Supporting Information (SI)

Regional and longitudinal estimation of product lifespan distribution: A case study for automobiles and a simplified estimation method

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Content of the Supporting Information

I. Estimation of distribution parameters by using the Weibull and generalized gamma distribution functions

This study used the Weibull distribution function to approximate the survival rate distribution of passenger cars in various countries. The Weibull distribution function is widely used in reliability analysis to model the failure rate of technical objects. It has also been demonstrated that the Weibull distribution function provides a good approximation of the actual lifespan distribution of automobiles and other consumer durable goods. On the other hand, a Japanese study reported that their likelihood ratio tests supported the hypothesis that vehicle lifespans follow the generalized gamma distribution function (Kagawa et al.(2011)). We therefore also estimated lifespan distribution function by using the generalized gamma distribution function and compared the results with the results by using the Weibull distribution function.

Assuming that the survival rate distribution of passenger cars follows the generalized gamma distribution function, we estimated the parameters of the distribution function by means of maximum likelihood method on the assumption that the errors follow the normal distribution. The survival rate function, $R(y)$, is expressed by using the generalized gamma distribution function as follows:

$$R(y) = 1 - F(y) = P\left(\left(\frac{y}{\alpha}\right)^{\beta}, \rho\right) \tag{S3}$$

$$P(z, a) = \frac{1}{\Gamma(a)} \int_0^z e^{-t} t^{a-1} \mathrm{dt} \tag{S4}$$

$$f(y) = \frac{\beta y^{\rho\beta - 1}}{(\alpha)^{\rho\beta} \Gamma(\rho)} \exp\left\{-\left(\frac{y}{\alpha}\right)^{\beta}\right\} \tag{S5}$$

where $F(y)$ is the cumulative distribution function of the generalized gamma distribution, α is the scale parameter, β and ρ are the shape parameters, P is the incomplete gamma function, Γ is the gamma function, $f(y)$ is the probability density function of the generalized gamma distribution.

Table S1 shows the estimation results of average lifespan of passenger cars for the 17 target countries by using the Weibull distribution and generalized gamma distribution functions. The coefficient of determination, R^2, indicated that the generalized gamma distribution function showed slightly better approximations of the survival rate distribution of passenger cars than the Weibull distribution function. The Weibull distribution function, however, also showed reasonably good approximations for the target countries of this study. The difference in the estimated average lifespan was 0.5 years on average. This difference is not significant for an aim of this study, which was to provide evidence for regional and longitudinal trends.

The generalized gamma distribution contains a special form of the Weibull distribution, which is the case of the shape parameter $\rho = 1$. As done by Kagawa et al.(2011), we conducted likelihood ratio tests with the null hypothesis as $\rho = 1$ and the alternative hypothesis as $\rho \neq 1$. The calculated test statistics, 2×{log(likelihood of generalized gamma)– log(likelihood of Weibull)}, were 0.6–23.9. The null hypothesis was not rejected for six countries: Australia, Brazil, Canada, France, Spain, and the United States at a 5% level of significance. Consequently, the alternative hypothesis, which means the generalized gamma distribution should be used, was not necessarily supported statistically for this study.

In addition, when the survival rate distribution was approximated by the generalized gamma distribution function, the estimated survival rate distribution showed inappropriate shapes for some countries, e.g. Denmark, Germany, and South Korea (see Figure S1). This is due to the limitation of the available primary data for the estimation and the number of the parameters of the generalized gamma distribution function. In our estimation, only the aggregated number of in-use cars was available for older cars (e.g. cars older than 14 years old) from some data sources and the distribution parameters were estimated under the constraint that the calculated aggregated number of in-use cars for older cars were consistent with the statistical data (equation (2) of the main text). Because the generalized gamma distribution function, which has three parameters, is more flexible, the approximated distribution with the optimized parameters was inappropriate for some countries. The Weibull distribution function has only two parameters and does not cause a problem like this.

[Literature]

Kagawa, S.; Nansai, K.; Kondo, Y.; Hubacek, K.; Suh, S.; Minx, J.; Kudoh, Y.; Tasaki, T.; Nakamura, S. Role of Motor Vehicle Lifetime Extension in Climate Change Policy. *Environ. Sci. Technol.* 2011, 45, 1184-1191.

Table S1 Estimated average lifespan of passenger cars by using the Weibull and generalized gamma distribution functions.

Country	Average lifespan, y_{av} (years)		Coefficient of determination, R^2		Log likelihood	
	Weibull	Generalized gamma	Weibull	Generalized gamma	Weibull	Generalized gamma
Australia	22.6	22.9	0.80	0.82	28.5	29.3
Austria	15.4	15.9	0.98	0.99	33.3	36.1
Brazil	18.5	19.0	0.98	0.99	39.8	40.5
Canada	15.4	15.9	0.77	0.78	15.0	15.3
Denmark	16.7	17.3	0.90	0.95	29.1	33.7
Finland	22.0	22.4	0.99	0.99	46.0	49.1
France	15.2	15.8	0.99	0.99	37.9	38.7
Germany	13.7	14.0	0.89	0.98	22.0	32.5
Ireland	13.0	13.6	0.95	0.97	21.5	25.6
Italy	14.1	14.6	0.99	1.00	38.5	45.0
Japan	13.3	13.7	0.95	0.97	25.0	28.0
The Netherlands	15.1	15.6	0.99	1.00	39.9	49.3
South Korea	13.0	13.1	0.96	0.99	26.2	38.1
Spain	18.0	18.7	0.92	0.93	33.7	34.3
Switzerland	14.1	14.6	0.98	1.00	34.0	43.1
U.K.	13.5	14.1	0.96	0.99	24.0	35.6
United States	16.2	16.6	0.92	0.94	25.7	27.0

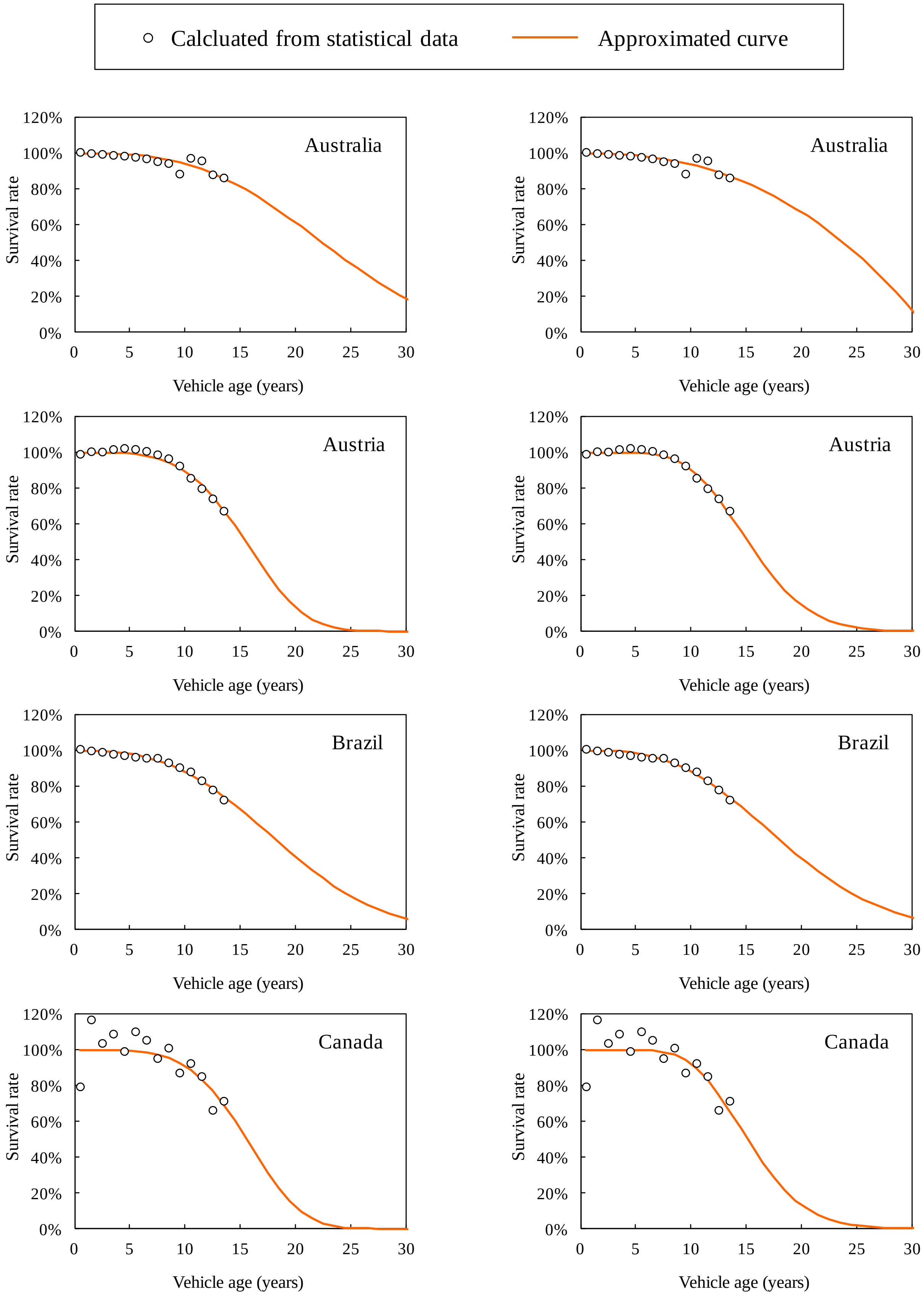


Figure S1 Approximation results of the survival rate distribution of passenger cars (left: Weibull distribution, right: generalized gamma distribution)

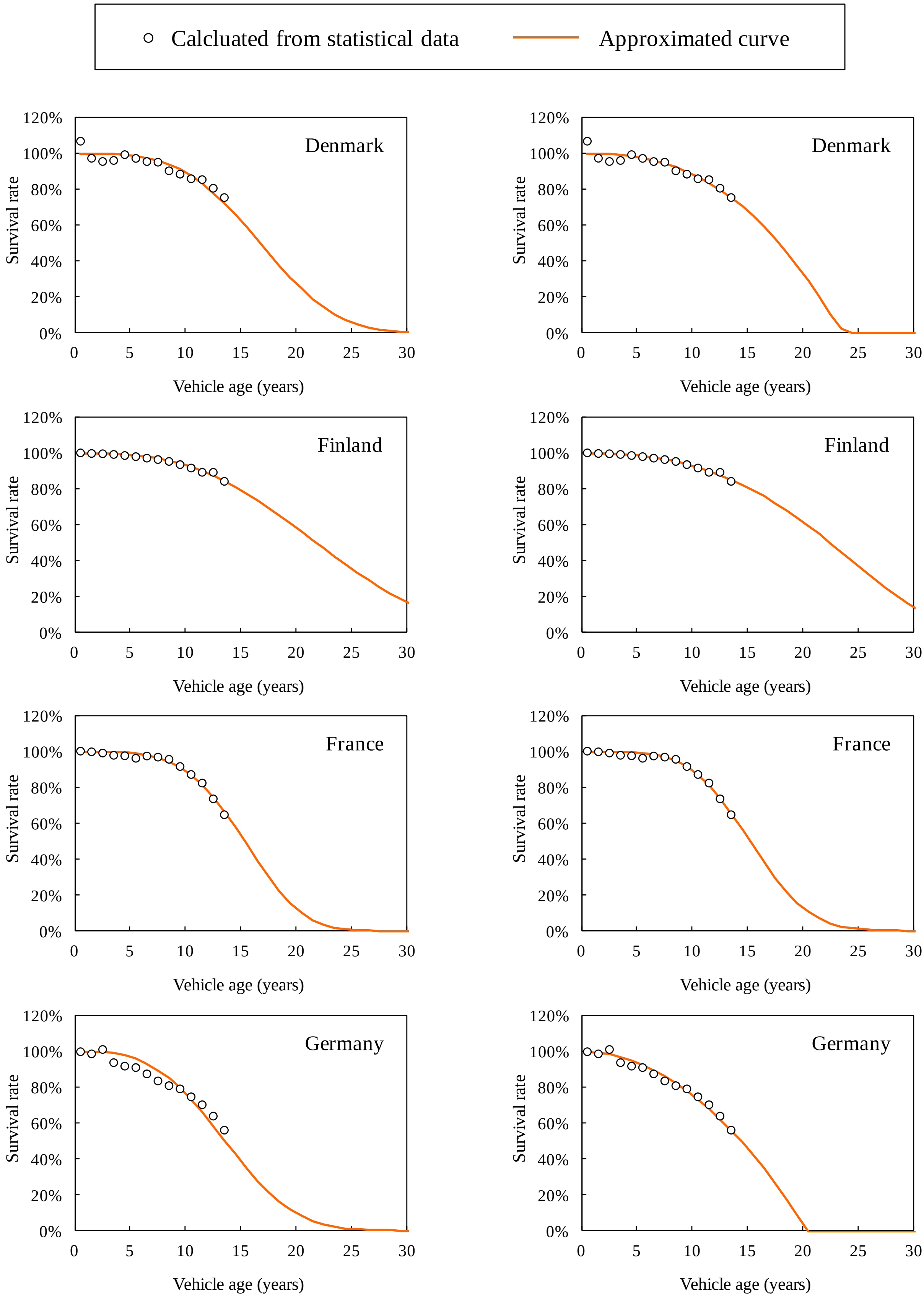


Figure S1 (cont'd) Approximation results of the survival rate distribution of passenger cars (left: Weibull distribution, right: generalized gamma distribution)

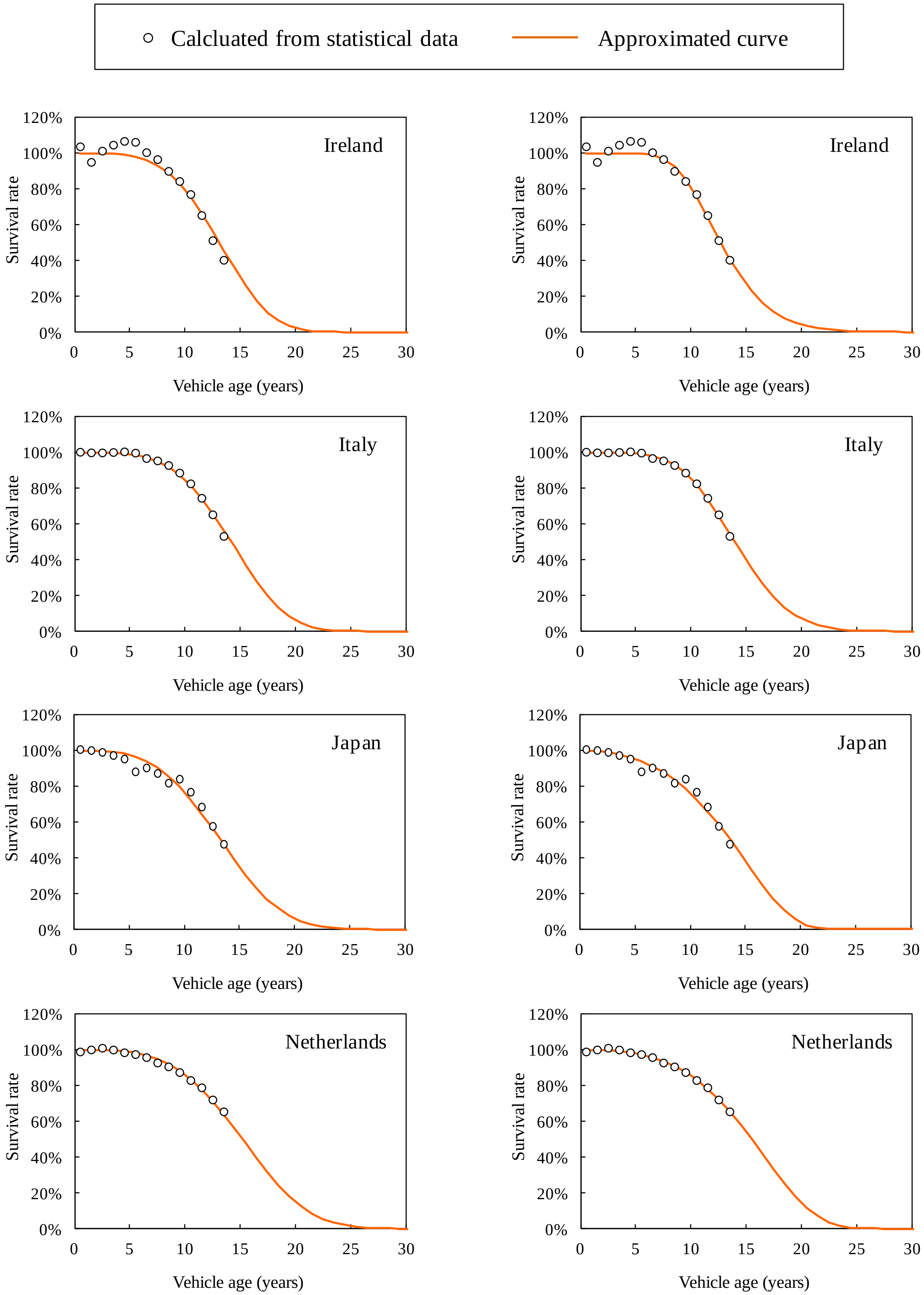
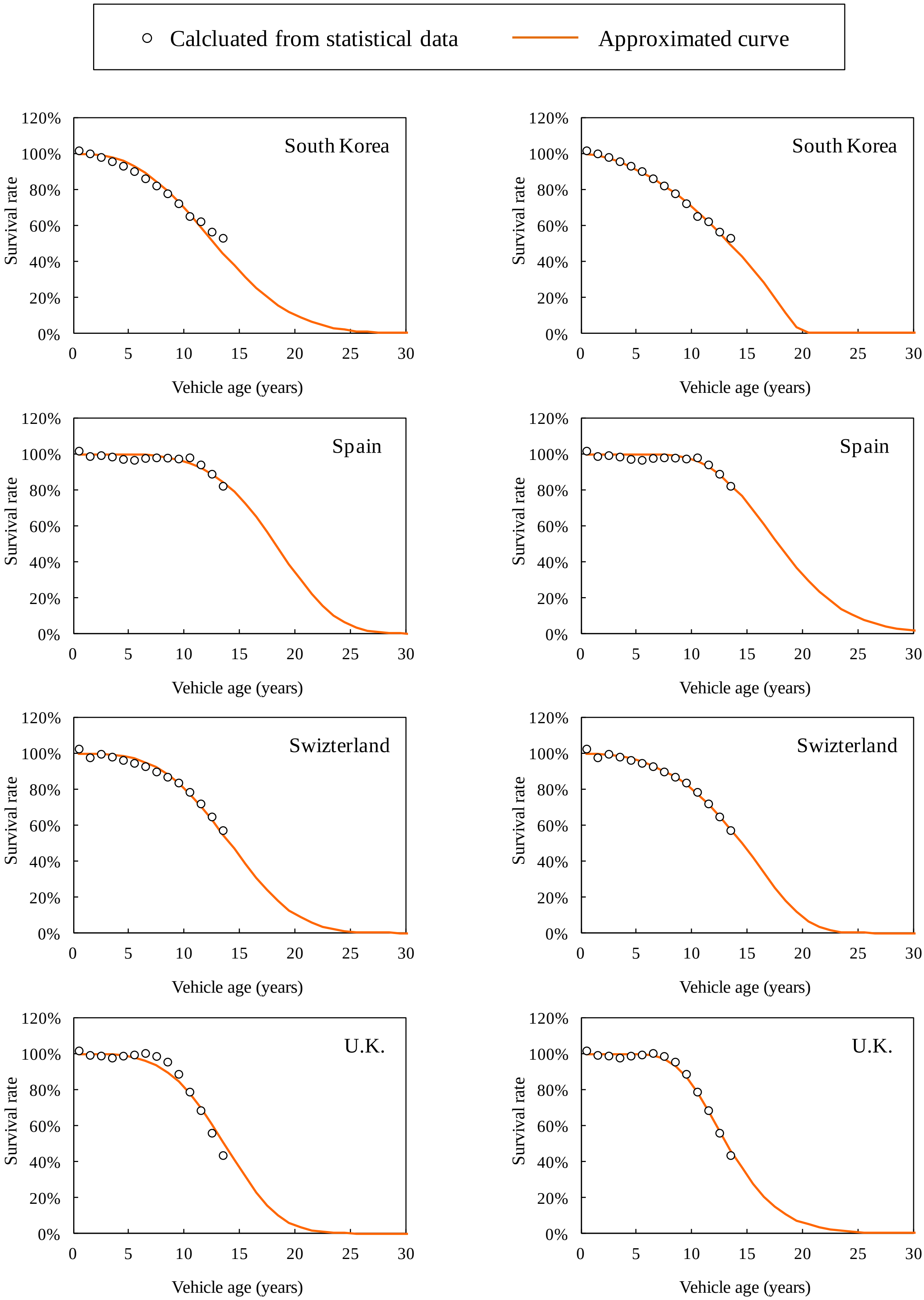


Figure S1 (cont'd) Approximation results of the survival rate distribution of passenger cars (left: Weibull distribution, right: generalized gamma distribution)



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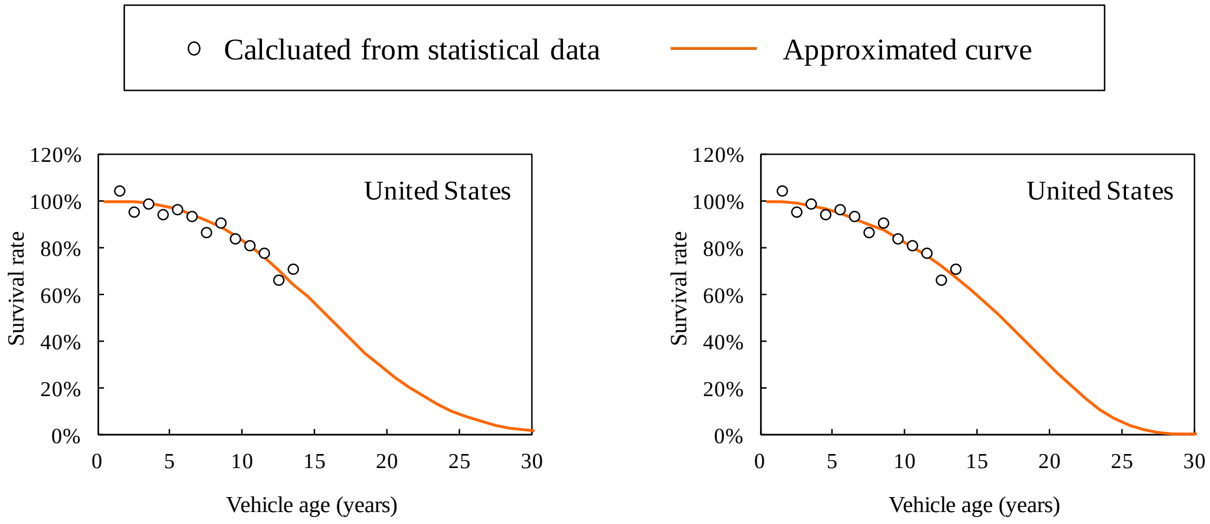

Figure S1 (cont'd) Approximation results of the survival rate distribution of passenger cars (left: Weibull distribution, right: generalized gamma distribution)

II. Information of data used in the study

The estimation in this study required the data for the number of in-use cars for each age, $N_t(i)$, new car sales in each year, S_{t-i}, and the total number of in-use cars, N_t. Following shows the information of the data used for the estimation.

The data for $N_t(i)$ was collected from the Parc database by R.L. Polk & Co., reports by Japan Automobile Manufacturers Association (JAMA)[1], and individual statistics in each country/region: European Automotive Manufacturers Association (ACEA)[2], Automobile Inspection and Registration Information Association of Japan (AIRIA)[3], and Korea Automobile Manufacturers Association (KAMA)[4]. After confirmed the consistency between the data sources, we used the data by R.L. Polk & Co. for the estimation of 17 different countries (Table 1 of the paper) and the data from the reports by JAMA[1] as well as the statistics by AIRIA[3] for the estimation in different years for four countries (Table 2 of the paper).

The data for S_{t-i} was collected from the NewReg database by R.L. Polk & Co., reports by JAMA[1], World Road Statistics by the International Road Federation (WRS-IRF), a report by FOURIN, Inc.,[5] which is a Japanese research company that specializes in the automotive industry, and individual statistics from each country: KAMA[6], Australian Bureau of Statistics[7], and Statistics Canada[8]. As covered data years were different between the data sources, we compiled the new car sales data from these data sources after checking the consistency and the continuity of data between data sources.

The data for N_t was collected from the Parc database by R.L. Polk & Co., reports by JAMA[1], WRS-IRF, and the report by FOURIN, Inc[5]. After confirmed the consistency between data sources, we used the data from R.L. Polk & Co for this study.

[Data sources]

1. Japan Automobile Manufacturers Association (JAMA), World Motor Vehicle Statistics
2. European Automotive Manufacturers Association (ACEA), European Motor Vehicle Parc
3. Automobile Inspection and Registration Information Association of Japan (AIRIA), Vehicle Ownership in Japan (in Japanese)
4. Korea Automobile Manufacturers Association (KAMA), Vehicle registration statistics by manufacturers, vehicle types, models, and vehicle age (in Korean)
5. FOURIN. World Motor Vehicle Statistics Yearbook 2008 (in Japanese)
6. Korea Automobile Manufacturers Association (KAMA), Sales statistics by vehicle types (in Korean)
7. Australian Bureau of Statistics, Sales of New Motor Vehicles, Australia
8. Statistics Canada, New Motor Vehicle Sales

III. Sensitivity of the estimated lifespan distribution to the values of the shape parameter.

Figure S2 shows an example of the differences in the approximated survival curve as a function of different values of the shape parameter: 2.5 to 4.5. Although the shape of the approximated survival curve differed slightly (by definition, since the shape parameter differed), we found a difference of only 0.1 year in the estimated average lifespan for b values ranging from 2.5 to 4.5, and the coefficient of determination was high ($R^2 > 0.98$) for all of the approximations. The sensitivity of the approximation results to changes in the value of the shape parameter was therefore low. This demonstrates the possibility of applying a constant value to the shape parameter for all the countries and years.

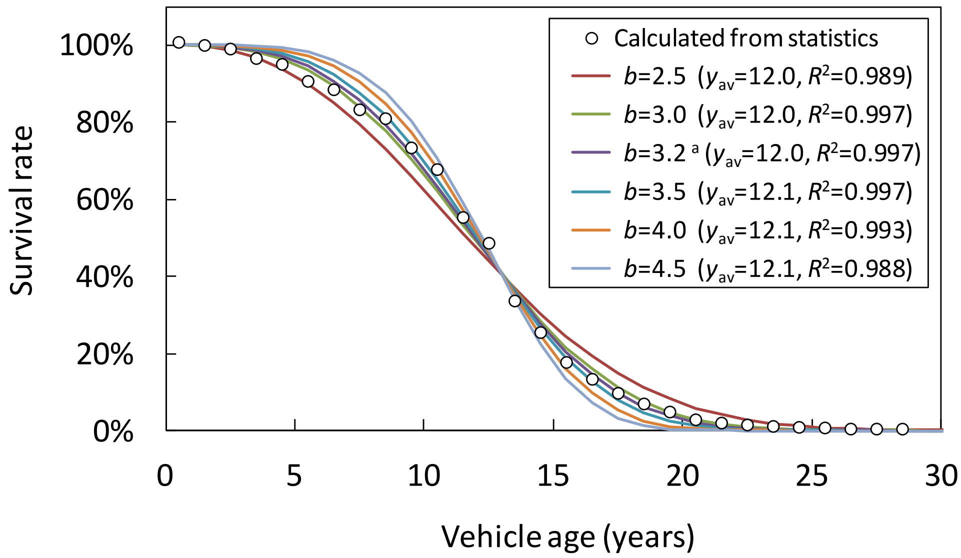

Figure S2. Approximated survival curves for different values of the shape parameter, *b* (passenger cars with engines of over 660 cm^3 in Japan, at the end of 2008). [a] Maximum likelihood estimator.

IV. Applicability of fixed shape parameter from the aspect of stock and end-of-life generation estimation.

In order to investigate the applicability of fixed shape parameters from the aspect of stock and end-of-life generation estimation, we examined the influence of assuming the shape parameter to be 3.6 on the estimated number of in-use and end-of-life cars.

We estimated the number of in-use cars at the end of 2008 (N_{2008}) and the number of end-of-life cars in 2008 (W_{2008}) using equations (S6-S8).

$$N_t = \sum_i \{S_{t-i} \times R_t(i + 0.5)\} \tag{S6}$$

$$W_t = \sum_i \{S_{t-i} \times f_t(i)\} \tag{S7}$$

$$f_t(i) = R_{t-i}(i - 0.5) - R_t(i + 0.5) \tag{S8}$$

where $f_t(i)$ is the discard rate for cars with an age of i years in the year t. We compared N_{2008} and W_{2008} between the estimations based on the parameters estimated by means of maximum likelihood method (Table 1 of the paper) and the average lifespan estimated by assuming the shape parameter to be constant at 3.6.

As shown in Figure S3, the differences in the estimated number of in-use and end-of-life cars were small, ranging from 0% to 7%. Thus, assuming that the shape parameter was constant at 3.6 did not greatly affect the estimation results. This is additional confirmation that the shape parameter for the lifespan distribution of passenger cars can be regarded as a constant value of 3.6 for all countries in our study. This makes the estimation of total stocks and end-of-life generation of passenger cars far more practical.

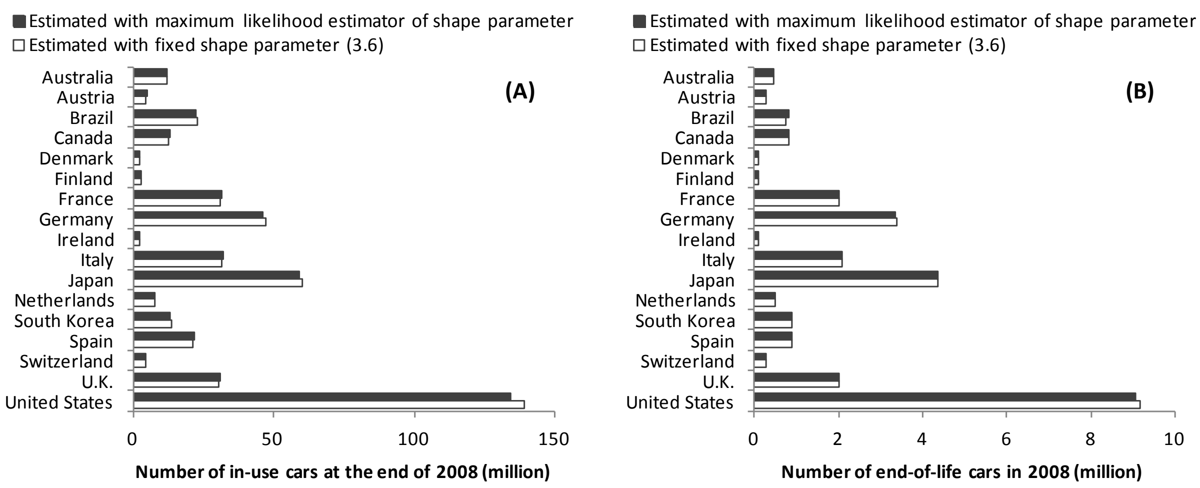

Figure S3. Influence of assuming the shape parameter to be a constant value (3.6) on the estimated number of (A) in-use cars and (B) end-of-life cars

V. Inaccuracy in stock and end-of-life estimates caused by assuming static average lifespans.

In order to show the inaccuracy in the stock and end-of-life generation estimation caused by assuming static average lifespans, we estimated the number of in-use and end-of-life passenger cars in each country by using the average lifespan for another country and years. The number of in-use cars (N_t) and end-of-life cars (W_t) were estimated using equations (S6-S8).

The results for the estimation using regionally-static average lifespans were shown in the paper (Figure 4). Figure S4 shows the inaccuracy in the stock and end-of-life generation estimation caused by assuming temporally-static average lifespans. The figure shows the ratio of the estimated number by fixing the average lifespan with the value for the year 2000 to the number estimated using the average lifespans specific for each year. Compared with the estimates using the temporally-specific average lifespans, assuming temporally-static average lifespans causes an inaccuracy of up to approximately 30% and 150% in the estimated number of in-use and end-of-life products, respectively. A large inaccuracy can be seen especially for countries in which average lifespan has been increasing or decreasing.

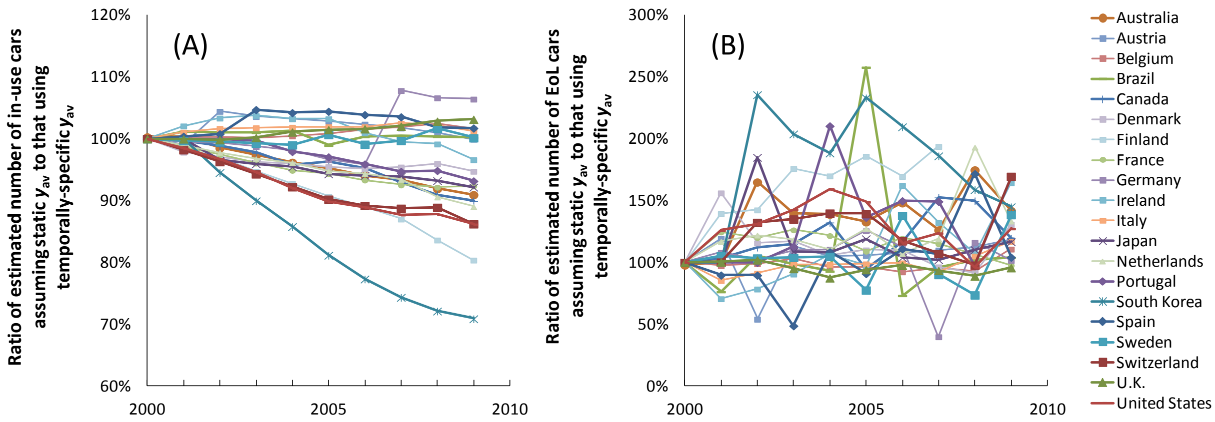

Figure S4. Difference in stock and end-of-life estimates between temporally-static and specific average lifespans. The figures show the ratio of estimated number of (A) in-use cars and (B) end-of-life cars using fixed average lifespan (fixed with the values for the year 2000) to the number estimated using the average lifespans specific for each year.