

# Quantum Capacitance Limited Vertical Scaling of Graphene Field Effect Transistor

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## Part I: Procedure for retrieving $\kappa$ and quantum capacitance

Denoting the total gate capacitances of two graphene devices respectively with  $C_{TG}$  and  $C_{TG}'$ , their oxide capacitances with  $C_{OX}$  and  $C_{OX}'$  (given by  $C_{OX} = \kappa \epsilon_0 / t_{ox}$ , with  $t_{ox}$  being the thickness of the  $Y_2O_3$  film), we have the following set of two equations for the two unknown  $\kappa$  and  $C_Q$

$$\begin{cases} C_{TG}^{-1}(V_{TG}) = C_Q^{-1}(V_{ch}) + \frac{t_{ox}}{\epsilon_0 \kappa} \\ C_{TG}'^{-1}(V_{TG}') = C_Q^{-1}(V_{ch}) + \frac{t_{ox}'}{\epsilon_0 \kappa} \end{cases} \quad (S1)$$

where  $\epsilon_0$  is the vacuum dielectric constant and  $\kappa$  is assumed to be independent of  $V_{TG}$ . The two unknowns can therefore be solved analytically to yield

$$\begin{cases} \kappa = \frac{t_{ox}' - t_{ox}}{\epsilon_0 [C_{TG}'^{-1}(V_{TG}') - C_{TG}^{-1}(V_{TG})]} \\ C_Q(V_{ch}) = \frac{t_{ox}' - t_{ox}}{t_{ox}' C_{TG}^{-1}(V_{TG}) - t_{ox} C_{TG}'^{-1}(V_{TG}')} \end{cases} \quad (S2)$$

Now the unknown  $\kappa$  and  $C_Q(V_{ch})$  are expressed in terms of the direct measurable parameters, *i.e.*  $t_{ox}$  from AFM measurement, and  $C_{TG}(V_{TG})$  from CV measurement. The point to note here is that the same  $V_{ch}$  corresponds to the same charge  $Q$  in the graphene channel which can be obtained from experimental  $C_{TG}(V_{TG})$  curves by integrating it. At any given  $V_{ch}$ ,  $C_{TG}(V_{TG})$

and  $C_{TG}'(V_{TG}')$  can be determined as follows:

1. Obtaining charge  $Q$  as a function of the gate voltage for the two devices by integrating the  $C_{TG}$ - $V_{TG}$  curves as shown in Figure 2d.
2. Obtaining  $V_{TG}$  and  $V_{TG}'$  for the same  $Q$  value as shown in Figure 2d.
3. Getting the corresponding  $C_{TG}$  and  $C_{TG}'$  values from the measured  $C_{TG}(V_{TG})$  and  $C_{TG}'(V_{TG}')$  curves as in Figure 2b, these obtained values of  $C_{TG}$  and  $C_{TG}'$  correspond to the same  $V_{ch}$  and charge  $Q$  and can be used to obtain  $V_{ch}$  independent  $\kappa$  using equation (S2).

It is worth noting that the quantum capacitance near the Dirac point varies from sample to sample owing to the fluctuations in the local density of state residue induced by charged impurities on graphene or in surroundings. Thus for real devices, equation (S2) will become invalid near the Dirac point. But we expect that equation (S2) gives a good description of graphene devices at relatively large gate voltage, where the gate effect largely dominates over the effect of residue charges.

## Part II: Derivation of equation (5)

Now we will prove that  $V_{TG, Dirac}$  linearly depend on  $V_{BG}$  following equation (5). Starting from the capacitive equivalent circuit (Figure 3b), [35] we can obtain the total charge  $Q_{ch}$  induced in the graphene channel by the top and bottom gates

$$Q_{ch} = -C_{OX}(V_{TG} - V_{ch}) - C_{BG}(V_{BG} - V_{ch}) \quad . \quad (S3)$$

On the other hand, because of the charge neutrality condition, the same amount of charge (but with opposite sign) is also induced *via* the quantum capacitance

$$Q_{ch} = -C_Q V_{ch} \quad . \quad (S4)$$

Due to the finite impurities in graphene channel, the Dirac point voltages for both the top and back gates are generally not equal to zero. In a general case we can define effective gate voltages  $V_{TG}^*$  and  $V_{BG}^*$  to replace  $V_{TG}$  and  $V_{BG}$  in equation (S3)

$$V_{TG}^* = V_{TG} - V_{TG, Dirac}^0 \quad (S5)$$

$$V_{BG}^* = V_{BG} - V_{BG,Dirac}^0, \quad (S6)$$

where  $V_{TG,Dirac}^0$  (or  $V_{BG,Dirac}^0$ ) is the top (or back) gate voltage at the Dirac point under a certain fixed back (or top) gate voltage. At the Dirac point of the top gate,  $V_{ch}$  is equal to zero, and then  $Q_{ch}$  is also zero. Substituting equation (S5)–(S6) into (S3), we then obtain the  $V_{BG}$  dependent Dirac point voltage

$$V_{TG,Dirac}(V_{BG}) = V_{TG,Dirac}^0 - \frac{C_{BG}}{C_{OX}}(V_{BG} - V_{BG,Dirac}^0), \quad (S7)$$

which is often rewritten as

$$C_{OX} / C_{BG} = -\Delta V_{BG} / \Delta V_{TG,Dirac}. \quad (S8)$$

### Part III: Extracting mobility of Graphene FET

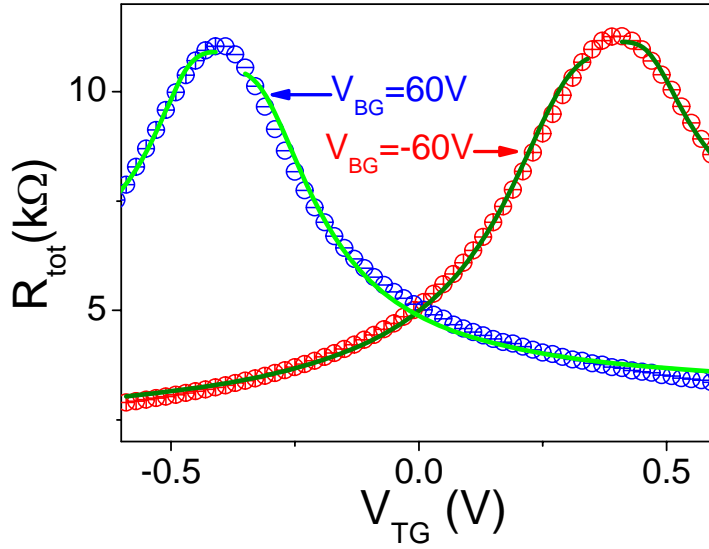


Figure S1: Total channel resistance  $R_{tot}$  vs  $V_{TG}$  for device 1 under various  $V_{BG}$ , the data were originated from Figure 3 (a). Circles represent experimental data, and lines indicate the fitting result.

The electron mobility  $\mu_e$  and hole mobility  $\mu_h$  of a graphene device were extracted according to a previously reported model,<sup>11</sup>

$$R_{total} = R_c + \frac{L}{W} \frac{1}{e\mu\sqrt{n_0^2 + n^2}} \quad (S9)$$

where,  $R_{total} = I_{ds} / V_{ds}$  is the total resistance of the graphene FET including the contact resistance  $R_c$  and the channel resistance,  $L$  is the length of the top gate and  $W$  is the width of the graphene channel covered by top gate,  $n$  and  $n_0$  are the top gate modulated carrier density and the residual density respectively. The Fermi velocity was chosen as  $1.15 \times 10^6$  m/s. The fitting results were shown in Figure S1 with the residual concentration  $n_0 = 4.5 \times 10^{11} / \text{cm}^2$ , and the mobility for electrons and holes were extracted to be about  $1900 \text{ cm}^2/\text{Vs}$  and  $1800 \text{ cm}^2/\text{Vs}$  respectively.