frac{1}{2}\left\{\left(k_{\mathrm{Re}-\mathrm{II}}+k_{m}+k_{2}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)\right)+\left(k_{\mathrm{Re}-\mathrm{IV}}+k_{m}+k_{3}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)\right)\right\}  \tag{S6}\\
& \delta=\frac{k_{3}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)-k_{2}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)}{2} \\
& \mu=\frac{1}{2} \sqrt{4 k_{m}{ }^{2}+\left(k_{3}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)-k_{2}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)\right)^{2}} \tag{S7}
\end{align*}
$$



Scheme S1. Relaxation processes in Ru-Re5 ${ }^{7+}$.

The decay functions of the excited states at time $t$ are calculated as the solution of eqs. S5 or S5' rate equations.

The initial concentrations of the excited states are assumed to be follows:

$$
\begin{align*}
& \operatorname{Ru}(0)=\operatorname{Ru} \\
& \operatorname{Re}-\mathrm{V}(0)=2\left(\operatorname{Re}-\mathrm{V}_{0}\right)  \tag{S8}\\
& \operatorname{Re}-\operatorname{III}(0)=\operatorname{Re}-\mathrm{III}_{0} \\
& \operatorname{Re}-\mathrm{IV}(0)=2\left(\operatorname{Re}-\mathrm{IV}_{0}\right)
\end{align*}
$$

The set of rate equations (eqs. S5) can be solved analytically. The solutions are the sums of the one to four exponential functions that have the common decay rate constants $\left(\alpha, \beta+\mu, \beta-\mu\right.$, and $\left.k_{\mathrm{Ru}}\right)$ for each chromophores. The solutions of eqs. S5 for the Re-V, Re-III, Re-IV, and Ru, are calculated as follows:
(1) the decay function of $\mathbf{R e}-\mathbf{V}$ is

$$
\begin{equation*}
\operatorname{Re}-\mathrm{V}(\mathrm{t})=2\left(\operatorname{Re}-\mathrm{V}_{0}\right) e^{-\alpha t} \tag{S9}
\end{equation*}
$$

(2) the decay function of Re-III is

$$
\begin{equation*}
\operatorname{Re}-\mathrm{III}(\mathrm{t})=\mathrm{A}_{1 \mathrm{Re}-\text { III }} e^{-\alpha t}+\mathrm{A}_{2 \mathrm{Re}-\mathrm{II}} e^{-(\beta+\mu) \mathrm{t}}+\mathrm{A}_{3 \mathrm{Re}-\text { III }} e^{-(\beta-\mu) \mathrm{t}} \tag{S10}
\end{equation*}
$$

where $\mathrm{A}_{1 \text { Re-III }}, \mathrm{A}_{2 \text { Re-III }}$ and $\mathrm{A}_{3 \text { Re-III }}$ are the pre-exponential factors that were defined as follows:

$$
\begin{align*}
& A_{1 \mathrm{Re}-\mathrm{III}}=\frac{2 k_{m} k_{1}\left(\mathrm{Ru}-\mathrm{Re}^{7+}\right)}{(\alpha-\beta+\mu)(\alpha-\beta-\mu)} T_{0} \\
& A_{2 \mathrm{Re}-\mathrm{III}}=-\frac{k_{m} k_{1}\left(\mathrm{Ru}-\mathrm{Re}^{7+}\right)}{(\alpha-\beta-\mu) \mu} T_{0}-\frac{\delta+2 k_{m}-\mu}{2 \mu} B_{0}  \tag{S11}\\
& A_{3 \mathrm{Re}-\mathrm{III}}=\frac{k_{m} k_{1}\left(\mathrm{Ru}-\mathrm{Re}^{7+}\right)}{(\alpha-\beta+\mu) \mu} T_{0}+\frac{\delta+2 k_{m}+\mu}{2 \mu} B_{0}
\end{align*}
$$

(3) the decay function of Re-IV is

$$
\begin{equation*}
\operatorname{Re}-\mathrm{IV}(\mathrm{t})=A_{1 \mathrm{Re}-\mathrm{IV}} e^{-\alpha t}+A_{2 \mathrm{Re}-\mathrm{IV}} e^{-(\beta+\mu) t}+A_{3 \mathrm{Re}-\mathrm{IV}} e^{-(\beta-\mu) t} \tag{S12}
\end{equation*}
$$

where $\mathrm{A}_{1 \text { Re-IV }}, \mathrm{A}_{2 \text { Re-IV }}$ and $\mathrm{A}_{3 \text { Re-IV }}$ are the pre-exponential factors that were calculated as follows:

$$
\begin{align*}
& A_{1 \mathrm{Re}-\mathrm{IV}}=-\frac{2(\alpha-\beta+\delta) k_{1}\left(\mathrm{Ru}-\mathrm{Re}^{7+}\right)}{(\alpha-\beta+\mu)(\alpha-\beta-\mu)} T_{0} \\
& A_{2 \mathrm{Re}-\mathrm{IV}}=-\frac{(\delta+\mu) k_{1}\left(\mathrm{Ru}-\mathrm{Re}^{7+}\right)}{2(\alpha-\beta-\mu) \mu} T_{0}+\frac{2 \mu+2 \delta-k_{m}}{2 \mu} B_{0}  \tag{S13}\\
& A_{3 \mathrm{Re}-\mathrm{III}}=\frac{(\delta-\mu) k_{1}\left(\mathrm{Ru}-\operatorname{Re} 5^{7+}\right)}{(\alpha-\beta+\mu) \mu} T_{0}+\frac{2 \mu-2 \delta+k_{m}}{2 \mu} B_{0}
\end{align*}
$$

(4) the decay function of $\mathbf{R u}$ is

$$
\begin{equation*}
\mathrm{Ru}(\mathrm{t})=f_{\mathrm{Ru} 1}(t) e^{-k_{\mathrm{Ru}} \mathrm{t}} \tag{S14}
\end{equation*}
$$

where

$$
\begin{equation*}
f_{\mathrm{Ru} 1}(\mathrm{t})=f_{\mathrm{Ru} 2}(\mathrm{t})+\int_{0}^{t} f_{\mathrm{Ru} 3}(s) d s \tag{S15}
\end{equation*}
$$

The first and the second term in eqs. S15 should be substituted with the following:

$$
\begin{align*}
& f_{\mathrm{Ru} 2}(\mathrm{t})=\mathrm{Ru}_{0} \\
& f_{\mathrm{Ru} 3}(s)=\frac{1}{2(\alpha-\beta-\mu)(\alpha+\beta-\mu) \mu^{2}} f_{\mathrm{Ru} 4}(s)  \tag{S16}\\
& f_{\mathrm{Ru} 4}(s)=A_{1 \mathrm{Ru}} e^{-\left(\alpha-k_{r}\right) s}+A_{2 \mathrm{Ru}} e^{-\left(\beta+\mu-k_{r}\right) t}+A_{3 \mathrm{Ru}} e^{-\left(\beta-\mu-k_{r}\right) t}
\end{align*}
$$

where $\mathrm{A}_{1 \mathrm{Ru}}, \mathrm{A}_{2 \mathrm{Ru}}$ and $\mathrm{A}_{3 \mathrm{Ru}}$ are the pre-exponential factors that were calculated as follows:

$$
\begin{align*}
& A_{1 \mathrm{Ru}}=2\left[k_{4}\left(\mathrm{Ru}-\mathrm{Re}^{7+}\right)-\frac{k_{1}\left(\mathrm{Ru}-\mathrm{Re}^{7+}\right)\left\{k_{2}\left(\mathrm{Ru}-\mathrm{Re}^{7+}\right) k_{m}-(\alpha-\beta+\delta) k_{3}\left(\mathrm{Ru}-\mathrm{Re}^{7+}\right)\right\}}{(\alpha-\beta+\mu)(\alpha-\beta-\mu)}\right] T_{0} \\
& A_{2 \mathrm{Ru}}=\frac{k_{1}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)\left\{(\delta+\mu) k_{3}\left(\mathrm{Ru}-\operatorname{Re} 5^{7+}\right)-k_{2}\left(\mathrm{Ru}-\mathrm{Re}^{7^{7+}}\right) k_{m}\right.}{(\alpha-\beta-\mu) \mu} T_{0} \\
& -\frac{2 k_{2}\left(\mathrm{Ru}-\mathrm{Re}^{7+}\right)\left(2 k_{m}-\mu+\delta\right)+k_{3}\left(\mathrm{Ru}-\mathrm{Re} 5^{7+}\right)\left(k_{m}-2 \mu-2 \delta\right)}{2 \mu} B_{0}  \tag{S17}\\
& A_{3 \mathrm{Ru}}=\frac{k_{1}\left(\mathrm{Ru}-\mathrm{Re}^{5^{7+}}\right)\left\{(\mu-\delta) k_{3}\left(\mathrm{Ru}-\mathrm{Re}^{7+}\right)+k_{2}\left(\mathrm{Ru}-\mathrm{Re}^{5^{7+}}\right) k_{m}\right.}{(\alpha-\beta+\mu) \mu} T_{0} \\
& +\frac{k_{2}\left(\mathrm{Ru}-\mathrm{Re}^{7+}\right)\left(2 k_{m}+\mu+\delta\right)+k_{3}\left(\mathrm{Ru}-\operatorname{Re}^{7+}\right)\left(k_{m}+2 \mu+2 \delta\right)}{2 \mu} B_{0}
\end{align*}
$$

Using the following equation for the exponential functions,

$$
\begin{equation*}
e^{-k t} \int_{0}^{t} e^{-(m-k) t} d t=\frac{1}{m-k}\left(e^{-k t}-e^{-m t}\right) \tag{S18}
\end{equation*}
$$

$\mathrm{Ru}(\mathrm{t})$ is represented by the sum of the four exponential functions:

$$
\begin{equation*}
\mathrm{Ru}(\mathrm{t})=C_{1 \mathrm{Ru}} e^{-k r t}+C_{2 \mathrm{Ru}} e^{-\alpha t}+C_{3 \mathrm{Ru}} e^{-(\beta+\mu) t}+C_{4 \mathrm{Ru}} e^{-(\beta-\mu) t} \tag{S19}
\end{equation*}
$$

where $\mathrm{C}_{1 \mathrm{Ru}}, \mathrm{C}_{2 \mathrm{Ru}}, \mathrm{C}_{3 \mathrm{Ru}}$, and $\mathrm{C}_{4 \mathrm{Ru}}$ are the pre-exponential factors that were calculated as follws:

$$
\begin{align*}
& C_{1 \mathrm{Ru}}=\mathrm{Ru}_{0}+\frac{A_{1 \mathrm{Ru}}}{\alpha-k_{\mathrm{Ru}}}+\frac{A_{2 \mathrm{Ru}}}{\beta+\mu-k_{\mathrm{Ru}}}+\frac{A_{3 \mathrm{Ru}}}{\beta-\mu-k_{\mathrm{Ru}}} \\
& C_{2 \mathrm{Ru}}=-\frac{A_{1 \mathrm{Ru}}}{\alpha-k_{\mathrm{Ru}}}  \tag{S20}\\
& C_{3 \mathrm{Ru}}=-\frac{A_{2 \mathrm{Ru}}}{\beta+\mu-k_{\mathrm{Ru}}} \\
& C_{4 \mathrm{Ru}}=-\frac{A_{3 \mathrm{Ru}}}{\beta-\mu-k_{\mathrm{Ru}}}
\end{align*}
$$

The observed emission intensity in the emission decay measurement should be proportional to the linear combination of the concentrations of the excited states $\operatorname{Re}-\mathrm{V}(\mathrm{t}), \operatorname{Re}-\mathrm{III}(\mathrm{t}), \operatorname{Re}-\mathrm{IV}(\mathrm{t})$ and $\mathrm{Ru}(\mathrm{t})$. Therefore the emission decay function should follow the sum of no more than four exponential functions, as follows:

$$
\begin{equation*}
I_{e m}(t)=A_{1} e^{-\alpha t}+A_{2} e^{-(\beta+\mu) t}+A_{3} e^{-(\beta-\mu) t}+A_{4} e^{-k_{r} t} \tag{S21}
\end{equation*}
$$

where

$$
\begin{align*}
& k_{o b s 1}=\alpha=k_{\mathrm{Re}-\mathrm{V}}+k_{1}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)+k_{4}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right) \\
& k_{o b s 2}=\beta+\mu=\frac{1}{2}\left(2 k_{\mathrm{Re}-\mathrm{III}}+2 k_{m}+k_{2}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)+k_{3}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)+\sqrt{4 k_{m}^{2}+\left(k_{2}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)-k_{3}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)\right)^{2}}\right.  \tag{S22}\\
& k_{o b s 3}=\beta-\mu=\frac{1}{2}\left(2 k_{\mathrm{Re}-\mathrm{IV}}+2 k_{m}+k_{2}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)+k_{3}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)-\sqrt{4{k_{m}{ }^{2}+\left(k_{2}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)-k_{3}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)\right)^{2}}^{k_{o b s 4}=k_{\mathrm{Ru}}}} .\right.
\end{align*}
$$

and $A_{1}, A_{2}, A_{3}$, and $A_{4}$ are the parameters that represent the contributions of each of the exponential components.

The energy-transfer rate constants are derived as a solution of the set of eqs $S 22$, as follows:

$$
\begin{align*}
& k_{1}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)=k_{o b s 1}-k_{\mathrm{Re}-\mathrm{V}}-k_{\mathrm{Ru}} \\
& k_{2}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)=\frac{1}{2}\left[\left(k_{o b s 2}+k_{o b s 3}\right)-2\left(k_{\mathrm{Re}-\mathrm{III}}+k_{m}\right)+\sqrt{\left(k_{o b s 2}-k_{o b s 3}\right)^{2}-4{k_{m}^{2}}^{2}}\right]  \tag{S23}\\
& k_{3}\left(\mathrm{Ru}-\operatorname{Re} 5^{7+}\right)=\frac{1}{2}\left[\left(k_{o b s 2}+k_{o b s 3}\right)-2\left(k_{\mathrm{Re}-\mathrm{IV}}+k_{m}\right)-\sqrt{\left(k_{o b s 2}-k_{o b s 3}\right)^{2}-4{k_{m}^{2}}^{2}}\right] \\
& k_{4}\left(\mathrm{Ru}-\operatorname{Re} 5^{7+}\right)=k_{\mathrm{Ru}}
\end{align*}
$$

Because all of the observed decay rate constants, $k_{\mathrm{obs} 1}, k_{\mathrm{obs} 2}, k_{\mathrm{obs} 3}$ and $k_{\mathrm{obs} 4}$ should be positive real numbers, the following limiting condition exists for the rate constants, $k_{2}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right), k_{3}\left(\operatorname{Ru}-\operatorname{Re} 5^{7+}\right)$ and $k_{\mathrm{m}}$ :
$\left(k_{o b s 2}-k_{o b s 3}\right)^{2}-4 k_{m}^{2} \geq 0$
Therefore, the energy-migration rate between the $\mathbf{R e - I I I}$ and $\mathbf{R e - I V}$ chromophores, $k_{\mathrm{m}}$, has the following upper limit:

$$
\begin{align*}
& k_{m} \leq \frac{k_{o b s 2}-k_{o b s 3}}{2} \\
& =\frac{1.4 \times 10^{7} s^{-1}-6.7 \times 10^{6} s^{-1}}{2}=3.6 \times 10^{6} \mathrm{~s}^{-1} \tag{S25}
\end{align*}
$$

