Supporting Information

High Resolution Hyperpolarized J-spectra with Parahydrogen Discrimination.

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Density operator evolution during a train of refocusing pulses. An AX two-spin systems.

We present here the spin dynamics of two weakly coupled spins during a train of 180º pulses, for two different initial states originating from thermally and PHIP hyperpolarized spins.

In the case of thermally polarized spins, the density operator in the high field approximation is:

$$\rho_T(0) \propto (I_1^z + I_2^z)$$

Right after a $45°|_y$excitation pulse we get

$$\rho_T(0+) \propto (I_1^x + I_2^x)\ ,$$

where the remaining term proportional to $(I_1^z + I_2^z)$ is neglected because it does not evolve into detectable magnetization during the train of 180º pulses. We are interested in the signal at the top of the successive echoes. Given that the coupling between spins is weak, the chemical shift evolutions are refocused at those particular time values, and can be neglected during the rest of the calculation. Thus, only evolutions under a weak *J*-coupling Hamiltonian are to be considered. The first evolution of ρ_T under such Hamiltonian, of the form $H_J = \pi J 2 I_1^z I_2^z$ (weak coupled system), evaluated at a time $t_E/2$ yields,

$$\rho_T(t_E/2) \propto (I_1^x + I_2^x)\cos(\pi J t_E/2) + (2I_1^y I_2^z + 2I_1^z I_2^y)\sin(\pi J t_E/2).$$

The first term in the above expression commutes with a $180°|_x$ pulse and the remaining term, $(2I_1^y I_2^z + 2I_1^z I_2^y)$, change its sign twice (i.e. both I_1^y and I_2^z invert their sign; therefore the product remains unaltered). Consequently, $\rho_T(t_E/2)$ is unaffected by the application of a $180°|_x$ pulse. Repeating the calculation for the second evolution period, we obtain for the density operator at the top of the first echo the following expression:

$$\rho_T(t_E) \propto (I_1^x + I_2^x)\cos(\pi J t_E) + \left(2I_1^y I_2^z + 2I_1^z I_2^y\right)\sin(\pi J t_E).$$

The detectable magnetization is calculated by means of the operator $I^+ = (I_1^x + I_2^x) + i\left(I_1^y + I_2^y\right)$ yielding

$$M_T(t_E) = Tr\{\rho_T(t_E)(I_1^x + I_2^x)\} \propto \cos(\pi J t_E). \tag{1}$$

Following the same procedure, the operator at the top of the second echo results

$$\rho_T(2t_E) \propto (I_1^x + I_2^x)\cos(\pi J 2t_E) + \left(2I_1^y I_2^z + 2I_1^z I_2^y\right)\sin(\pi J 2t_E),$$

the corresponding magnetization being,

$$M_T(2t_E) = Tr\{\rho_T(2t_E)(I_1^x + I_2^x)\} \propto \cos(\pi J 2t_E). \tag{2}$$

Notice that the signal at the top of the echoes appears in the x-axis (real part in the NMR jargon) and modulated by a cosine function. This implies that the signal starts from a maximum value and is modulated by the action of the *J*-couplings, irrespective of the action of the refocusing pulses.

Now we turn our attention to an initial operator after a hydrogenation with p-H_2. After averaging for a reaction time much longer than any characteristic time of the spin system (i.e. >>$1/T_2$, $1/T_1$, etc) we obtain

$$\rho_P(0) \propto (2I_1^z I_2^z)$$

The application of a $45°|_y$ pulse transforms the operator into

$$\rho_P(0+) \propto (2I_1^z I_2^z) + (2I_1^x I_2^x) + (2I_1^x I_2^z + 2I_1^z I_2^x). \quad (3)$$

The first two terms on the right hand side are discarded since they will not contribute to the detectable magnetization in the whole sequence. Computing the evolution under the *J*-coupling Hamiltonian for the last term of Eq. (3), for a period of time $t_E/2$ we get:

$$\rho_P(t_E/2) \propto \left(I_1^y + I_2^y\right)\sin(\pi J t_E/2) + (2I_1^x I_2^z + 2I_1^z I_2^x)\cos(\pi J t_E/2).$$

After the first $180°|_x$ pulse the first term in the right hand side of the above expression changes its sign. Concerning the bilinear term, only I_1^z and I_2^z are affected by the inverting pulse, leading to the global sign change of the term. As a result, the effect of the pulse is a sign change in the density operator,

$$\rho_P(t_E/2) \propto -\left(I_1^y + I_2^y\right)\sin(\pi J t_E/2) - (2I_1^x I_2^z + 2I_1^z I_2^x)\cos(\pi J t_E/2).$$

Continuing with the second free evolution, at the top of the first echo, the density operator can be written as:

$$\rho_P(t_E) \propto -\left(I_1^y + I_2^y\right)\sin(\pi J t_E) - (2I_1^x I_2^z + 2I_1^z I_2^x)\cos(\pi J t_E),$$

and the magnetization of the first echo is

$$M_P(t_E) = Tr\{\rho_P(t_E) I^+\} \propto -i\sin(\pi J t_E). \quad (4)$$

Notice that in this case the signal will appear in the imaginary channel and modulated by a sine function (see Fig. 1 in the manuscript). After a new evolution under H_J we obtain

$$\rho_P(3t_E/2) \propto -(I_1^y + I_2^y)\sin(\pi J 3t_E/2) - (2I_1^x I_2^z + 2I_1^z I_2^x)\cos(\pi J 3t_E/2).$$

Again, the application of $180°|_x$ pulse globally changes the sign of the above operator to yield,

$$\rho_P(3t_E/2) \propto (I_1^y + I_2^y)\sin(\pi J 3t_E/2) + (2I_1^x I_2^z + 2I_1^z I_2^x)\cos(\pi J 3t_E/2).$$

Finally, after another period of evolution we obtain the operator at the top of the second echo in the train,

$$\rho_P(2t_E) \propto \left(I_1^y + I_2^y\right)\sin(\pi J 2t_E) + (2I_1^x I_2^z + 2I_1^z I_2^x)\cos(\pi J 2t_E).$$

The detectable signal is

$$M_P(2t_E) = Tr\{\rho_P(t_E) I^+\} \propto i\sin(\pi J 2t_E). \quad (5)$$

Following this reasoning for N echoes of the sequence, we find that the thermal signal of the K-th echo will be proportional to $\cos(\pi J K t_E)$ whereas the PHIP signal will be proportional to $(-1)^K i\sin(\pi J K t_E)$. In other words, the top of the echoes will present a cosine modulation for the thermal case while in the case of PHIP it will show a sine modulation with a sign alternation superimposed, as shown in the simulations of Fig. 2 in the manuscript.

The effect of the Fast Fourier Transform.

The sign change between odd-even echoes has a remarkable influence when an FFT of the time domain signal is carried out, rendering spectra with a frequency offset of half the spectral width (or equivalently, shifted $N/2$ acquired points). In general, time domain signals after digitalization are represented by numerical series ranging from 1 to N, with a total acquisition time given by $T = N\,dw$, being dw the time between acquired points, or dwell time. In the particular case of a J-spectrum the dwell time is replaced by the echo time, t_E. The FFT algorithm will return another numerical series with the same dimension (disregarding zero filling) ranging also from 1 to N, but representing frequencies, where the maximum frequency is $\nu_m=1/t_E$ and the spectral resolution is $1/T$. In order to describe the effect of the Fourier transformation on thermal or PHIP originated signals the reasoning presented for different data collection methods in Magnetic Resonance Imaging are followed. In particular an $N/2$ aliasing is normally present in the acquisition of images based on Echo Planar Imaging (EPI) pulse sequences

Suppose that a continuous signal $f(t)$ is discretized in equally spaced points, with separation t_E. A simple way to achieve this is by using a "Dirac Comb". The discrete version of the function $f(t)$ is,

$$f_d(t) = f(t)\sum_{n=0}^{\infty}\delta(t-nt_E).$$

The Fourier transformed signal will be denoted as $\{f_d(t)\} = F_d(\omega)$, which can be written as:

$$F_d(\omega) = F(\omega)\odot\sum_{n=0}^{\infty}\delta\left(\omega-\frac{n}{t_E}\right) = \sum_{n=0}^{\infty}F\left(\omega-\frac{n}{t_E}\right), \tag{6}$$

where $\odot$ denotes convolution. In the case of PHIP, the time domain signal can be written as:

$$f_d(t) = f(t)\sum_{n=0}^{\infty}(-1)^n\delta(t-nt_E) = f(t)\sum_{n=0}^{\infty}\delta(t-2nt_E) - f(t)\sum_{n=0}^{\infty}\delta(t-(2n+1)t_E),$$

where the sum was separated for even and odd values. By defining $\tau = t - t_E$, this expression can be recast as:

$$f_d(t) = f(t)\sum_{n=0}^{\infty}\delta(t-2nt_E) - f(\tau+t_E)\sum_{n=0}^{\infty}\delta(\tau-2nt_E).$$

Using the Shift Theorem, namely $F\{f(t+t_0)\} = F(\omega)e^{-i2\pi\omega t_0}$, the Fourier Transform results in,

$$F_d(\omega) = \sum_{n=0}^{\infty}F\left(\omega-\frac{n}{2t_E}\right) - e^{-i2\pi\omega t_E}F(\omega)\odot\sum_{n=0}^{\infty}\delta\left(\omega-\frac{n}{2t_E}\right) = \sum_{n=0}^{\infty}F\left(\omega-\frac{n}{2t_E}\right) - e^{-i2\pi\omega t_E}\sum_{n=0}^{\infty}F\left(\omega-\frac{n}{2t_E}\right) =$$

$$= \left(1-e^{-i2\pi\omega t_E}\right)\sum_{n=0}^{\infty}F\left(\omega-\frac{n}{2t_E}\right).$$

Noticing that $\frac{n}{2t_E} = \frac{n}{t_E} - \frac{n}{2t_E}$, the signal can be expressed as:

$$F_d(\omega) = \left(1-e^{-i2\pi\omega t_E}\right)\sum_{n=0}^{\infty}F\left(\left(\omega+\frac{n}{2t_E}\right)-\frac{n}{t_E}\right). \tag{7}$$

Comparing Eq. (6) and Eq. (7) it is evident that, besides a phase factor correction of order of unity, the main difference lies in a shift of the central frequency for PHIP signals by $n/(2t_E)$. Therefore, both signals will appear separated by half the spectral window in a *J*-spectrum, providing a natural mechanism for discriminating hyperpolarized and thermally polarized signals (see Fig. 2 in manuscript).