High Dimensional Fluctuations in Liquid Water: Combining Chemical Intuition with Unsupervised Learning

Supplementary Information

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Figure S1: This figure shows the probability density estimates of the logarithm of SOAP distances between $1_{in}2_{out}$ environments(blue), $2_{in}1_{out}$ environments(red) as well as distances between $1_{in}2_{out}$ and $2_{in}1_{out}$ environments(yellow). Panel a shows that there is a complete overlap between all estimates when only oxygen atoms are used in computing the SOAP distances. In panel b however, we observe that the three distributions exhibit bigger differences when hydrogen atoms are included in the analysis.



Figure S2: 2D UMAP projection of the environments for three datasets colored by d_{ice} .



Figure S3: This figure which shows the scatter plot of the free energy values of $\vec{\mathbf{O}}$ environments versus $(\vec{\mathbf{O}}, \vec{\mathbf{H}}_{ave}, \vec{\mathbf{H}}_{dif})$. There is close to a linear relationship with a correlation coefficient of 0.7 and an RMSE between the two free energies of $\sim 2k_BT$.



Figure S4: Free energy surface of MB-pol constructed in 2D UMAP manifold reveals a single basin without an appreciable barrier consistent with the results from TIP4P/2005 shown in the manuscript.



Figure S5: The panels a)-d) show the scatter plots of $\text{Log}(d_{ice})$ at 3.7 Å (including the hydrogen atom SOAP descriptors) versus the chemical-based collective variables for $q_{tet}, d_5, \rho_{voro}$ and LSI.



Figure S6: The panels a)-d) show the scatter plots of $\text{Log}(d_{ice})$ at 6.0 Å (including only the oxygen atoms for the SOAP descriptor) versus collective variables for $q_{tet}, d_5, \rho_{voro}$ and LSI.



Figure S7: Left panel is the fraction of defects for $3k_BT$ cuts of the free energy. Right panel shows the q_{tet} distribution of points high in free energy. Also shown are the weighted q_{tet} distributions of points high in free energy restricted to defective(red) environments and non-defects(orange).



Figure S8: Probability density estimate of $Log(d_{ice})$ for non-defects, under-coordinated defects and over-coordinated defects.



Figure S9: Figure shows the difference in distribution of $\text{Log}(d_{ice})$ of non-defects and defects for the three variations of the SOAP descriptors: (**O**), (**OH**_{ave}), (**O**, **H**_{ave}, **H**_{dif}). The descriptor including both \mathbf{H}_{ave} and \mathbf{H}_{dif} is found to have the greatest difference between defects and non-defects.



Figure S10: Probability density estimates of $\text{Log}(d_{ice})$ and $\text{Log}(d_{dod})$ restricted to defects and non-defects for radial cutoffs of 3.7 Å and 6.0 Å. The top panels are constructed using only oxygen atoms (**O**) while bottom panels include the hydrogen atoms in computing the distance (**O**, $\mathbf{H}_{ave}, \mathbf{H}_{dif}$).



Figure S11: Density plot of $\log(d_{ice})$ versus ρ_{voro} for HD and LD environments of supercooled water.