Supporting Information for the manuscript:

Constructing Hyperbolic Metamaterials with Arbitrary Medium

Li-Zheng Yin, Feng-Yuan Han, Jin Zhao, Di Wang, Tie-Jun Huang, and Pu-Kun Liu*

State Key Laboratory of Advanced Optical Communication Systems and Networks,

Department of Electronics, Peking University, Beijing 100871, China

*Correspondence to pkliu@pku.edu.cn

1. Proof of the inexistence of structure-induced TE SSP

We derive the field distribution under the same configuration with Fig. 1(a) in the manuscript. The corresponding Borgnis potential functions U = 0 and V has the form

$$V = \begin{cases} A_1 e^{-jk_z z} e^{-jk_{1y} y} \sin(k_{mx} x), y > 0 \\ A_2 e^{-jk_z z} e^{-jk_{2y} y} \sin(k_{mx} x), y < 0 \end{cases} \tag{S1}$$

where k_{1y} and k_{2y} represent the normal wave vector components in the region of $y > 0$ and $y < 0$, respectively. A_1 and A_2 are their corresponding amplitude. The definitions of all the other parameters are the same as the TM counterpart in the original manuscript. The boundary conditions between the two regions can be expressed as

$$E_{1x} = E_{2x} |_{y=0}, \tag{S2a}$$

$$H_{2x} - H_{1x} = \sigma_e E_z |_{y=0} . \tag{S2b}$$

Substituting Equations S1 into Equation S2,

$$\frac{\partial V}{\partial y} |_{y=0^+} = \frac{\partial V}{\partial y} |_{y=0^-}, \tag{S3a}$$

$$\frac{\partial^2 V}{\partial x \partial z}|_{y=0^-} - \frac{\partial^2 V}{\partial x \partial z}|_{y=0^+} = \sigma_e(\frac{\partial^2 U}{\partial z^2} - \varepsilon_r k_0^2 U)|_{y=0} . \quad \text{(S3b)}$$

By simplifying Equation S3, the dispersion relation finally arrives at

$$k_{1y} = k_{2y}, \quad \text{(S4a)}$$

$$A_1 = A_2. \quad \text{(S4b)}$$

Equation S4 tells that the electric and magnetic fields are completely continuous at the interface of $y = 0$, which indicates that TE mode structure-induced SSP cannot exist in the two-parallel-plate waveguides.

2. Derivation of Meander-line PMs Dispersion

In this work, we use the meander line as a practical realization of the PM whose structure schematic is illustrated in Figure S1b. Before solving the mode characteristics and dispersion relation of the meander line, firstly, we analyze an infinite array of parallel strips with period p and width w. The EMWs propagating on the infinite periodic strips are TEM with respect to the direction of the strips (x direction).[1] Therefore, the potential function $\Phi(x, y, z)$ satisfies the Helmholtz equation

$$\nabla^2\Phi + k^2\Phi = 0, \quad \text{(S5)}$$

$k = \sqrt{\varepsilon_r} k_0$ represents the wave vector of the Eigen electromagnetic modes, where ε_r is the permittivity of the dielectric surrounding the infinite periodic strips and k_0 is the wave vector in free space. Considering the periodicity in y direction, the general solutions of Equation S5 can be written as

$$\Phi(x, y, z) = \sum_{\xi} A_{\xi} e^{-j\beta_{\xi} z} e^{\pm jkx}, \quad \text{(S6)}$$

where $\beta_{\xi} = \frac{\varphi + 2\pi\xi}{p}$, ζ represents the order of the harmonics, p represents spatial period and φ represents the phase between the adjacent spatial units. As indicated in Figure S1b, the meander line can be viewed as a part of the array of parallel strips, with the ends of the adjacent strips shorted alternatively at the specified position. In this work, we only analyze the fundamental mode with ζ equals zero. Therefore, considering the multiple reflections on each strip, the voltage and current in the m-th spatial period have the form

$$U_m = (a\cos kx + b\sin kx)e^{-jm\varphi}, \tag{S7a}$$

$$I_m = \frac{j}{Z(\varphi)}(b\cos kx - a\sin kx)e^{-jm\varphi}, \tag{S7b}$$

in which $Z(\varphi)$ is the wave impedance of the infinite periodic strips and is the function of φ. If each spatial period consists of N conductors, then, the corresponding voltage and current can be further expressed as

$$U_m = \sum_{n=0}^{N-1}(a_n \cos kx + b_n \sin kx)e^{-jm\phi_n}, \tag{S8a}$$

$$I_m = \sum_{n=0}^{N-1}\frac{j}{Z(\phi_n)}(b_n \cos kx - a_n \sin kx)e^{-jm\phi_n}. \tag{S8b}$$

In this case, the spatial period of the strips $ps = \frac{p}{N}$, so the phase difference between the adjacent strips is $\phi_n = \beta_0 p + \frac{2n\pi}{N}$, $n = 0,1,2,\cdots N-1$. Considering the alternative shorted endpoints of the periodic strips, the boundary conditions of the meander line can be expressed as

$$U_{m-1} = U_m \,|_{x=endpoint}, \tag{S9a}$$

$$I_{m-1} = -I_m \,|_{x=endpoint} . \tag{S9b}$$

Substituting Equation S8 into Equation S9, we get the system of equations

$$\begin{bmatrix} (e^{j\phi}-1)\cos\frac{kh}{2} & (1-e^{j\phi})\sin\frac{kh}{2} & (e^{j\phi}+1)e^{-jm\pi}\cos\frac{kh}{2} & (e^{j\phi}+1)e^{-jm\pi}\sin\frac{kh}{2} \\ (e^{-j\phi}-1)\cos\frac{kh}{2} & (e^{-j\phi}-1)\sin\frac{kh}{2} & -(e^{-j\phi}+1)e^{-jm\pi}\cos\frac{kh}{2} & -(e^{-j\phi}+1)e^{-jm\pi}\sin\frac{kh}{2} \\ \frac{(1+e^{j\phi})}{Z(\phi)}\sin\frac{kh}{2} & \frac{(1+e^{j\phi})}{Z(\phi)}\cos\frac{kh}{2} & -\frac{(e^{j\phi}-1)}{Z(\phi+\pi)}e^{-jm\pi}\sin\frac{kh}{2} & -\frac{(e^{j\phi}-1)}{Z(\phi+\pi)}e^{-jm\pi}\cos\frac{kh}{2} \\ \frac{(e^{-j\phi}+1)}{Z(\phi)}\sin\frac{kh}{2} & -\frac{(e^{-j\phi}+1)}{Z(\phi)}\cos\frac{kh}{2} & -\frac{(e^{-j\phi}-1)}{Z(\phi+\pi)}e^{-jm\pi}\sin\frac{kh}{2} & \frac{(e^{-j\phi}-1)}{Z(\phi+\pi)}e^{-jm\pi}\cos\frac{kh}{2} \end{bmatrix} \begin{bmatrix} a_1 \\ b_1 \\ a_2 \\ b_2 \end{bmatrix} = 0. \tag{S10}$$

By solving Equation S10, the dispersion of the meander line can be obtained

$$\frac{1+\cos^2 kh}{1-\cos^2 kh} = \frac{1}{2\sin^2\phi}\left[\frac{Z(\phi)}{Z(\phi+\pi)}(1-\cos\phi)^2 + \frac{Z(\phi+\pi)}{Z(\phi)}(1+\cos\phi)^2\right]. \tag{S11}$$

In this work, the wave impedance of the TEM mode equals that of the dielectric. Therefore, the dispersion relation Equation S11 can be further simplified as

$$\phi = \pm kh. \tag{S12}$$

Equation S12 indicates that the phase difference between the adjacent unit cells is mainly determined by the length of the strips. This equation can also be interpreted that the EMW or the current is propagation along the strip uniformly at the speed of light in the dielectric. The effective conductance of the meander line can therefore be determined as

$$\sigma = \frac{2\omega\varepsilon_0\varepsilon_r}{j\sqrt{(kh/p_s)^2 - \varepsilon_r k_0^2}}. \tag{S13}$$

Under this circumstance, arbitrary effective conductance σ can be obtained by adjusting the length and period of the strips.

3. Numerical Method to Obtain the Effective Conductance

It should be noted that Equation S12 is not strictly accurate because the Equation S9 is only a set of approximate boundary conditions in which the precondition $p << b$ must be met. Besides, the finite specific thickness of the HMMs also limits the length of the strips, which further restricts the realizable upper limit of the effective conductance. Therefore, in this work, we use the numerical method to accurately obtain the relation between the effective conductance and structure parameters. The dispersion of the PMs

$$\sigma = \frac{2\omega\varepsilon_0\varepsilon_r}{j\sqrt{k_z^2 - \varepsilon_r k_0^2}}. \tag{S14}$$

tell us the effective conductance is determined by the corresponding propagation constant k_z, which can be calculated by the Eigenvalue solver of COMSOL Multiphysics 5.4.

In order to obtain a larger range of effective conductance with specified strip width b, we turn eyes to the width w and period p_s of the strips. It's evident that w and p_s are a negative correlation with effective conductance. By confirming $p = 4w$, we calculate

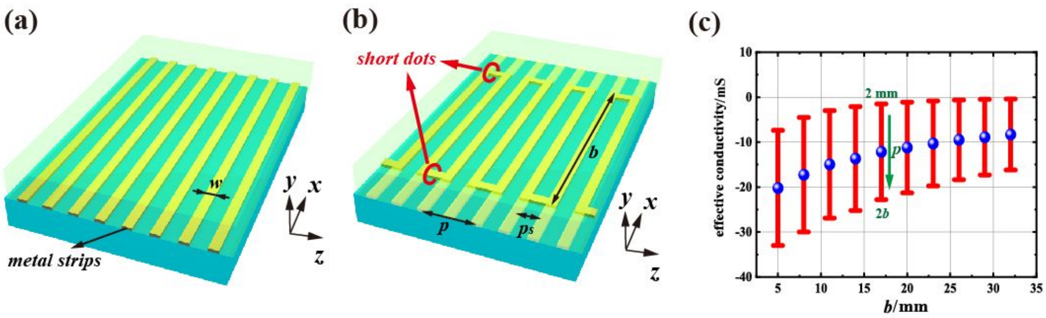

Figure S1. Schematic of the (a) multi-conductor transmission lines and (b) meander lines. (c) The effective conductance of the meander lines versus strip width b. At each specified b, the red line represents the dynamic range of the effective conductance when p changes from 2 mm to 2*b*.

the practical range of the effective conductance at each specified strip width *b* when *p* changes from 2 mm to 2*b*. Figure S1c illustrates the corresponding results with strip width *b* ranging from 5 mm to 32 mm, from which we can find that a large range of the effective conductance can also be obtained by only changing the period *p*.

Reference:

1. Weiss, J. A. Dispersion and field analysis of a microstrip meander-line slow-wave structure. *IEEE Trans. Microwave Theory Tech.* **1974,** *22* (12), 1194-1201.