Supporting information
Uniform Huygens metasurfaces with post-fabrication phase pattern recording functionality

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The Supporting information file includes:

Supplementary Text: 4 pages

Supplementary Figure: Figures S1 to S2

Transmission zeros evolution in the vicinity of EP

Here, we show in more detail the transition from conventional operation regime (i) to Huygens regime (ii) and plot the complex plane calculation for three values of the parameter L highlighting this transition. Transmission amplitude ($Log_{10}(|T|)$) and phase ($Arg(T)$) were computed with JCMwave suite for $L = 370\ nm$, $L = 371.55\ nm$, and $L = 372\ nm$ (Fig. S1). Results in Fig. S1a.and c. correspond to the operation regime (i) and (ii) where two spectrally separated zeros of the first order, characterized by a phase vortex with a winding number 1, are observed[1]). Fig. S1 b. shows that for a certain parameter value ($L \approx 371.55\ nm$) the two zeros degenerate and merge together forming a zero of the second order, i.e. a transmissionless EP characterized by a winding number of 2 with a 4π phase accumulation encircling the singularity.

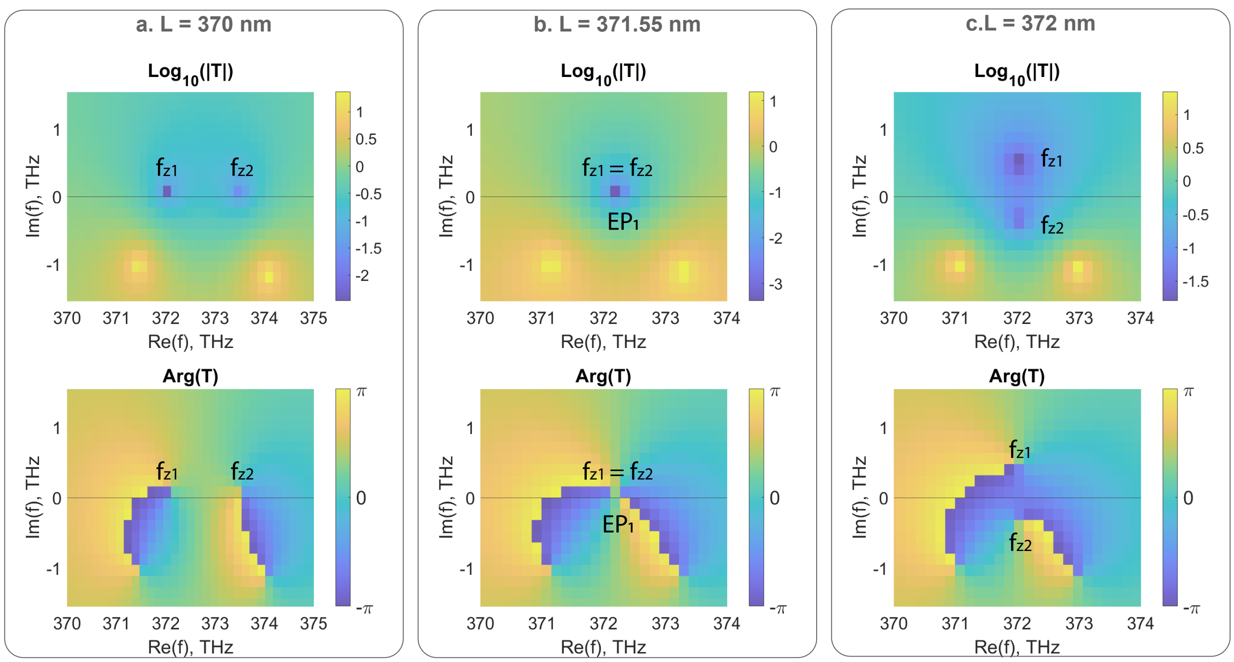

Figure S 1: Transmission Amplitude and phase maps of an As_2S_3 metasurface excited with complex-frequency illumination impinging at normal incidence on metasurfaces designed with 3 specific values of the particles size L: a. For $L = 370\ nm$ the two zeros are spectrally separated and their eigenfrequencies are real-valued. b. For $L = 371.55\ nm$ the two complex zeros spectrally coincide forming transmissionless EP on the real axis. c. For $L = 372\ nm$ zeros have the same real part, but different imaginary parts, overall their complex eigenfrequencies being conjugated.

Fitting metasurface response to the parameter change in the vicinity of EP

The typical property of exceptional points in optical systems is the enhanced system's sensitivity to parameters change. This is because the system's response to this change depends non-linearly (as a square-root function) on the external change. We demonstrate this property for the considered structure by fitting the zeros trajectories in the vicinity of EP by the square root dependency on the size variation: $f = f_{EP_{1,2}} + \alpha_{1,2}\sqrt{L - L_{EP_{1,2}}}$. Fitted constant values are $\alpha_1 = -0.2045$ for the first zero and $\alpha_2 = 0.9896$ for the second zero. The comparison of the fitted curves with the numerically obtained transmission zero eigenfrequencies is shown in Fig. S2.

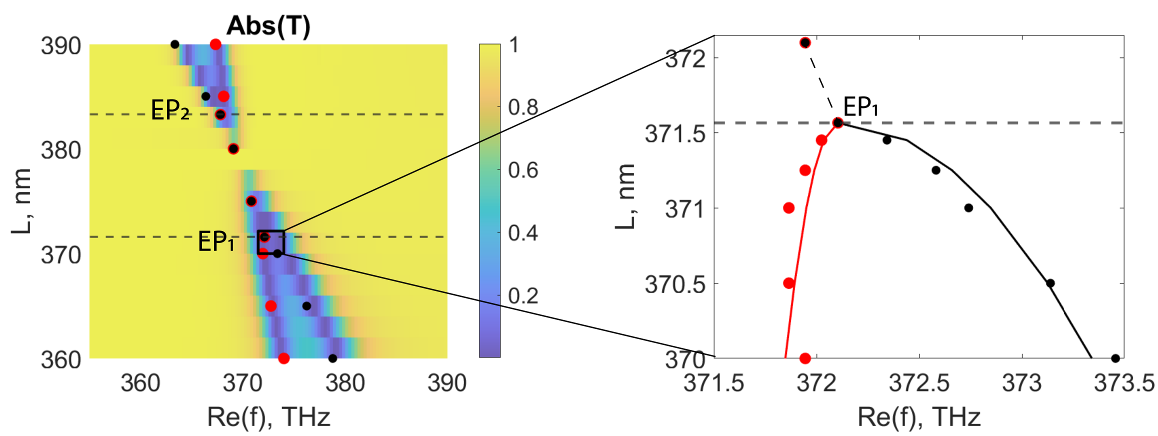

Figure S 2: Metasurfaces's transmission spectrum amplitude with the selected area around the exceptional point is shown with a black rectangle. Zeros evolution in this area is shown in more detail in the figure on the right: numerical fit of zeros real parts trajectories in the vicinity of EP_1 indicates a typical square-root dependence of the system's response as a function of the small external parameter change (in this case, resonator's size). $f = f_{EP_{1,2}} + \alpha_{1,2}\sqrt{L - L_{EP_{1,2}}}$, are $\alpha_1 = -0.2045$ and $\alpha_2 = 0.9896$.

References

(1) Colom, R.; Mikheeva, E.; Achouri, K.; Zuniga-Perez, J.; Bonod, N.; Martin, O. J. F.; Burger, S.; Genevet, P. Crossing of the branch cut: the topological origin of a universal 2π-phase retardation in non-Hermitian metasurfaces. *arXiv:2202.05632 [physics]* **2022**,