

## Supporting Information

# Complex Phase Behavior in Particle-Forming AB/AB' Diblock Copolymer Blends with Variable Core Block Lengths

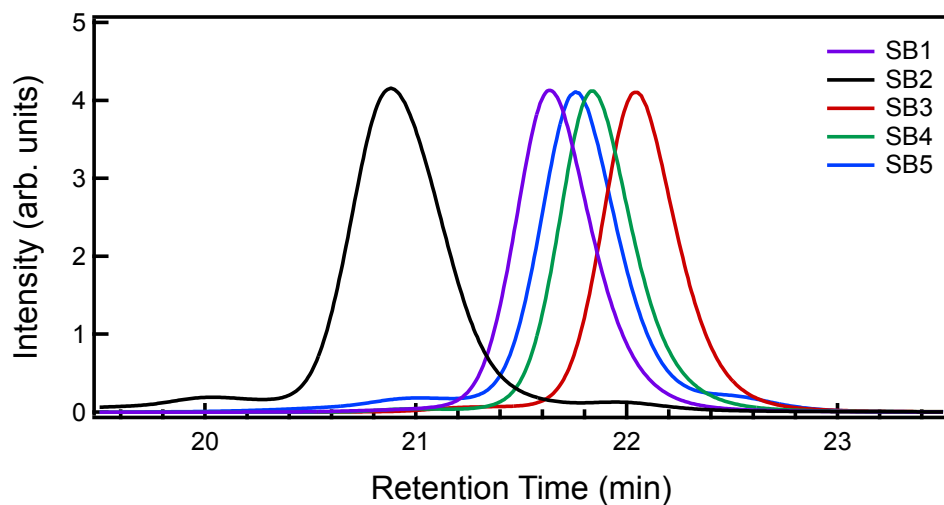
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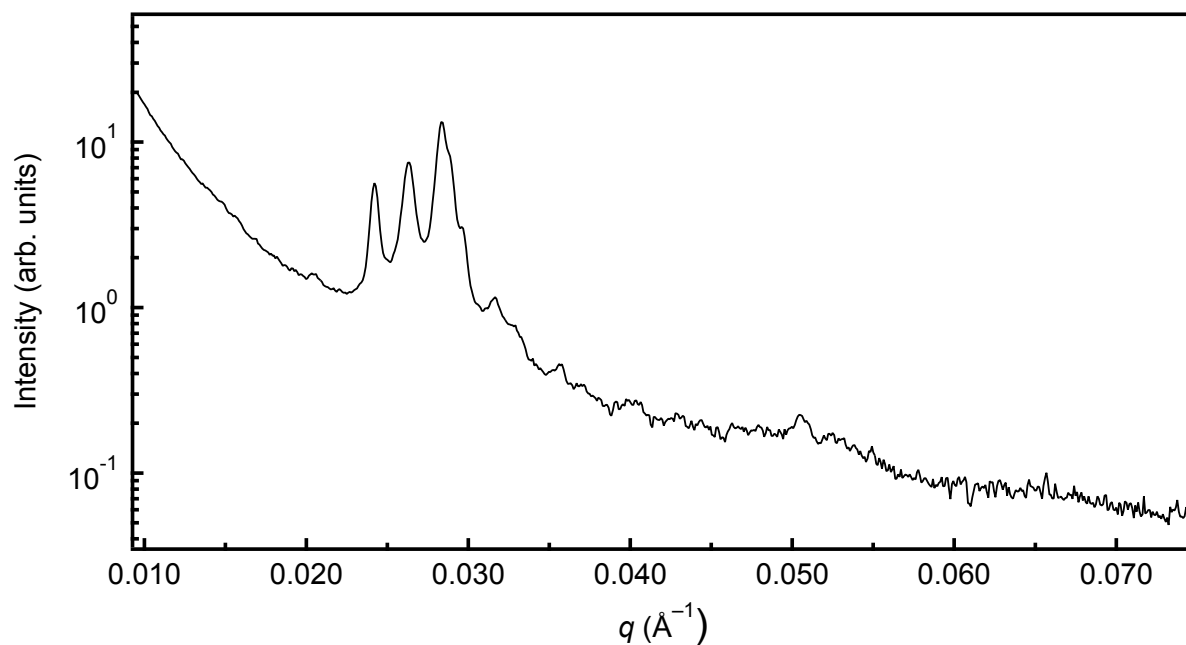
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**Figure S1.** Size exclusion chromatography light scattering traces collected in tetrahydrofuran.



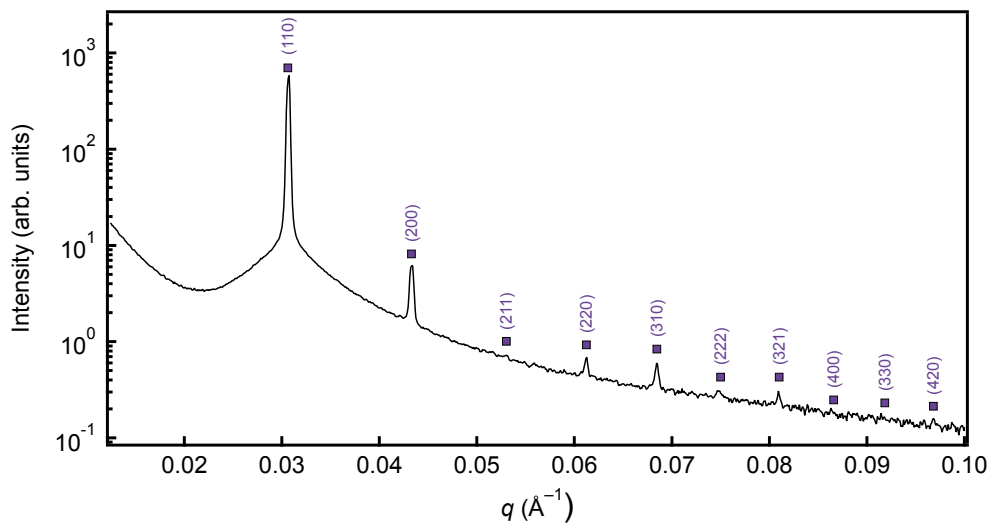
**Figure S2.** Extended 1D SAXS trace for the C14 Laves phase observed in SB1/SB2 blends with  $\phi_2 = 0.075$  at 150 °C following the thermal processing outlined in Figure 2 and the main text. Indexing and residuals can be found in Table S1.

**Table S1.** Observed and calculated peak positions for the C14 Laves phase in Figures 2 and S2; data were collected from SB1/SB2 blends with  $\phi_2 = 0.075$  at 150 °C. Peak positions were calculated as  $q_{hkl} = 2\pi [(4/3) (h^2 + hk + k^2)/a^2 + l^2/c^2]^{1/2}$  based on  $P6_3/mmc$  space group symmetry with lattice parameters  $a = 520.0$  Å and  $c = 847.5$  Å.

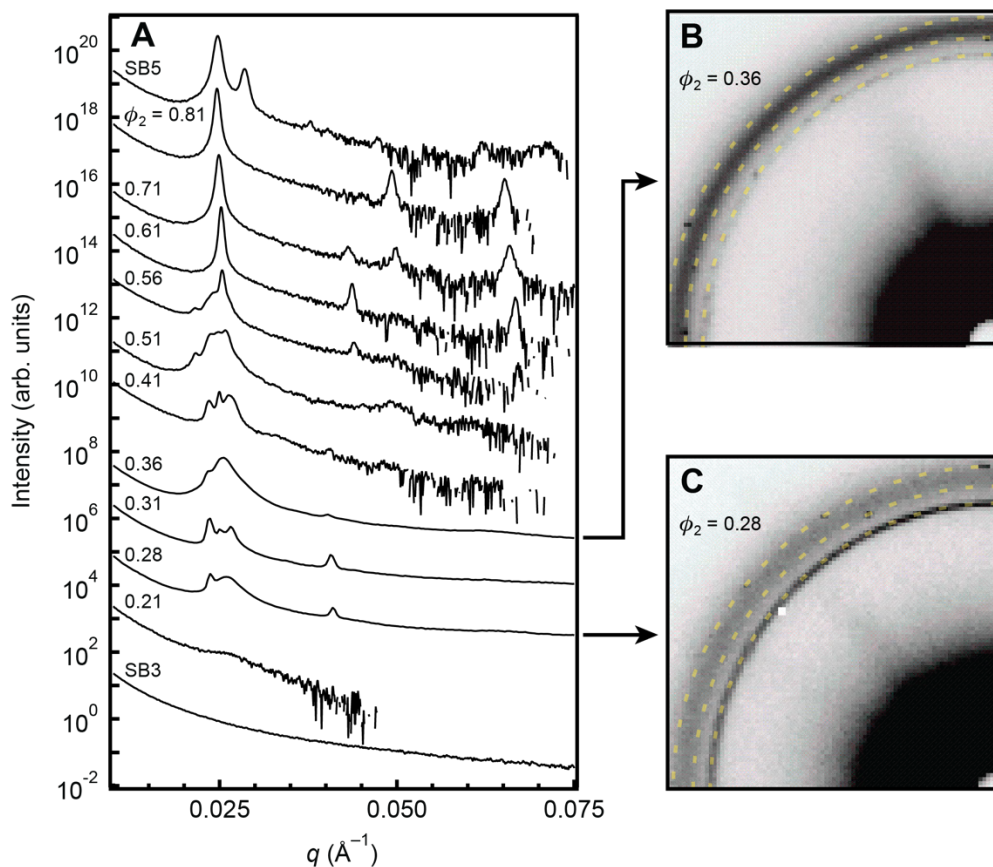
Miller Indices ( <i>hkl</i> )	$q_{\text{obs}}$ (1/Å)	$q_{\text{calc}}$ (1/Å)	% Residual ( $\Delta q / q_{\text{calc}} \times 100$ )
(100)	0.013948	0.013952	0.03
(002)	0.014846	0.014827	−0.13
(101)	0.015744	0.015800	0.35
(102)	0.020325	0.020359	0.17
(110)	0.024188	0.024166	−0.09
(103)	0.026292	0.026254	−0.14
(200)	—	0.027905	—
(112)	0.028319	0.028352	0.12
(201)	0.028858	0.028873	0.05
(004)	0.029577	0.029653	0.26
(202)	0.031643	0.031599	−0.14
(104)	0.032900	0.032772	−0.39
(203)	0.035685	0.035683	−0.01
(210)	0.036942	0.036914	−0.07

**Table S2.** Observed and calculated peak positions for the  $\sigma$  phase coexisting with  $\text{HEX}_C$  in Figures 3 and 4. Data were collected from an SB3/SB4 blend with  $\phi_2 = 0.80$  at 150 °C. Peak positions were calculated as  $q_{\text{hkl}} = 2\pi [(h^2 + hk + k^2)/a^2 + l^2/c^2]^{1/2}$  based on  $P4_2/mnm$  space group symmetry with lattice parameters  $a = 917.8 \text{ \AA}$  and  $c = 484.9 \text{ \AA}$ .

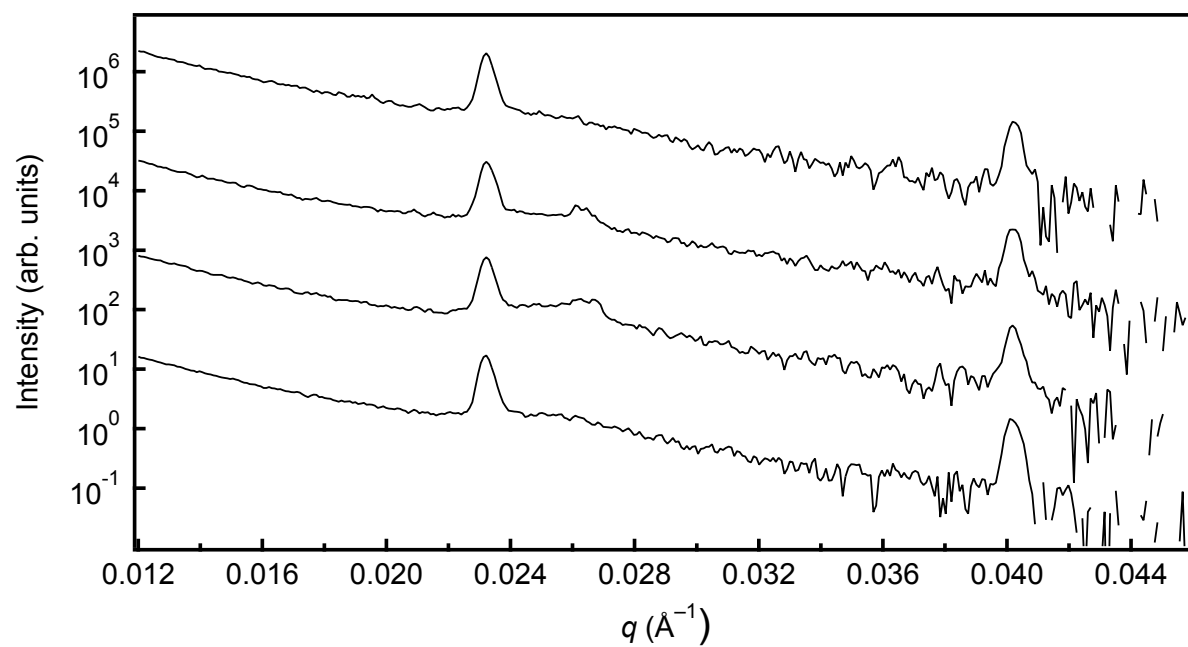
Miller Indices ( <i>hkl</i> )	$q_{\text{obs}}$ (1/Å)	$q_{\text{calc}}$ (1/Å)	% Residual ( $\Delta q/q_{\text{calc}} \times 100$ )
(110)	—	0.009682	—
(200)	—	0.013692	—
(101)	—	0.014654	—
(210)	—	0.015308	—
(111)	—	0.016174	—
(220)	—	0.019363	—
(211)	0.020048	0.020055	0.04
(310)	0.021753	0.021649	−0.48
(221)	0.023369	0.023298	−0.30
(301)	0.024267	0.024283	0.07
(320)	0.024626	0.024684	0.23
(311)	0.025255	0.025230	−0.10
(002)	0.025917	0.025913	−0.02
(400)	0.027320	0.027384	0.23
(112)	—	0.027663	—
(321)	0.027858	0.027877	0.07
(410)	0.028217	0.028227	0.03
(330)	0.029025	0.029045	0.07
(202)	0.029295	0.029308	0.04
(212)	—	0.030097	—
(420)	—	0.030616	—
(411)	0.03109	0.031058	−0.10
(331)	0.031809	0.031804	−0.02
(222)	0.032347	0.032348	0.00
(421)	0.033155	0.033245	0.27
(312)	0.033694	0.033766	0.21
(430)	—	0.034230	—
(510)	—	0.034908	—
(322)	0.035849	0.035788	−0.17
(501)	0.036657	0.036600	−0.16
(520)	—	0.036867	—
(511)	0.037195	0.037235	0.11



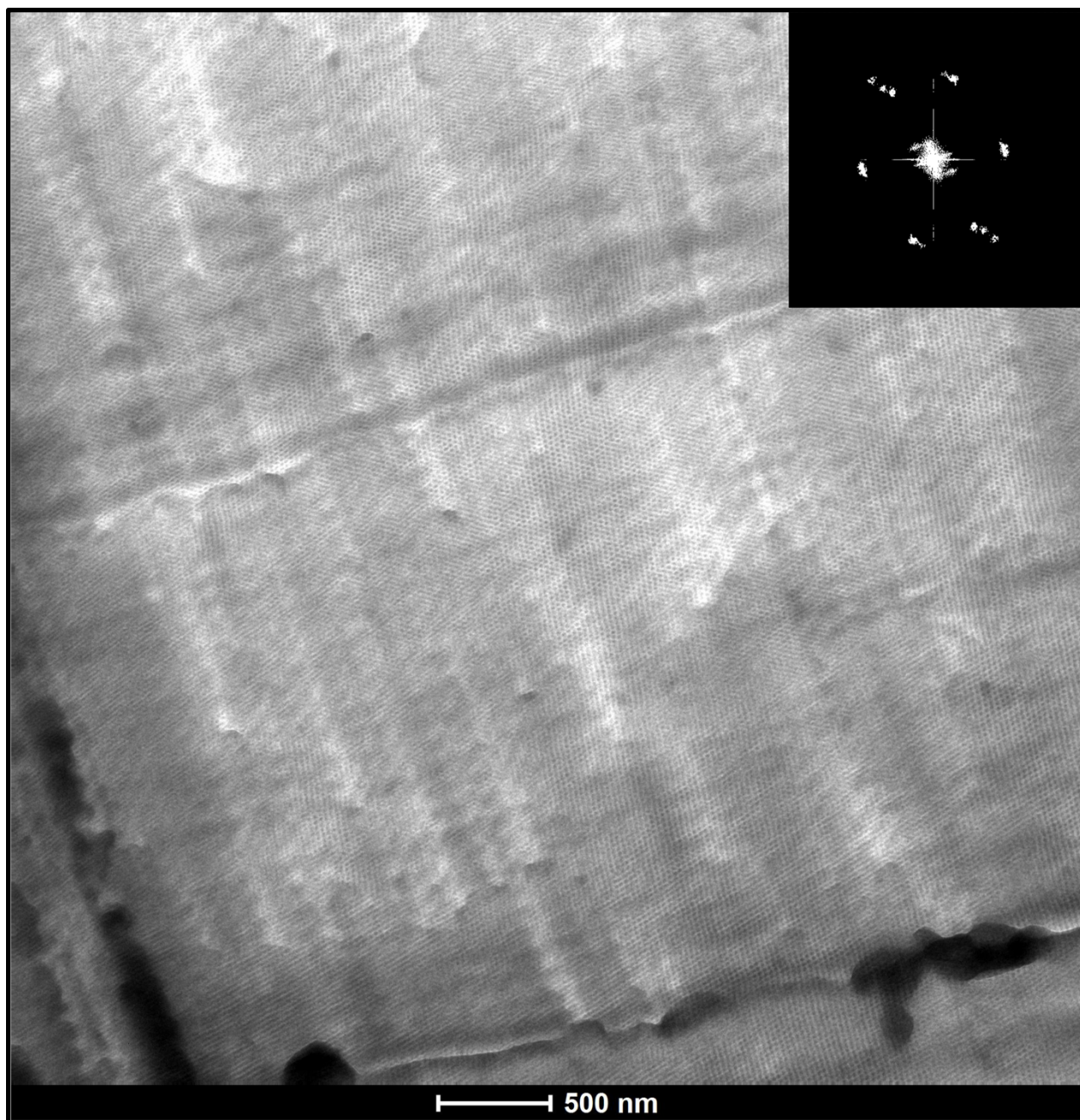
**Figure S3.** 1D SAXS trace collected from a SB3/SB4 blend with  $\phi_2 = 0.80$  following a 169 h anneal at 180 °C. The trace is indexed to a BCC phase.



**Figure S4.** (A) 1D and (B,C) 2D SAXS data collected from SB3/SB5 blends following extended annealing (66-92 h) at 120 °C. 2D SAXS data in B and C was collected following 92 h of annealing at 120 °C. 2D data is indexed to the HCP phase (yellow dashed lines).

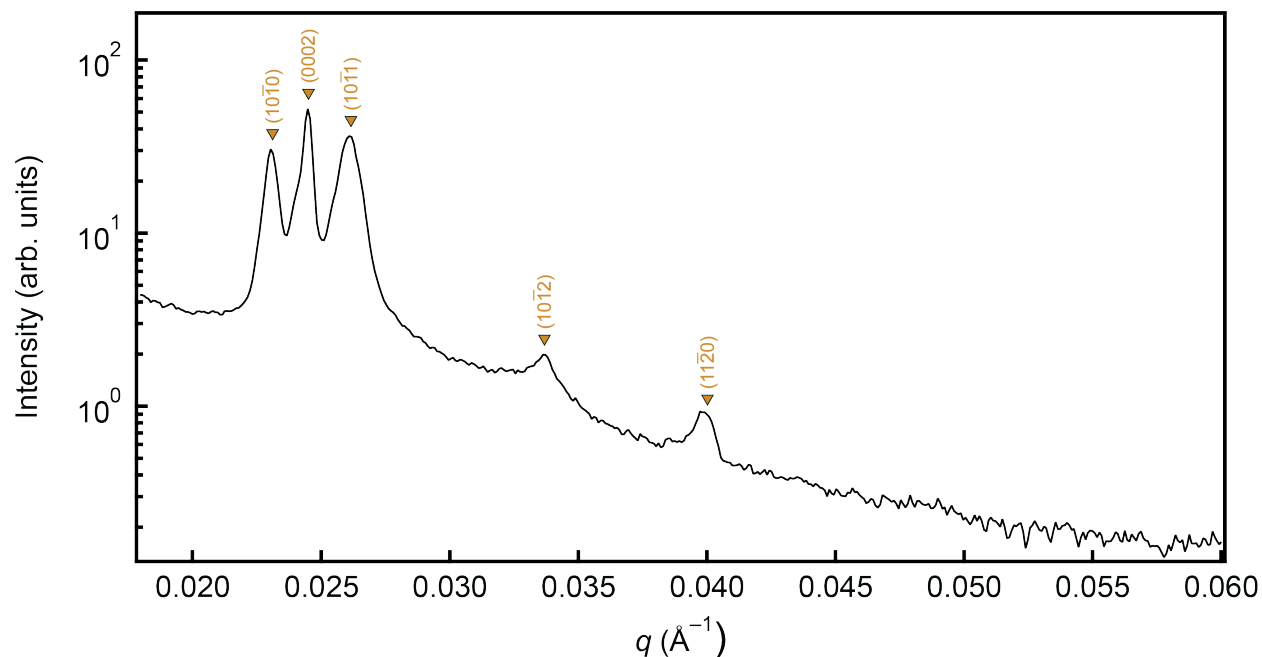


**Figure S5.** 1D SAXS traces collected from different locations on a SB3/SB5 blend with  $\phi_2 = 0.31$  following 66 h of annealing at 150 °C.

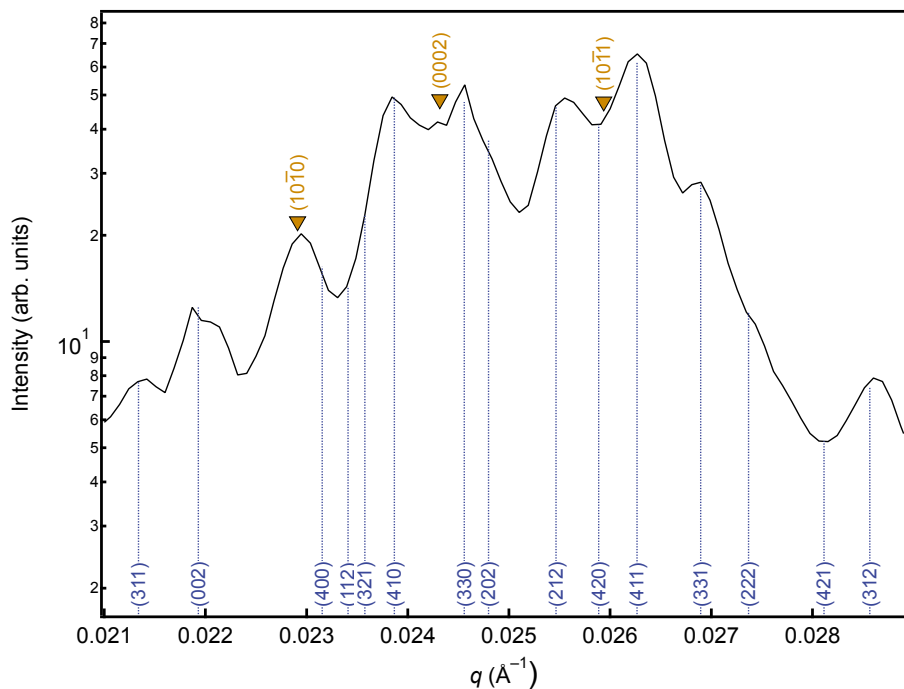


**Figure S6.** Transmission electron microscopy (TEM) micrograph collected from a thin ( $\sim 70$  nm) microtomed section of a SB3/SB5 blend with  $\phi_2 = 0.31$ . Prior to microtoming, the sample was annealed for 66 h at 150 °C, after which the SAXS patterns in Figures 5A,C and S5 were collected and the sample was vitrified in liquid nitrogen. The inset in the upper right corner is a Fourier transform of the image displaying the 6-fold rotation symmetry present over large areas.





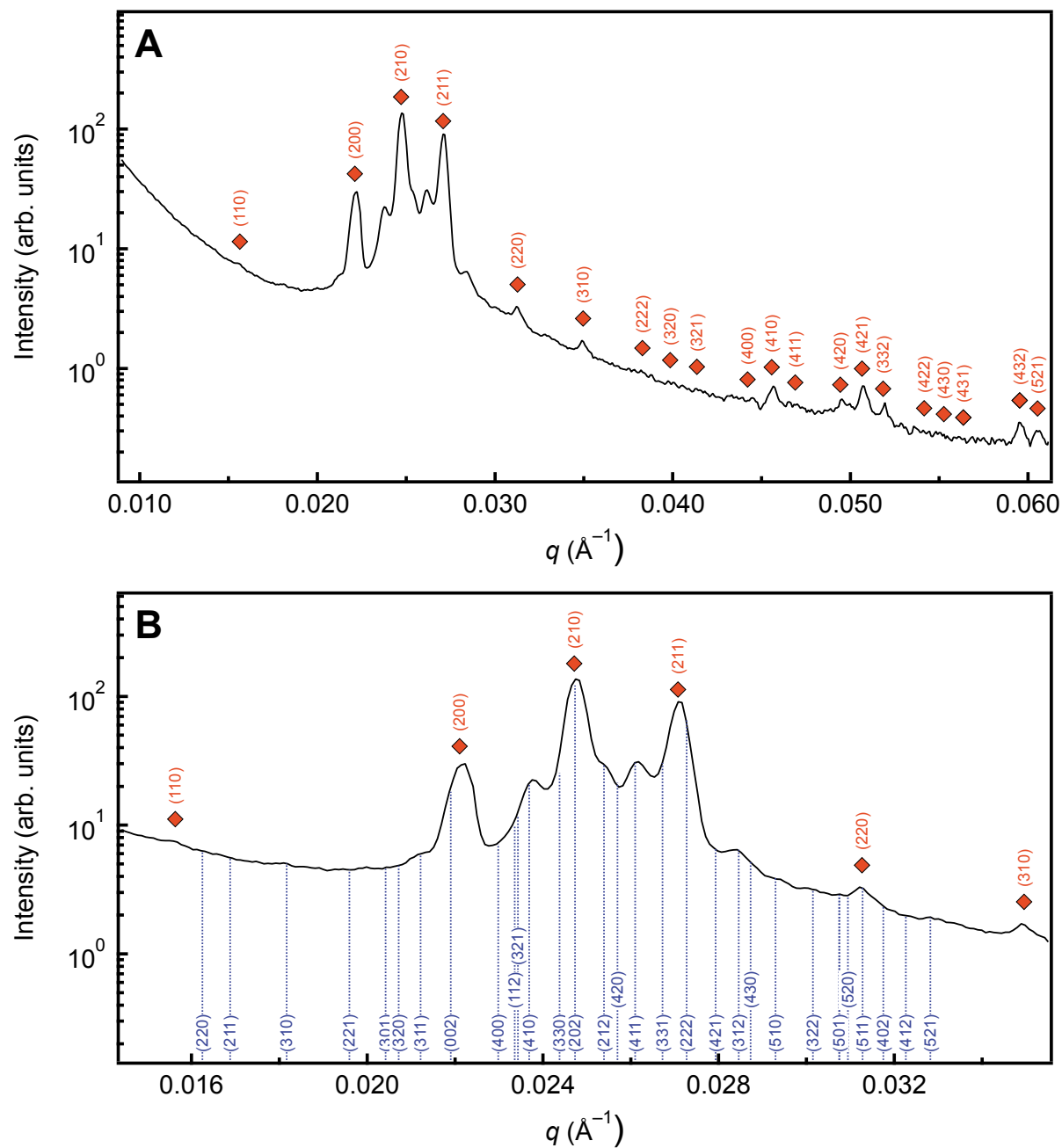
**Figure S7.** 1D SAXS trace collected from a SB3/SB5 blend with  $\phi_2 = 0.41$  following 92 h of annealing at 180 °C. The trace is indexed to a HCP phase. Additional patterns can be found in Figures 5 and S9. Indexing and residuals can be found in Table S4.



**Figure S8.** 1D SAXS trace collected from a SB3/SB5 blend with  $\phi_2 = 0.46$  after annealing at 180 °C for 66 h. Lines and inverted triangles denote peaks associated with the  $\sigma$  and HCP phases, respectively. Indexing and residuals can be found in Table S3.

**Table S3.** Observed and calculated peak positions for the  $\sigma$ /HCP phase coexistence evidenced in Figures 5 and S10; data were collected from SB3/SB5 blends with  $\phi_2 = 0.46$  at 180 °C. Peak positions for the HCP phase were calculated as  $q_{hkl} = 2\pi [(4/3) (h^2 + hk + k^2)/a^2 + l^2/c^2]^{1/2}$  based on  $P6_3/mmc$  space group symmetry with lattice parameters  $a = 316.7 \text{ \AA}$  and  $c = 517.1 \text{ \AA}$ . Peak positions for the  $\sigma$  phase were calculated as  $q_{hkl} = 2\pi [(h^2 + k^2)/a^2 + l^2/c^2]^{1/2}$  based on  $P4_2/mnm$  space group symmetry with lattice parameters  $a = 1085.5 \text{ \AA}$  and  $c = 573.0 \text{ \AA}$ .

Miller Indices ( <i>hkl</i> ) or ( <i>hkl</i> )	$q_{\text{obs}}$ (1/Å)	$q_{\text{calc}}$ (1/Å)	% Residual ( $\Delta q / q_{\text{calc}} \times 100$ )
<b>HCP</b>			
(10 $\bar{1}$ 0)	0.022946	0.022910	−0.16
(0002)	0.024293	0.024300	0.03
(10 $\bar{1}$ 1)	—	0.025933	−0.26
(10 $\bar{1}$ 2)	0.033539	0.033397	−0.43
(11 $\bar{2}$ 0)	0.039643	0.039682	0.10
(10 $\bar{1}$ 3)	—	0.043052	—
(20 $\bar{2}$ 0)	0.046016	0.045820	−0.43
(11 $\bar{2}$ 2)	0.046555	0.046531	−0.05
(20 $\bar{2}$ 1)	0.047363	0.047404	0.09
(0004)	0.048799	0.048599	−0.41
(20 $\bar{2}$ 2)	0.051761	0.051865	0.20
<b><math>\sigma</math></b>			
(211)	0.016932	0.016964	0.19
(310)	0.018279	0.018304	0.14
(221)	0.019625	0.019705	0.40
(301)	0.020613	0.020537	−0.37
(320)	—	0.020870	—
(311)	0.021331	0.021337	0.03
(002)	0.021959	0.021931	−0.13
(400)	—	0.023153	—
(112)	—	0.023409	—
(321)	—	0.023575	—
(410)	0.023844	0.023866	0.09
(330)	0.024562	0.024558	−0.02
(202)	0.024832	0.024799	−0.13
(212)	0.02555	0.025465	−0.33
(420)	—	0.025886	—
(411)	0.026268	0.026264	−0.01
(331)	0.026896	0.026895	−0.01



**Figure S9.** 1D SAXS trace collected from a SB3/SB5 blend with  $\phi_2 = 0.51$  after annealing at 180 °C for 66 h. Lines and diamonds denote peaks associated with  $\sigma$  and A15 phases, respectively. Indexing and residuals can be found in Table S6.

**Table S4.** Observed and calculated peak positions for the A15/ $\sigma$  phase coexistence evidenced in Figures 5 and S11; data were collected from SB3/SB5 blends with  $\phi_2 = 0.51$  at 180 °C. Peak positions for the A15 phase were calculated as  $q_{hkl} = 2\pi [(h^2 + k^2 + l^2) / a^2]^{1/2}$  based on  $Pm\bar{3}n$  space group symmetry with a lattice parameter of  $a = 568.5$  Å. Peak positions for the  $\sigma$  phase were calculated as  $q_{hkl} = 2\pi [(h^2 + k^2)/a^2 + l^2/c^2]^{1/2}$  based on  $P4_2/mnm$  space group symmetry with lattice parameters  $a = 1093.3$  Å and  $c = 573.7$  Å.

Miller Indices ( <i>hkl</i> )	$q_{obs}$ (1/Å)	$q_{calc}$ (1/Å)	% Residual ( $\Delta q / q_{calc} \times 100$ )
<b>A15</b>			
(110)	0.015586	0.015630	0.28
(200)	0.022228	0.022104	−0.56
(210)	0.024742	0.024713	−0.12
(211)	0.027165	0.027072	−0.34
(220)	0.031205	0.031260	0.18
(310)	0.034885	0.034950	0.19
(222)	0.038207	0.038285	0.20
(320)	0.039733	0.039849	0.29
(321)	0.041348	0.041353	0.01
(400)	0.0444	0.044208	−0.43
(410)	0.045657	0.045569	−0.19
(411)	0.047004	0.046890	−0.24
(420)	0.049517	0.049426	−0.18
(421)	0.050684	0.050647	−0.07
(332)	0.051941	0.051839	−0.20
<b><math>\sigma</math></b>			
(410)	0.023754	0.023695	−0.25
(330)	—	0.024382	—
(202)	—	0.024735	—
(212)	0.02546	0.025394	−0.26
(420)	—	0.025701	—
(411)	0.026178	0.026103	−0.29
(331)	—	0.026728	—
(222)	—	0.027275	—
(421)	—	0.027937	—
(312)	0.028422	0.028460	0.13
(430)	—	0.028734	—
(510)	0.02941	0.029303	−0.36
(322)	0.030038	0.030150	0.37

**Table S5.** Observed and calculated peak positions for the QC evidenced in Figures 5 and S12; data were collected from SB3/SB5 blends with  $\phi_2 = 0.51$  at 150 °C. Peak positions were calculated as described by Iwami and Ishimasa [1] based on a  $P12_6/mmc$  space group symmetry, a tiling edge length  $a = 572.9$  Å, and a periodicity of  $c = 575.6$  Å.

Miller Indices ( $a_1 a_2 a_3 a_4 a_5$ )	$q_{\text{obs}}$ (1/Å)	$q_{\text{calc}}$ (1/Å)	% Residual ( $\Delta q / q_{\text{calc}} \times 100$ )
(11000)	—	0.012233	—
(00002)	0.021780	0.021832	0.24
(12100)	0.023844	0.023633	−0.89
(01102)	0.024921	0.025026	0.42
(220 $\bar{1}$ 1)	0.025999	0.026032	0.13
(11102)	0.028063	0.027856	−0.74
(12202)	—	0.036531	—
(00004)	0.043664	0.043664	0.00
(01104)	0.045298	0.045345	0.10

**Table S6.** Observed and calculated peak positions for the A15 phase coexisting with HEX<sub>C</sub> evidenced in Figures 5 and S13; data was collected from SB3/SB5 blends with  $\phi_2 = 0.56$  at 180 °C. Peak positions for the A15 phase were calculated as  $q_{\text{hkl}} = 2\pi [(h^2 + k^2 + l^2) / a^2]^{1/2}$  based on  $Pm\bar{3}n$  space group symmetry with a lattice parameter of  $a = 577.0$  Å.

Miller Indices ( $hkl$ )	$q_{\text{obs}}$ (1/Å)	$q_{\text{calc}}$ (1/Å)	% Residual ( $\Delta q / q_{\text{calc}} \times 100$ )
(110)	—	0.015400	—
(200)	0.021753	0.021779	0.12
(210)	0.024357	0.024349	−0.03
(211)	0.026601	0.026673	0.27
(220)	—	0.030800	—

### Calculation of mean particle radii:

For a periodic particle packing, the mean particle radius  $\langle R \rangle$  can be calculated as:

$$\langle R \rangle = \left( \frac{3V_{UC}}{4\pi\rho_{P,UC}} \right)^{1/3} \quad (S1)$$

where  $V_{UC}$  is the unit cell volume, which can be readily determined *via* SAXS, and  $\rho_{P,UC}$  is the number of particles per unit cell set by the packing. In the most general form, unit cell volume can be calculated as:

$$V_{UC} = a b c (1 - \cos^2\alpha - \cos^2\beta - \cos^2\gamma + 2 \cos\alpha \cos\beta \cos\gamma)^{1/2} \quad (S2)$$

where  $a$ ,  $b$ , and  $c$  are lattice constants and the angles  $\alpha$ ,  $\beta$ , and  $\gamma$  are lattice parameters. These values are determined from SAXS from the relations:

$$q_{hkl,cubic} = 2\pi \left( \frac{(h^2 + k^2 + l^2)}{a^2} \right)^{1/2} \quad (S3)$$

$$q_{hkl,hexagonal} = 2\pi \left( \frac{4(h^2 + hk + k^2)}{3a^2} + \frac{l^2}{c^2} \right)^{1/2} \quad (S4)$$

$$q_{hkl,tetragonal} = 2\pi \left( \frac{(h^2 + k^2)}{a^2} + \frac{l^2}{c^2} \right)^{1/2} \quad (S5)$$

for cubic, hexagonal, or tetragonal lattices, where  $q_{hkl}$  is the scattering wavevector for plane  $(hkl)$ ,  $\alpha = \beta = \gamma = 90^\circ$  for a cubic or tetragonal lattice, and  $\alpha = \beta = 90^\circ$  and  $\gamma = 120^\circ$  for a hexagonal lattice.

From these relations,  $\langle R \rangle$  can be calculated for the phases observed in this work as:

$$\langle R \rangle_{BCC} = R_{BCC} = \frac{3^{1/3} 2^{1/2} \pi^{2/3}}{q_{110}} \quad (S6)$$

$$\langle R \rangle_{FCC} = R_{FCC} = \frac{3^{5/6} \pi^{2/3}}{2^{1/3} q_{111}} \quad (S7)$$

$$\langle R \rangle_{\text{HCP}} = R_{\text{HCP}} = \frac{2^{2/3} \pi^{2/3} (c/a)^{1/3}}{q_{10\bar{1}0}} \quad (\text{S8})$$

$$\langle R \rangle_{\sigma} = \frac{2\pi^{2/3}}{5^{1/3} (c/a)^{2/3} q_{002}} \quad (\text{S9})$$

$$\langle R \rangle_{\text{A15}} = \frac{3^{1/3} \pi^{2/3}}{2^{1/6} q_{110}} \quad (\text{S10})$$

$$\langle R \rangle_{\text{C14}} = \frac{2^{1/3} \pi^{2/3} (c/a)^{1/3}}{3^{1/3} q_{10\bar{1}0}} \quad (\text{S11})$$

Owing to a lack of translational symmetry,  $\langle R \rangle$  for a dodecagonal quasicrystal (QC) or a liquid-like packing (LLP) can only be estimated. For a QC, this is best done by taking the (00002) reflection as the  $\sigma$   $q_{002}$  peak owing to the close structural relationship between the two phases and the invariance of this reflection on transition to the  $\sigma$  phase.<sup>2,3</sup> For LLP the principal reflection can be taken as the  $q_{110}$  peak of the BCC phase by a similar argument. On calculating  $\langle R \rangle$ , the core radius can be calculated as:

$$\langle R_{\text{core}} \rangle = f_{\text{core}}^{1/3} \langle R \rangle \quad (\text{S12})$$

where  $f_{\text{core}}$  is volume fraction of the core domain assuming complete segregation of both blocks. This core radius can then be used to calculate the underlying spherical form factor.

A similar strategy can be used to calculate the radius of cylinders in the hexagonally-packed cylinder (HEX<sub>C</sub>) phase. However, the functional form changes slightly owing to periodicity in only two dimensions.  $\langle R \rangle$  is instead calculated as:

$$R_{\text{cyl}} = \left( \frac{A_{\text{UC}}}{\pi \rho_{\text{C,UC}}} \right)^{1/2} \quad (\text{S13})$$

where  $A_{UC}$  is the unit cell area, which can be readily determined *via* SAXS, and  $\rho_{C,UC} = 1$  is the number of cylinders per unit cell. The unit cell area can be calculated as:

$$A_{UC} = \frac{3^{1/2} a^2}{2} \quad (S14)$$

The lattice parameter  $a$  is determined from SAXS according to the relation:

$$q_{hk,hexagonal2D} = 2\pi \left( \frac{4(h^2 + hk + k^2)}{3a^2} \right)^{1/2} \quad (S15)$$

From these equations,  $\langle R \rangle$  can be calculated for the  $HEX_C$  phase as:

$$R_{cyl} = \frac{2^{3/2} \pi^{1/2}}{3^{1/4} q_{10}} \quad (S16)$$

On calculating  $R_{cyl}$ , the cylinder core radius can then be calculated as:

$$R_{core,cyl} = f_{core}^{1/2} R_{cyl} \quad (S17)$$

where  $f_{core}$  is again the volume fraction of the core domain assuming complete segregation of both blocks. This core radius can then be used to calculate the underlying cylindrical form factor.



## Dispersity:

Core block dispersity  $\bar{D} \approx 1.12$  was estimated for SB3/SB4 blends with  $\phi_2 = 0.80$  approximating the constituent diblock copolymers as monodisperse via the relation:

$$\bar{D} = \frac{M_w}{M_n} \quad (\text{S18})$$

where  $M_w$  and  $M_n$  are the weight- and number-average block molecular weights calculated as:

$$M_n = x_1 M_{n,1} + x_2 M_{n,2} \quad (\text{S19})$$

$$M_w = \frac{x_1 M_{n,1}^2 + x_2 M_{n,2}^2}{x_1 M_{n,1} + x_2 M_{n,2}} \quad (\text{S20})$$

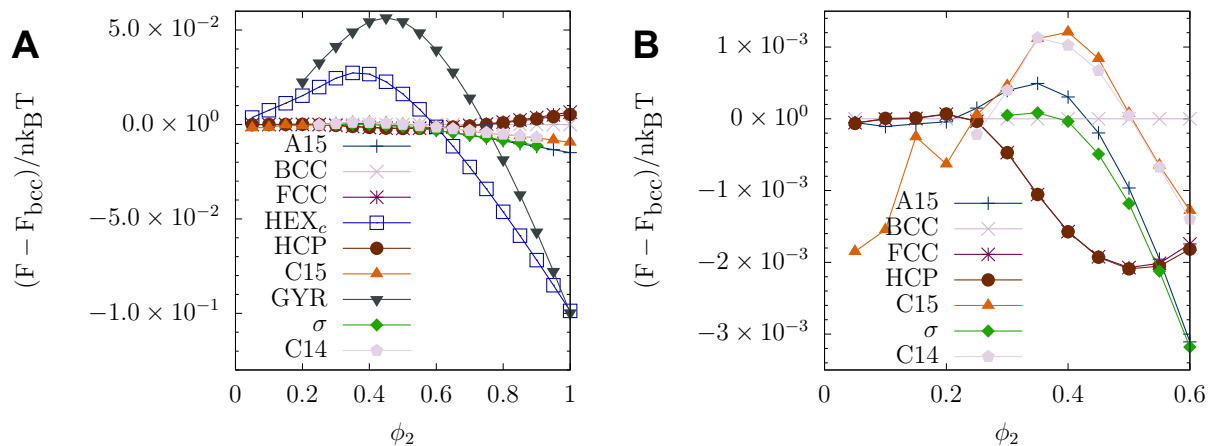
where values of  $M_n$  for each block and polymer can be found in Table S1 and  $x_i$  is the mole fraction of copolymer  $i$  in the core domain. A similar approach can be used to estimate the dispersity of the corona as  $\bar{D} \approx 1.001$ . Notably, this approach only provides an estimate and neglects the dispersity of each polymer, each determined via size exclusion chromatography (SEC) to be  $\bar{D} \approx 1.01$ .

To compare this value with previously investigated 1,4-polyisoprene-*block*-poly( $\pm$ -lactide) (IL) diblock copolymers, we assume a I precursor dispersity of 1.06, a reasonable estimate for low molecular weight I synthesized *via* anionic polymerization,<sup>4</sup> and independent block dispersities. Using the rule for the sum of variances of statistical distributions (*i.e.*,  $\bar{D}_{\text{PIPLA}} = w_{\text{PI}}^2 (\bar{D}_{\text{PI}} - 1) + w_{\text{PLA}}^2 (\bar{D}_{\text{PLA}} - 1) + 1$ , where  $w_i$  is the mass fraction of block  $i$ ), it can be found that many of the  $\sigma$ -forming IL diblock copolymers reported in the literature likely had core (L) block dispersities greater than the  $\bar{D}_{\text{core}} = 1.2$  we found in this work to be sufficient to drive  $\sigma$ -formation.<sup>5–12</sup> It should be noted that the accuracy of such calculations is limited by that of the assumed I precursor dispersity and the resolution of the instruments used for dispersity measurements.<sup>13</sup> However, there are several reasons one might anticipate a higher dispersity for the second block. First, ring-opening polymerization of lactides generally results in higher dispersity ( $\sim 1.1$ – $2.0$ ) than anionic polymerization of isoprene ( $< 1.1$ ).<sup>5,14,15</sup> Second, it is anticipated that some fraction of PI precursor will be present in the final diblock copolymer due to a combination of early termination,

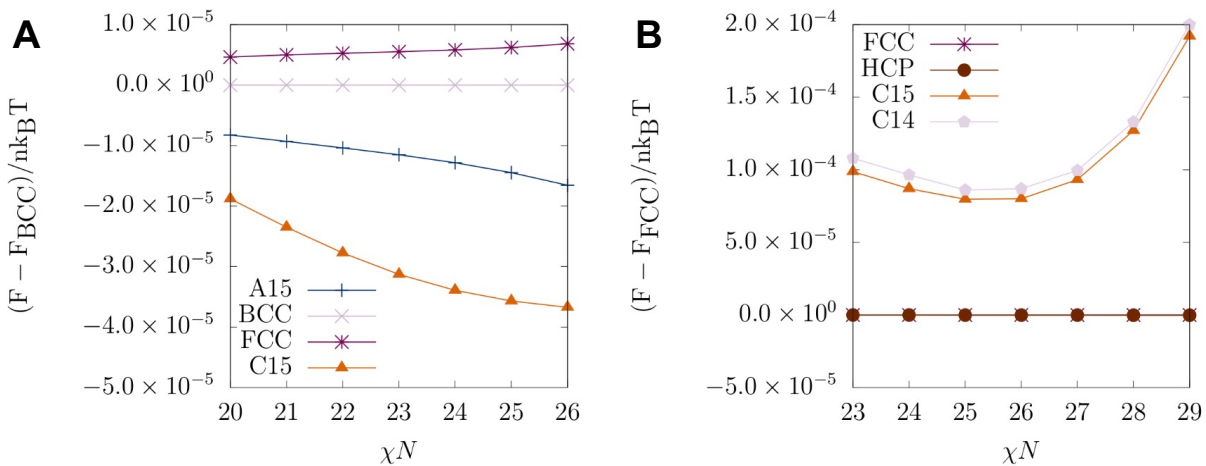
incomplete reinitiation, and challenges inherent to purification.<sup>13</sup> Third, when considering the low molecular weight of these polymers, generally characterized by an average block length of 45 isoprene and 6 lactide monomer units, it is clear that the loss or gain of only a few lactide monomer units over the course of the polymerization would have a dramatic effect on the block dispersity.

### Self-consistent mean-field theory (SCFT):

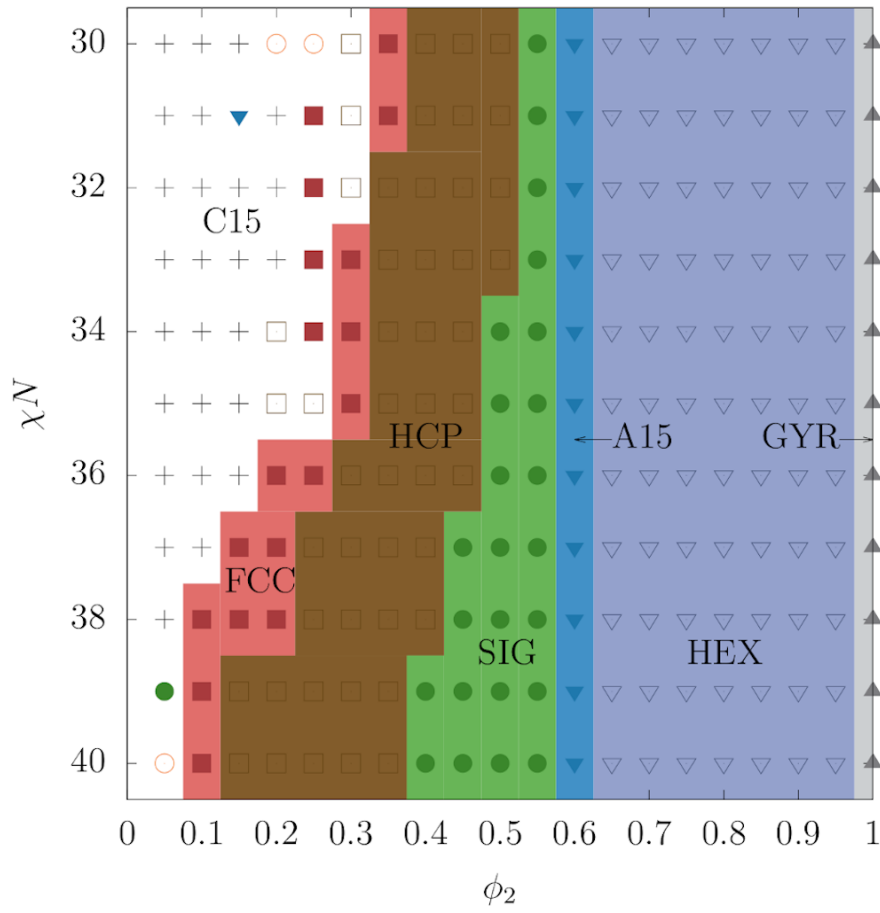
Calculations based on self-consistent mean-field theory were performed using the open-source Polymer Self-Consistent Field software package (PSCF).<sup>16</sup> We performed binary blend canonical ensemble calculations at  $N_2/N_1 = 1.4$ ,  $N_{B,1} = N_{B,2}$ ,  $f_{A,1} = 0.12$ ,  $f_{A,2} = 0.38$  and a conformational asymmetry of  $\varepsilon = b_B/b_S$  ( $v_S/v_B$ ) = 1.7. Note this differs modestly from experiments, where  $f_{A,2} = 0.388$ , but is within experimental error and, thus, has a negligible impact on the applicability of the calculations to the experimental results. We scanned a range of  $\chi N$  (20–30) and  $\phi_2$  (0–1), mimicking the parameter space explored experimentally. The candidate phases tested in the canonical ensemble calculations include the body-centered cubic (BCC), face-centered cubic (FCC), hexagonally close-packed (HCP), hexagonally-packed cylinder (HEX<sub>C</sub>), double gyroid (GYR), and disordered (DIS) phases as well as the Frank–Kasper (FK)  $\sigma$ , A15, C14, and C15 phases. SCFT calculations were performed using a grid size of  $64 \times 64 \times 64$  for the three-dimensional structures except for the  $\sigma$  and two-dimensional HEX<sub>C</sub> phases, which were performed at a grid size of  $96 \times 96 \times 48$  and  $64 \times 64$ , respectively. Calculations were performed with a contour length step size of  $\Delta s = 0.01$  and a convergence criterion of  $10^{-5}$  as defined by Arora *et al.*<sup>17</sup> As shown in Figure S10, the free energies were almost degenerate at  $\phi_2 = 0.15$  and 0.25. To better resolve the phase behavior at those compositions, we repeated the calculations using a more stringent convergence criterion of  $10^{-6}$ . Figure S11 shows the results under this stricter convergence criterion, revealing that C15 and HCP phases offer the lowest free energy at low  $\phi_2$ . Then, we performed grand canonical ensemble calculations between neighboring phases to resolve phase coexistence. Further calculation details can be found elsewhere.<sup>18</sup>



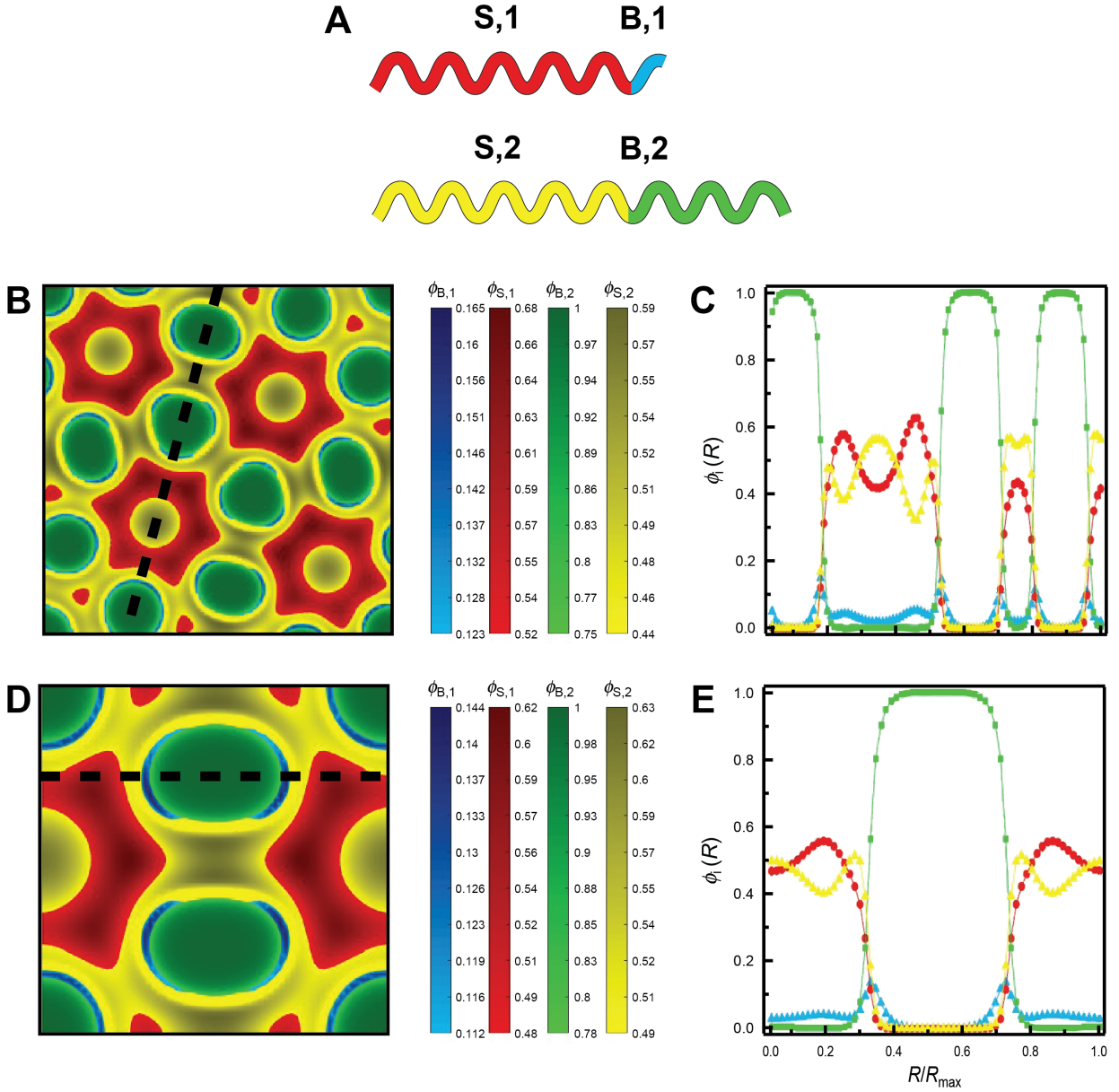
**Figure S10.** Normalized free energy relative to BCC *versus*  $\phi_2$  at  $\chi N = 28$  for (A) all studied phases and (B) only particle phases.



**Figure S11.** Normalized free energy relative to (A) BCC and (B) FCC *versus*  $\chi N$  at (A)  $\phi_2 = 0.15$  and (B)  $\phi_2 = 0.25$  for select phases under the more stringent convergence criterion of  $10^{-6}$ .



**Figure S12.** Binary blend phase diagram generated from canonical ensemble SCFT calculations over an extended range of  $\chi N$  ( $N_2/N_1 = 1.4$ ,  $N_{S,1} = N_{S,2}$ ,  $f_{B,1} = 0.12$ , and  $f_{B,2} = 0.38$ ). Symbols correspond to the double gyroid (gray ▲), hexagonally-packed cylinder (purple ▽), A15 (blue ▼),  $\sigma$  (green ●), hexagonal close-packed (□), face-centered cubic (red ■), C14 (orange o), and C15 phases (black +).



**Figure S13.** (B,D) Composition maps and (C,E) 1D composition profiles for the (B,C)  $\sigma$  and (D,E) A15 phases in the (001) planes. Data were calculated *via* SCFT for SB3/SB5 blends with (B,C)  $\phi_2 = 0.55$  and (D,E) 0.60 at  $\chi\langle N \rangle = 30$ .  $\phi_i$  is the volume fraction of block  $i$  at each position.  $R/R_{\max}$  is the nondimensional distance along the dashed lines in (B,D), where 0 corresponds to the (C) bottom or (E) left edge of the composition map. The schematic in (A) shows the relative copolymer block lengths. Blue and red shading correspond to the B and S blocks of SB3, whereas green and yellow shading correspond to the B and S blocks of SB5, respectively.

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