Microstreaming inside model cells induced by ultrasound and microbubbles

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SUPPORTING INFORMATION

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Theory of acoustic streaming

This section outlines the theoretical foundation of acoustic streaming, which served as a basis for the numerical modelling results presented in the manuscript.

Throughout, homogeneous isotropic fluids were assumed, for which the continuity and momentum equations for the fluid motion are described below. Bold and normal-emphasis fonts are used to represent vector and scalar quantities, respectively.

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{u}) = 0, \tag{1a}$$

$$\rho\left(\frac{\partial \boldsymbol{u}}{\partial t} + \boldsymbol{u} \cdot \nabla \boldsymbol{u}\right) = -\nabla p + \mu \nabla^2 \boldsymbol{u} + \left(\mu_b + \frac{1}{3}\mu\right)\nabla\nabla \cdot \boldsymbol{u}, \tag{1b}$$

where ρ is the fluid density, t is time, $\boldsymbol{u}$ is the fluid velocity, p is the pressure, and μ and μ_b are respectively the dynamic and bulk viscosity coefficients of the fluid.

Taking the first and second order into account, we write the perturbation series of fluid density, pressure, and velocity:[1]

$$\rho = \rho_0 + \rho_1 + \rho_2, \tag{2a}$$

$$p = p_0 + p_1 + p_2, \tag{2b}$$

$$\boldsymbol{u} = \boldsymbol{u_1} + \boldsymbol{u_2}, \tag{2c}$$

where the subscripts 0, 1 and 2 represent the static (absence of sound and fluid), first-order and second-order quantities, respectively. Substituting equations (2) into equations (1) and considering the equations to the first-order, equations (1) for solving the first-order acoustic velocity take the form,

$$\frac{\partial \rho_1}{\partial t} + \rho_0 \nabla \cdot \boldsymbol{u_1} = 0, \tag{3a}$$

$$\rho_0 \frac{\partial \boldsymbol{u_1}}{\partial t} = -\nabla p_1 + \mu \nabla^2 \boldsymbol{u_1} + \left(\mu_b + \frac{1}{3}\mu\right)\nabla\nabla \cdot \boldsymbol{u_1}. \tag{3b}$$

Repeating the above procedure, considering the equations to the second-order and taking the time average of equations (1) using equations (2), the continuity and momentum equations for solving the second-order time-averaged acoustic streaming velocity can be turned into

$$\nabla \cdot \overline{\rho_1 \boldsymbol{u_1}} + \rho_0 \nabla \cdot \overline{\boldsymbol{u_2}} = 0, \tag{4a}$$

$$-\nabla \overline{p_2} + \mu \nabla^2 \overline{\boldsymbol{u_2}} + \left(\mu_b + \frac{1}{3}\mu\right)\nabla\nabla \cdot \overline{\boldsymbol{u_2}} - \boldsymbol{F} = 0, \tag{4b}$$

$$\boldsymbol{F} = -\rho_0 \overline{\boldsymbol{u_1} \nabla \cdot \boldsymbol{u_1} + \boldsymbol{u_1} \cdot \nabla \boldsymbol{u_1}}, \tag{4c}$$

where the upper bar denotes a time-averaged value and $\boldsymbol{F}$ is the Reynolds stress force.[2]

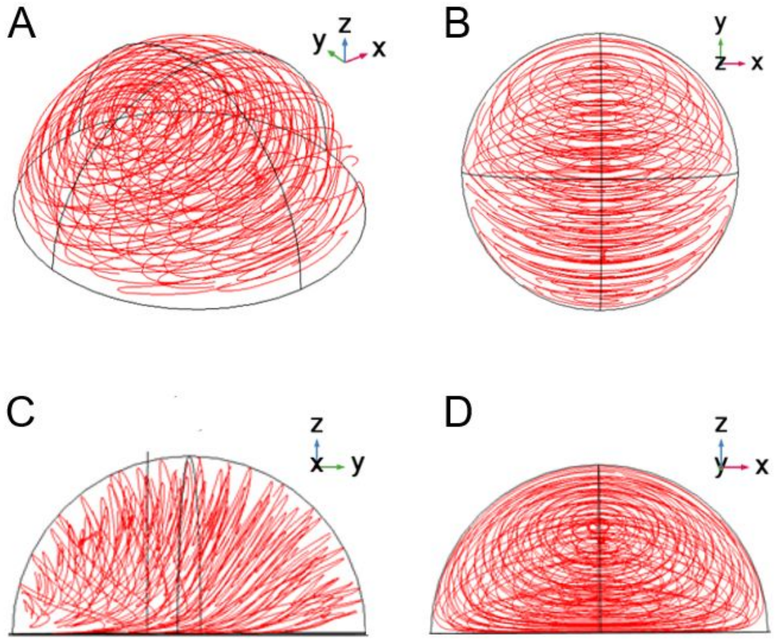

Figure 1 modelled flow velocity vector fields in vesicles near the side boundary (vesicle 1, y-coordinate of -135 μm): (a) three-dimensional view; (b) top view (xy); (c) side view (yz); and (d) front view (xz).

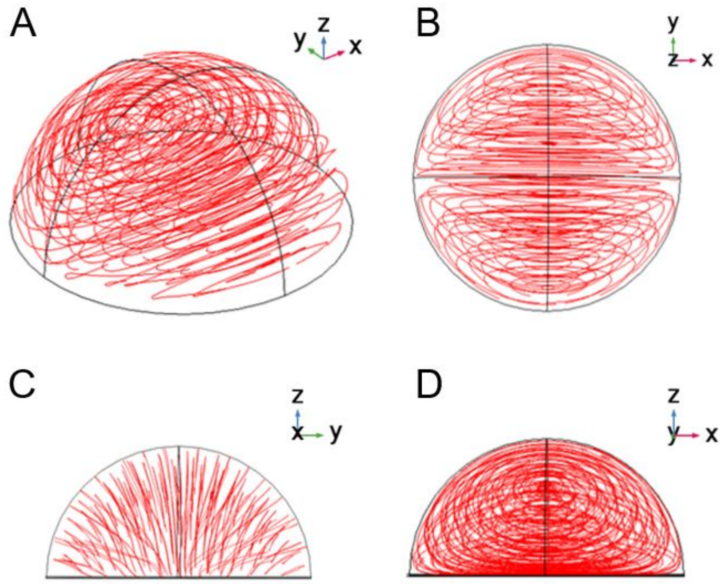

Figure 2 modelled flow streamlines in vesicles far from the side boundary (vesicle 2, y-coordinate of 0): (a) three-dimensional view; (b) top view (xy); (c) side view (yz); and (d) front view (xz).

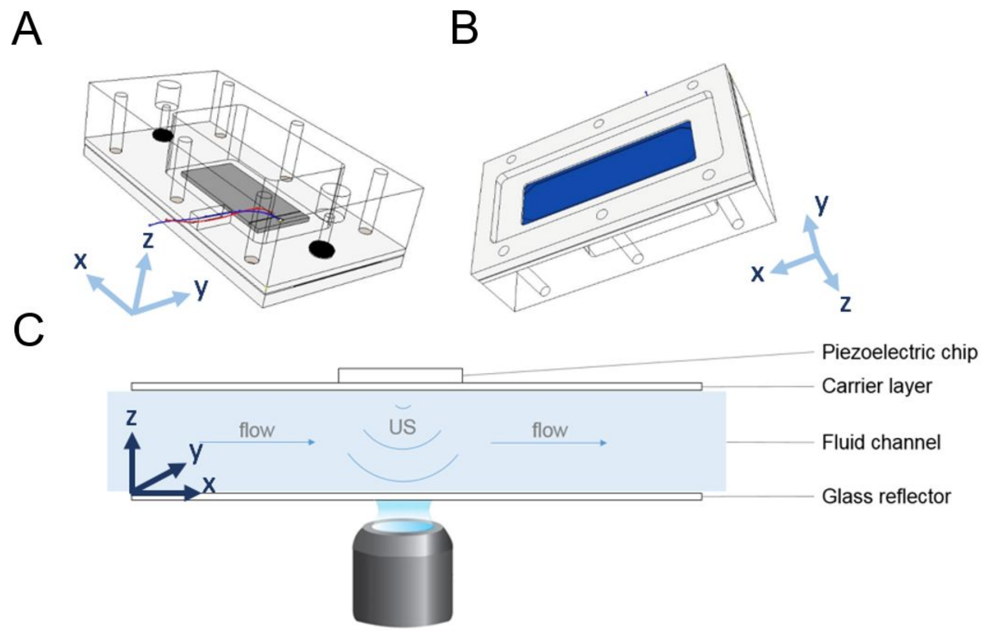

Figure 3 - A. Top view of the acoustofluidic device, showing the grey piezoelectric chip with electrical wires. B. Bottom view of the device showing the glass slide (in blue) that encloses the fluid channel. C. Side view of the channel, showing the device geometry and the constitutive layers. Not to scale.

References

1 H. Bruus, *Lab Chip*, 2012, **12**, 20–28.
2 S. J. Lighthill, *J. Sound Vib.*, 1978, **61**, 391–418.