Supporting Information

Wetting States on Circular Micropillars with Convex Sidewalls after Liquids Contact Groove Base

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Equation (4) can be further re-written as

$$\frac{d[(w-x)\sin\theta]}{dx} = 2b(w-x), \tag{s1}$$

where $\frac{dx}{ds} = \cos\theta$ was used in deriving this equation from Eq. (4). With the aid of Eq. 2(b), it follows from Eq. (s1) that

$$\sin\theta = \frac{c}{(w-x)} - b(w-x), \tag{s2}$$

where c is a constant and has the following expression:

$$c = r\sin\theta_2 + br^2. \tag{s3}$$

Given that $\theta = \theta_1$ at a_1, it follows from Eqs. (s2) and (s3) that

$$b = \frac{r\sin\theta_2 - w\sin\theta_1}{w^2 - r^2}. \tag{s4}$$

Noting that

$$\frac{dy}{dx} = \tan\theta, \tag{s5}$$

with the assistance of Eq. (s2), we have

$$\frac{dy}{dx}=\frac{c-b(w-x)^2}{\sqrt{(1+\frac{bc}{2})(w-x)^2-[b^2(w-x)^4+c^2]}}. \tag{s6}$$

Let x_p and y_p represent x and y coordinates of a representative point p on a_1b_1, where x_p ranges from 0 to (w- r). In view of Eq. (s6), y_p is given below:

$$y_p(x_p)=\int_0^{x_p}\frac{c-b(w-x)^2}{\sqrt{(1+\frac{bc}{2})(w-x)^2-[b^2(w-x)^4+c^2]}}dx. \tag{s7}$$

This equation gives a solution to Eqs. (4) and (2). When $x_p=w-r$, we have

$$y_{b_1}=\int_0^{w-r}\frac{c-b(w-x)^2}{\sqrt{(1+\frac{bc}{2})(w-x)^2-[b^2(w-x)^4+c^2]}}dx, \tag{s8}$$

where (w-r) and y_{b_1}, respectively, represent x and y coordinates of b_1. In the case of circular micropillars with concave sidewalls, it is observed from Eq. (2b) that θ_2 is a function of φ. For a given sidewall profile, φ can also be determined from r. In this sense, φ is also a function of r. Therefore, by Eq. (s4), b is actually a function of w and r for given microstructures. Since Eq. (s8) gives another relation that b, w and r have to meet, b and w both can be considered to be functions of r only. Thus, once θ_{01}, θ_{02}, r, and the equation of the sidewall profile are given, the solution to Eqs. (s4) and (s8) give unique values to b and w. Subsequently, a unique value of y_p can be obtained from Eq. (s7). Consequently, Eq. (s7) is also a unique solution to Eqs. (4) and (2). On the other hand, since the right-hand side of Eq. (s7) is an elliptical integral, we cannot get a straightforward expression of y_p. However, once θ_{01}, θ_{02}, r, and the equation of the sidewall profile are given, a numerical value can be found for y_p by numerically integrating this elliptical integral.