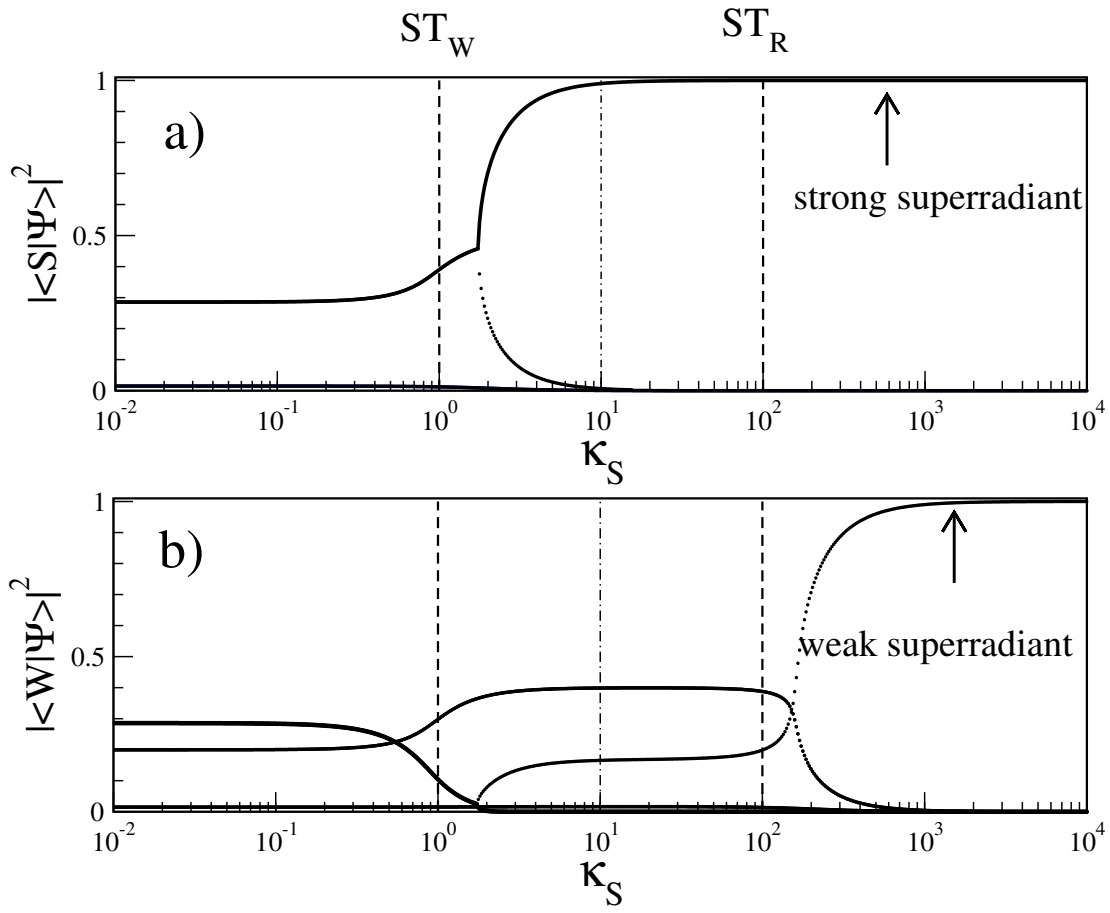

Figure 6: a) Probability to be at the strongly coupled end $|\langle S|\Psi\rangle|^2$, for all energy eigenstates $|\Psi\rangle$. b) Probability to be at the weakly coupled end $|\langle W|\Psi\rangle|^2$, for all energy eigenstates. Here $q = 100$.

Supporting Information Available

Participation Ratio

In Fig. 6a, we show the probabilities to occupy the end sites ($|S\rangle, |W\rangle$) for all energy eigenstates. As one can see, on approaching the strong transition, ST_S, the superradiant state, $|SR\rangle$, increases its probability to occupy the strongly coupled end, and this probability becomes one at the switching point. Of course, at the same point the probability to be at the strongly coupled end of the other states becomes zero, so that transport to the strong sink (S) is completely inhibited, and it switches to the weak sink (W).

One can also observe that after the ST_W, the same effect occurs at very large coupling

thus inhibiting also the transport to the weakly coupled end. This is at variance with the classical behavior, where for very large coupling one gets, $\eta_S = \eta_W = 1/2$.

In order to study the localization of the eigenstates, $|\Psi\rangle$, we consider their participation ratio in the site basis $|n\rangle$,

$$PR = \frac{1}{\sum_n |\langle n|\Psi\rangle|^4}.$$

In Fig. 7a we show the PR of all energy eigenstates. As one can see, for $\kappa_S < 1$ most of the states are approximately delocalized over the whole system (the maximal PR corresponds to the total number of sites, 6). For $1 < \kappa_S < q$ both superradiant and subradiant states tend to localize. However, while the PR of the strong superradiant state SR_S becomes ≈ 1 at the switching point $\kappa_S \sim \sqrt{q}$ (complete localization), the PR of the other states does not decrease below ≈ 3, which means that they are approximately extended over the system, thus allowing transport.

Even if the strong superradiant state becomes extremely localized immediately after the ST_S, its width is very large. (See Fig. 7b.) This competition between localization and decay width determines the switching point of transport.

The critical switching point

Let us first define the strong/weak partial width as follow,

$$\Gamma_{S,W} = \gamma_{S,W} \sum_k \langle k|\ W_{S,W}\ |k\rangle, \tag{12}$$

where the sum is taken over the subradiant states, $|k\rangle$. Let us also consider the range of parameters between the two STs, namely $1 < \kappa_S < q$, and the effective Hamiltonian,

$$\mathcal{H} = -i\gamma_s W_S + (H_0 - i\gamma_w W_W)\,, \tag{13}$$

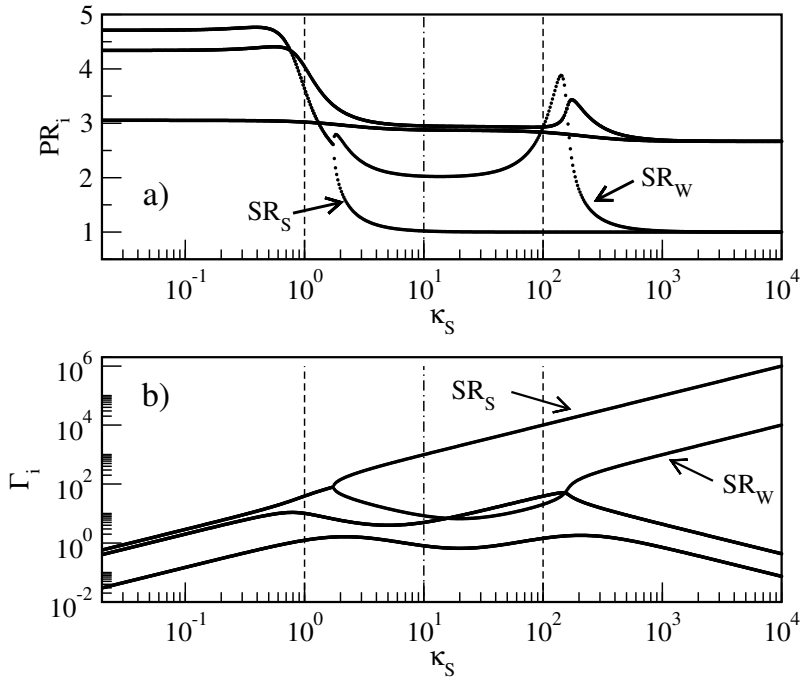

Figure 7: a) Participation ratio versus κ_S for all states. b): Decay width versus κ_S for all states. Here $q = 100$.

where we consider $-i\gamma_{_S} W_W$ as the unperturbed Hamiltonian, and $H_0 - i\gamma_{_W} W_W$, as the perturbation.

Let us consider, for definiteness, the site W as the first site $j = 1$ and the strongly coupled S as the last $j = N$. Eigenvalues for W_S are 1 and 0, the latter $N - 1$ times degenerate; the eigenvector correspondent to 1 is $|S\rangle$, since,

$$W_S|S\rangle = |S\rangle,$$

while, as a degenerate basis we can choose the site basis $|j\rangle$, for $j = 1, ..., N - 1$, since $W_S|j\rangle = 0$.

Eigenvalues and eigenvectors of the $N - 1$ degenerate system can be obtained at zero order by solving the eigenvalues problem,

$$\langle k|H_0 - i\gamma_{_W} W_W|j\rangle = \epsilon_k \delta_{jk}. \tag{14}$$

Since H_0 is of the order of Ω, and $\gamma_{_W}/\Omega \ll 1$, we can use first order perturbation theory in $\gamma_{_W}/\Omega$ to get the eigenvalues,

$$\epsilon_q^I = \epsilon_q^0 - i\gamma_{_W}\langle q|W_W|q\rangle, \tag{15}$$

and the eigenvectors,

$$|q\rangle^I = |q\rangle - \frac{i\gamma_{_W}}{2\Omega N}\sum_{q'\neq q}\frac{\sin(q\pi/N)\sin(q'\pi/N)}{\sin[(q+q')\pi/N]\sin[(q-q')\pi/N]}|q'\rangle \equiv |q\rangle - \frac{i\gamma_{_W}}{2\Omega N}\sum_{q'\neq q}c_{q,q'}|q'\rangle, \tag{16}$$

where we have chosen the eigenbasis by the restriction of H_0 to the $N-1$ dimensional space,

$$\langle k|q\rangle = \sqrt{\frac{2}{N}}\sin\left(\frac{\pi kq}{N}\right),$$

with eigenvalues $\epsilon_q^0 = -2\Omega\cos(\pi q/N)$. The same eigenvectors can be considered (with the same name) in the N-th dimensional space, by simply adding a 0 in the N-th component, so to be orthogonal to $|S\rangle$.

To have the eigenstates at first order of the perturbation theory in $\Omega/\gamma_{_S}$ and $\gamma_{_W}/\gamma_{_S}$, one should take into account the interaction between $|q\rangle$ and $|S\rangle$, mediated by the perturbation $H_0 - i\gamma_{_W}W_W$, so that,

$$|q\rangle^I = |q\rangle - \frac{i\gamma_{_W}}{2\Omega N}\sum_{q'\neq q}c_{q,q'}|q'\rangle + \frac{\langle q|H_0 - i\gamma_{_W}W_W|S\rangle}{-i\gamma_{_S} - \epsilon_q^0}. \tag{17}$$

From this one gets,

$$\begin{aligned}\langle W|q\rangle^I &= \langle W|q\rangle + O(\gamma_{_W}/\Omega),\\ \langle S|q\rangle^I &= i\sqrt{\tfrac{2}{N}}\tfrac{\Omega}{\gamma_{_S}}\sin\left(\tfrac{\pi q}{N}\right) + O(\Omega/\gamma_{_S}).\end{aligned} \tag{18}$$

From Eq. (18) the partial widths easily follows,

$$\begin{aligned} \Gamma_W &= \tfrac{2\gamma_W}{N} \textstyle\sum_q \sin^2(\pi q/N), \\ \Gamma_S &= \tfrac{2\Omega^2}{N\gamma_S} \textstyle\sum_q \sin^2(\pi q/N). \end{aligned} \tag{19}$$

Equating Eq. (18) and Eq. (19), one gets,

$$\gamma_S \gamma_W \simeq \Omega^2,$$

or

$$\kappa_S = q/\kappa_S.$$

Lindblad master Equation

The dynamics of the system to second order in the system-bath coupling can be described by the Lindblad master equation in the Born-Markov and secular approximations as

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}\left(\mathcal{H}\rho - \rho\mathcal{H}^\dagger\right) + L_p(\rho), \tag{20}$$

where the superoperator, L_p, acts on ρ as follow,

$$\begin{aligned} L_p(\rho) &= \textstyle\sum_{\omega,m} \gamma(\omega)[A_m(\omega)\rho A_m^\dagger(\omega) - \tfrac{1}{2}A_m^\dagger(\omega)A_m(\omega)\rho \\ &\quad -\tfrac{1}{2}\rho A_m^\dagger(\omega)A_m(\omega)]. \end{aligned} \tag{21}$$

The Lindblad generators, $A_m(\omega)$, are given by,

$$A_m(\omega) = \sum_{E-E'=\hbar\omega} c_m^*(E)c_m(E')|E\rangle\langle E'|, \tag{22}$$

where the summation is over all transitions with frequency, $\omega = (E - E')/\hbar$, and $|E\rangle$ is the eigenstate of the Hamiltonian, H_0, of the closed system with eigenvalue, E. The coefficients, $c_m(E)$, are the expansion coefficients of the energy eigenstate in the sites basis, $|m\rangle$,

$$|E\rangle = \sum_m c_m(E)|m\rangle.$$

The rates, $\gamma(\omega)$, are given by,

$$\gamma(\omega) = 2\pi[J(\omega)(1 + n_T(\omega)) + J(-\omega)n_T(-\omega)],$$

where $n_T(\omega)$ is the bosonic distribution function at the temperature T,

$$n_T(\omega) = \frac{1}{e^{\hbar\omega/k_BT} - 1},$$

and $J(\omega)$ is the Ohmic spectral density, which we choose of the form,

$$J(\omega) = \begin{cases} 0 & \text{for} \quad \omega < 0 \\ \dfrac{E_R\omega}{\hbar\omega_c} e^{-\omega/\omega_c} & \text{for} \quad \omega > 0. \end{cases} \tag{23}$$

Eq. (23) implicitly defines the reorganization energy, E_R, and the cut-off frequency, ω_c.

Note that this form of master equation does not assume weak coupling to the sinks, only to the phonon bath. We also assumed that a strong coupling to the sinks does not influence the phonon coupling. Further work is in progress to check the validity of this assumption.

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