

# Supporting Information for: Transport in Asymmetrically Coupled Donor-Based Silicon Triple Quantum Dots

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## S1: Equivalent circuit model

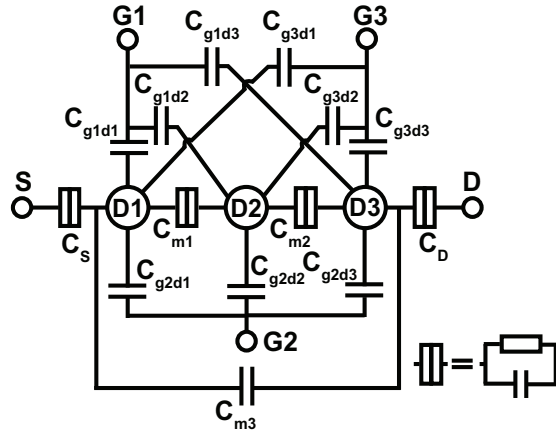


Figure S1: An equivalent circuit model of the triple quantum dot transport device used to model the charge stability diagram and finite bias triangles. The dots are assumed to be serially tunnel coupled. All cross capacitances between the gates and dots are included in the model.

## S2: Extraction of the excited state energy

In Figure S2 we compare the modelled and measured bias triangles over the full range of  $VG2$  at a SD bias of +10 mV. As discussed in the letter, the base of the bias triangles corresponds to either resonances between D2 and D3 (labelled 1 and 2), or, D1 and D2 (labelled 3 and 4). In the data we observe only resonances between D2 and D3 due to asymmetric tunnel coupling between adjacent dots. There is an additional resonance line marked by the red star that corresponds to the first visible excited state resonance between D2 and D3 involving the  $(m+1, n+1, o)$  and  $(m+1, n, o+1)$  charge states. We can assign this because the gate voltage difference,  $\Delta VG3$ , between this resonance and its corresponding ground state resonance (labelled 2) stays fixed as a function of  $VG2$  as shown by arrows in Figure S2.

To calculate the energy of this excited state we require the lever arm,  $\alpha_{g3d2}$ , which converts a change in  $VG3$  to a change in energy of the electrochemical potential of D2. Figure S3 shows the finite bias triangles at  $VG2 = -320$  mV recorded at  $V_{SD} = +10$  mV. The lever arm is extracted from the bias triangle corresponding to QP5 (outlined in red) following a similar method used for double quantum dots (1). At the green circle, the electrochemical potential of D2 and D3 are aligned and are  $\Delta\mu_3$  below the electrochemical potential of the drain. Here,  $\Delta\mu_3 = \mu_3(0, 1, 1) - \mu_3(1, 0, 1)$  is the difference in electrochemical potential of D3 as a result a charge transfer between D1 and D2 and is constant as a function of gate voltage. From the energy level diagram we find,

$$\begin{aligned} \mu_2(1, 1, 0, VG1, VG2, VG3) &= \mu_3(1, 0, 1, VG1, VG2, VG3) \\ &= \mu_3(0, 1, 1, VG1, VG2, VG3) - \Delta\mu_3 \\ &= \mu_D - \Delta\mu_3 = -\Delta\mu_3, \end{aligned} \tag{1}$$

where we choose  $\mu_D = 0$ .

In figure S3, the size of the bias triangle for QP5 is at its maximum and therefore at the orange circle the electrochemical potentials of D1 and D2, and, D2 and D3 are aligned (see manuscript). Furthermore, the electrochemical

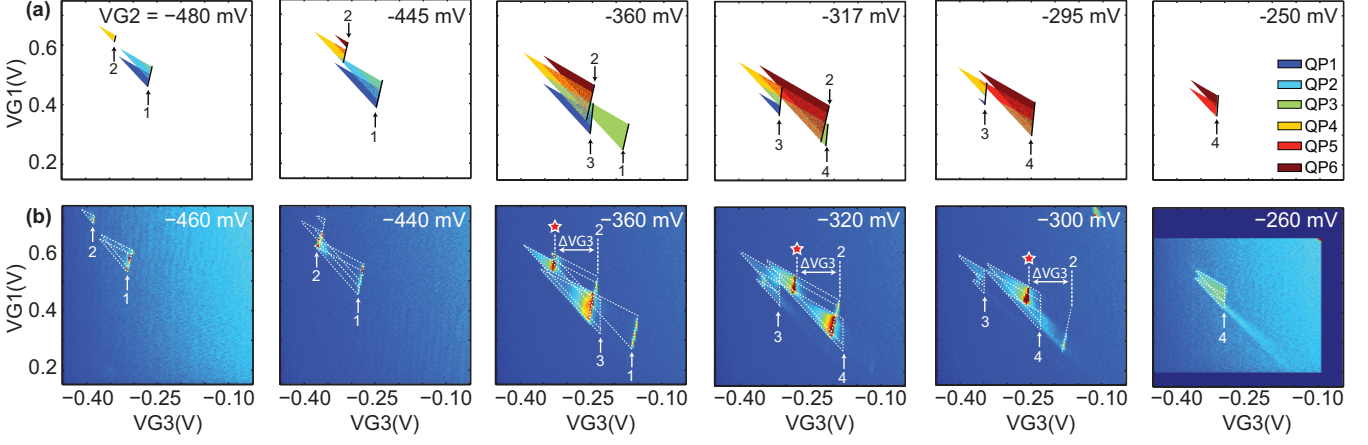


Figure S2: (a) Regions of finite bias transport as a function of  $VG2$  at a SD bias of 10 mV. The different coloured bias triangles correspond to the 6 distinct QPs. (b) Corresponding measurement data recorded at a SD bias of 10 mV. Each plot shows the current as a function of  $VG1$  and  $VG3$ . The red star indicates the first visible excited state.

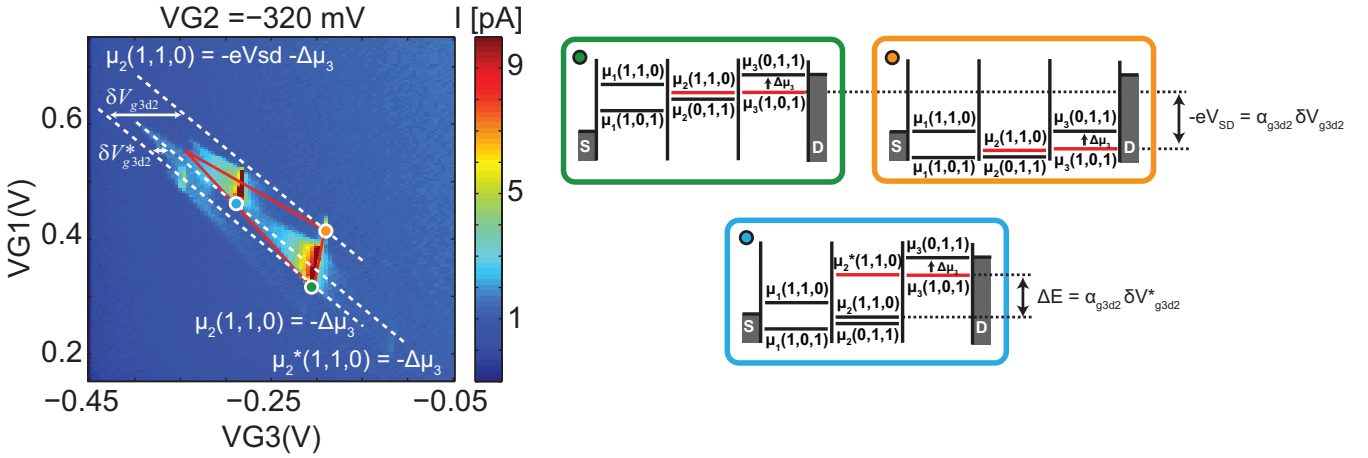


Figure S3: Extraction of the lever arm,  $\alpha_{g3d2}$ , and the energy,  $\Delta E$ , of the first visible excited state. The bias triangle outlined in red corresponds to QP5. The energy level diagrams show the alignment of the electrochemical potentials at the green, orange, and blue circles.

potentials of D2 and D3 are  $\Delta\mu_3$  below that of the source,  $\mu_S$ . Here, the electrochemical potential of D2 differs from that defined by equation 1 (green circle) by the addition of the gate voltage difference  $\delta V_{g3d2}$ ,

$$\begin{aligned}
 \mu_2(1, 1, 0, VG1, VG2, VG3 + \delta V_{g3d2}) &= \mu_3(1, 0, 1, VG1, VG2, VG3 + \delta V_{g3d2}) \\
 &= \mu_3(0, 1, 1, VG1, VG2, VG3 + \delta V_{g3d2}) - \Delta\mu_3 \\
 &= \mu_1(1, 1, 0, VG1, VG2, VG3 + \delta V_{g3d2}) - \Delta\mu_3 \\
 &= \mu_S - \Delta\mu_3 \\
 &= -eV_{SD} - \Delta\mu_3,
 \end{aligned} \tag{2}$$

where  $\mu_S = -eV_{SD}$ . Subtracting equation 1 from equation 2 we find that,

$$-eV_{SD} = \alpha_{g3d2} \cdot \delta V_{g3d2}, \tag{3}$$

defining the gate lever arm,  $\alpha_{g3d2}$ . From Figure S3 we extract  $\delta V_{g3d2} = 96$  mV and find  $\alpha_{g3d2} = 0.10e$ .

At the blue circle, an excited state on either D2 or D3 is in resonance with the ground state of the other dot. The electrochemical potential of this excited state is defined as,

$$\mu_2^* = \mu_2 + \Delta E \tag{4}$$

where  $\Delta E$  is the single particle level spacing. At the blue circle the electrochemical potentials of the D2 excited state and D3 are aligned and are  $\Delta\mu_3$  below that of the drain. Here, the electrochemical potential of D2 differs from that defined by equation 1 (green circle) by the addition of the gate voltage difference  $\delta V_{g3d2}^*$ ,

$$\mu_2^*(1, 1, 0, VG1, VG2, VG3 + \delta V_{g3d2}^*) = -\Delta\mu_3, \tag{5}$$

This difference in gate voltage,  $\delta V_{g3d2}^*$ , converted to an energy using the gate lever arm, gives the excited state splitting,

$$\Delta E = \alpha_{g3d2} \delta V_{g3d2}^*. \quad (6)$$

From figure S3 we extract  $\delta V_{g3d2}^* = 34$  mV, giving  $\Delta E = 3.4$  meV.

## References

- (1) van der Wiel, W. G.; De Franceschi, S.; Elzerman, J. M.; Fujisawa, T.; Tarucha, S.; Kouwenhoven, L. P. *Rev. Mod. Phys.* **2002**, 75, 1-22