Supporting Information

Theoretical analysis of sessile evaporating droplet on curved substrate with interfacial cooling effect

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1. The toriodal coordinate system

For droplet with low Bond number, the gravitational effect can be ignored, hence the droplet have the shape of spherical cap, here the curved substrate is also assumed to have the shape of spherical cap, hence their boundaries can be exactly mapped in toroidal coordinates(α, β), as shown in Figure S1. θ_{CA} is the contact angle of droplet with substrate surface, θ_{Sub} is the tangential angle of curved substrate over the horizontal bottom, $\theta = \theta_{\mathrm{CA}} + \theta_{\mathrm{Sub}}$ is the cutting angle of droplet edge over the horizontal substrate bottom. The bottom of the substrate is at the temperature T_{W}, heat is transferred to the droplet surface for evaporation through substrate and droplet. The ambient temperature and vapor concentration are T_{∞} and C_{∞} respectively.

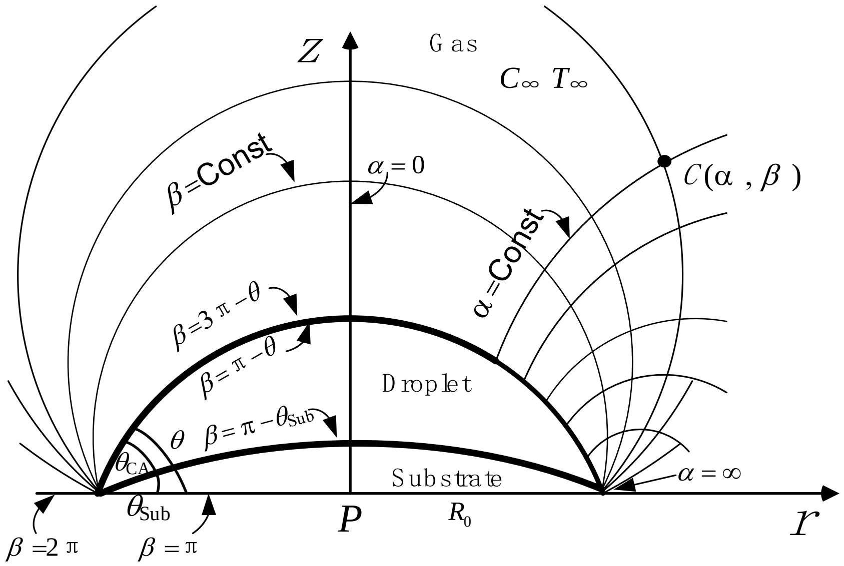

Figure S1. Schematic diagram of a sessile evaporating droplet on curved substrate in toroidal coordinate

The relationship between the toroidal coordinate (α, β) and the cylindrical coordinate (r, z) is shown below

$$r / \sinh \alpha = z / \sin \beta = R(\cosh \alpha - \cos \beta)^{-1} \tag{1}$$

where R is base radius of droplet.

The convective heat transfer inside the droplet and the vapor is ignored, hence both heat transfer and vapor transfer are diffusion-controlled, the temperature and vapor concentration are governed by the Laplace equation $\nabla^2 T = 0$ and $\nabla^2 C = 0$.

2. The boundary conditions

The boundary conditions of vapor concentration and temperature are as follows:

For vapor region around the droplet ($0 \le \alpha < \infty, 2\pi \le \beta \le 3\pi - \theta$)

(1) In the region far from the droplet, temperature and vapor concentration are T_∞, C_∞ respectively

(2) At the axis of symmetry:

$$(\partial C(\alpha, \beta) / \partial \alpha)_{\alpha=0} = 0$$

(3) At the gas-solid interface, no penetration for vapor into the solid substrate:

$$(\partial C(\alpha,\beta)/\partial\beta)_{\beta=2\pi}=0$$

(4) At the gas-liquid interface, the heat transfer is coupled with the evaporative mass transfer, i.e. evaporative cooling effect is considered.

$$(q(\alpha))_{\beta=\pi-\theta}=L(J(\alpha))_{\beta=3\pi-\theta}$$

where $q(\alpha)$ is heat flux, L is liquid latent heat of vaporization, $J(\alpha)$ is evaporative flux.

For temperature region within the droplet ($0\le\alpha<\infty,\pi-\theta\le\beta\le\pi-\theta_{\mathrm{Sub}}$)

(1) At the axis of symmetry:

$$(\partial T_{\mathrm{L}}(\alpha,\beta)/\partial\alpha)_{\alpha=0}=0$$

(2) At the solid−liquid interface, the heat transfer inside the substrate and droplet is coupled together, hence there is no temperature jump across the interface.

$$T_{\mathrm{L}}(\alpha,\pi-\theta_{\mathrm{Sub}})=T_{\mathrm{S}}(\alpha,\pi-\theta_{\mathrm{Sub}})$$

The heat flux is identical from both sides

$$\frac{(\cosh\alpha-\cos\beta)}{R}\frac{\partial T_{\mathrm{L}}(\alpha,\beta)}{\partial\beta}\Big|_{\beta=\pi-\theta_{\mathrm{Sub}}}=k_R\frac{(\cosh\alpha-\cos\beta)}{R}\frac{\partial T_{\mathrm{S}}(\alpha,\beta)}{\partial\beta}\Big|_{\beta=\pi-\theta_{\mathrm{Sub}}}$$

where $k_{\mathrm{R}}=k_{\mathrm{S}}/k_{\mathrm{L}}$ is relative thermal conductivity of substrate and droplet.

For temperature region within the substrate ($0\le\alpha<\infty,\pi-\theta_{\mathrm{Sub}}\le\beta\le\pi$)

(1) At the axis of symmetry:

$$(\partial T_{\mathrm{S}}(\alpha,\beta)/\partial\alpha)_{\alpha=0}=0$$

(2) On the bottom, the temperature is constant

$$T_{\mathrm{S}}(\alpha,\pi)=T_{\mathrm{W}}$$

3. The vapor concentration field and temperature field

The above boundary conditions are applied to solve the governing equation

$\nabla^2 T=0$ and $\nabla^2 C=0$. According to Nguyen et al.[24], the vapor concentration and temperature field can be obtained in the toroidal coordinate:

Vapor concentration outside the droplet:

$$C(\alpha,\beta)=C_{\infty}+\sqrt{\cosh\alpha-\cos\beta}\int_0^{\infty}E_{\mathrm{C}}^{*}(\tau)P_{i\tau-0.5}(\cosh\alpha)\cosh[(2\pi-\beta)\tau]d\tau \tag{2}$$

Temperature field inside the droplet:

$$T_{\mathrm{L}}(\alpha,\beta)=T_{\mathrm{W}}+\sqrt{\cosh\alpha-\cos\beta}\int_0^{\infty}P_{i\tau-0.5}(\cosh\alpha)[M(\tau)\cosh(\tau\beta)+N(\tau)\sinh(\tau\beta)]d\tau \tag{3}$$

Temperature field inside the substrate

$$T_{\mathrm{S}}(\alpha,\beta)=T_{\mathrm{W}}+\sqrt{\cosh\alpha-\cos\beta}\int_0^{\infty}E_{\mathrm{S}}^{*}(\tau)P_{i\tau-0.5}(\cosh\alpha)\sinh[(\pi-\beta)\tau]d\tau \tag{4}$$

where τ , $P_{i\tau-0.5}(\cosh\alpha)$ are the integration dummy, Legendre functions of the first kind ,respectively, T_{W} is the temperature of the substrate bottom, $E_{\mathrm{C}}^{*}(\tau)$ and $E_{\mathrm{S}}^{*}(\tau)$ are functions of the integration dummy, and independent of the toroidal coordinates α and β.

At the solid-liquid interface, according to $T_{\mathrm{L}}(\alpha,\pi-\theta_{\mathrm{Sub}})=T_{\mathrm{S}}(\alpha,\pi-\theta_{\mathrm{Sub}})$, the following can be obtained as

$$M(\tau)\cosh[\tau(\pi-\theta_{\mathrm{Sub}})]+N(\tau)\sinh[\tau(\pi-\theta_{\mathrm{Sub}})]=E_{\mathrm{S}}^{*}(\tau)\sinh(\theta_{\mathrm{Sub}}\tau) \tag{5}$$

The temperature gradient at the liquid side:

$$\begin{aligned}&\frac{\partial T_{\mathrm{L}}(\alpha,\beta)}{\partial\beta}\Big|_{\beta=\pi-\theta_{\mathrm{Sub}}}=\\&\frac{\sin\theta_{\mathrm{Sub}}}{2\sqrt{\cosh\alpha+\cos\theta_{\mathrm{Sub}}}}\int_0^{\infty}P_{i\tau-0.5}(\cosh\alpha)\{M(\tau)\cosh[\tau(\pi-\theta_{\mathrm{Sub}})]+N(\tau)\sinh[\tau(\pi-\theta_{\mathrm{Sub}})]\}d\tau+\\&\sqrt{\cosh\alpha+\cos\theta_{\mathrm{Sub}}}\int_0^{\infty}\tau P_{i\tau-0.5}(\cosh\alpha)\{M(\tau)\sinh[\tau(\pi-\theta_{\mathrm{Sub}})]+N(\tau)\cosh[\tau(\pi-\theta_{\mathrm{Sub}})]\}d\tau\end{aligned} \tag{6}$$

The temperature gradient at the solid side:

$$\frac{\partial T_{\mathrm{S}}(\alpha,\beta)}{\partial\beta}\Big|_{\beta=\pi-\theta_{\mathrm{Sub}}}=\frac{\sin\theta_{\mathrm{Sub}}}{2\sqrt{\cosh\alpha+\cos\theta_{\mathrm{Sub}}}}\int_0^{\infty}P_{i\tau-0.5}(\cosh\alpha)E_{\mathrm{S}}^{*}(\tau)\sinh(\theta_{\mathrm{Sub}}\tau)d\tau+ \\ \sqrt{\cosh\alpha+\cos\theta_{\mathrm{Sub}}}\int_0^{\infty}(-\tau)P_{i\tau-0.5}(\cosh\alpha)E_{\mathrm{S}}^{*}(\tau)\cosh(\theta_{\mathrm{Sub}}\tau)d\tau \tag{7}$$

According to

$$\frac{(\cosh\alpha-\cos\beta)}{R}\frac{\partial T_{\mathrm{L}}(\alpha,\beta)}{\partial\beta}\Big|_{\beta=\pi-\theta_{\mathrm{Sub}}}=k_{\mathrm{R}}\frac{(\cosh\alpha-\cos\beta)}{R}\frac{\partial T_{\mathrm{S}}(\alpha,\beta)}{\partial\beta}\Big|_{\beta=\pi-\theta_{\mathrm{Sub}}}$$

The following can be obtained

$$\int_0^{\infty}\tau P_{i\tau-0.5}(\cosh\alpha)\{M(\tau)\sinh[\tau(\pi-\theta_{\mathrm{Sub}})]+N(\tau)\cosh[\tau(\pi-\theta_{\mathrm{sub}})]+K_{\mathrm{R}}E_{\mathrm{S}}^{*}(\tau)\cosh(\theta_{\mathrm{Sub}}\tau)\}d\tau \\ =(K_{\mathrm{R}}-1)\frac{\sin\theta_{\mathrm{Sub}}}{2(\cosh\alpha+\cos\theta_{\mathrm{Sub}})}\int_0^{\infty}P_{i\tau-0.5}(\cosh\alpha)E_{\mathrm{S}}^{*}(\tau)\sinh(\theta_{\mathrm{Sub}}\tau)d\tau \tag{8}$$

Here the followings are defined as

$$F(\theta,\tau)=\int_0^{\infty}\frac{\sinh\alpha}{(\cosh\alpha+\cos\theta)^{3/2}}P_{i\tau-0.5}(\cosh\alpha)d\alpha=\sin(\theta\tau)/[\sinh(\pi\tau)\sin\theta] \tag{9}$$

$$\frac{\partial F(\theta,\tau)}{\partial\theta}=\frac{3\sin\theta}{2}\int_0^{\infty}\frac{\sinh\alpha}{(\cosh\alpha+\cos\theta)^{5/2}}P_{i\tau-0.5}(\cosh\alpha)d\alpha \tag{10}$$

Then Eq. (8) becomes

$$M(\tau)\sinh[\tau(\pi-\theta_{\mathrm{Sub}})]+N(\tau)\cosh[\tau(\pi-\theta_{\mathrm{Sub}})]=E_{\mathrm{S}}^{*}(\tau)H(\theta_{\mathrm{Sub}},\tau) \tag{11}$$

where

$$H(\theta_{\mathrm{Sub}},\tau)=\frac{\dfrac{dF(\theta_{\mathrm{Sub}},\tau)}{d\theta}(k_{\mathrm{R}}-1)\sinh(\theta_{\mathrm{Sub}}\tau)}{3\tau F(\theta_{\mathrm{Sub}},\tau)}-k_{\mathrm{R}}\cosh(\theta_{\mathrm{Sub}}\tau) \tag{12}$$

Together with Eqs. (5) and (11), the followings can be obtained as

$$N(\tau)=E_{\mathrm{S}}^{*}(\tau)\{H(\theta_{\mathrm{Sub}},\tau)\cosh[(\pi-\theta_{\mathrm{Sub}})\tau]-\sinh(\theta_{\mathrm{Sub}}\tau)\sinh[(\pi-\theta_{\mathrm{Sub}})\tau]\} \tag{13}$$

$$M(\tau)=E_{\mathrm{S}}^{*}(\tau)\{-H(\theta_{\mathrm{Sub}},\tau)\sinh[(\pi-\theta_{\mathrm{Sub}})\tau]+\sinh(\theta_{\mathrm{Sub}}\tau)\cosh[(\pi-\theta_{\mathrm{Sub}})\tau]\} \tag{14}$$

Then Eq. (3) becomes

$$T_{\mathrm{L}}(\alpha,\beta)=T_{\mathrm{W}}+\sqrt{\cosh\alpha-\cos\beta}$$
$$\int_0^{\infty}P_{i\tau-0.5}(\cosh\alpha)E_{\mathrm{S}}^{*}(\tau)\{-H(\theta_{\mathrm{Sub}},\tau)\sinh[(\pi-\theta_{\mathrm{Sub}}-\beta)\tau]+\sinh(\theta_{\mathrm{Sub}}\tau)\cosh[(\pi-\theta_{\mathrm{sub}}-\beta)\tau]\}d\tau \tag{15}$$

At the gas-liquid interface:

$$(q(\alpha))_{\beta=\pi-\theta}=\frac{k_{\mathrm{L}}}{R}(\cosh\alpha+\cos\theta)\frac{\partial T_{\mathrm{L}}(\alpha,\beta)}{\partial\beta}\Big|_{\beta=\pi-\theta}$$
$$=\frac{k_{\mathrm{L}}}{R}(\cosh\alpha+\cos\theta)^{2/3}\int_0^{\infty}E_{\mathrm{S}}^{*}(\tau)P_{i\tau-0.5}(\cosh\alpha)\{\frac{\sin\theta}{2(\cosh\alpha+\cos\theta)}\{-H(\theta_{\mathrm{Sub}},\tau)\sinh[(\theta-\theta_{\mathrm{Sub}})\tau]$$
$$+\sinh(\theta_{\mathrm{Sub}}\tau)\cosh[(\theta-\theta_{\mathrm{Sub}})\tau]\}-\tau\{-H(\theta_{\mathrm{Sub}},\tau)\cosh[(\theta-\theta_{\mathrm{Sub}})\tau]+\sinh(\theta_{\mathrm{Sub}}\tau)\sinh[(\theta-\theta_{\mathrm{Sub}})\tau]\}\}d\tau \tag{16}$$

Defining

$$f(k_{\mathrm{R}},\tau)=-H(\theta_{\mathrm{Sub}},\tau)\sinh[(\theta-\theta_{\mathrm{Sub}})\tau]+\sinh(\theta_{\mathrm{Sub}}\tau)\cosh[(\theta-\theta_{\mathrm{Sub}})\tau] \tag{17}$$

$$g(k_{\mathrm{R}},\tau)=-H(\theta_{\mathrm{Sub}},\tau)\cosh[(\theta-\theta_{\mathrm{Sub}})\tau]+\sinh(\theta_{\mathrm{Sub}}\tau)\sinh[(\theta-\theta_{\mathrm{Sub}})\tau] \tag{18}$$

Then heat flux across the gas-liquid interface in Eq. (16)

$$(q(\alpha))_{\beta=\pi-\theta}=\frac{k_{\mathrm{L}}}{R}(\cosh\alpha+\cos\theta)^{2/3}\times$$
$$\int_0^{\infty}E_{\mathrm{S}}^{*}(\tau)P_{i\tau-0.5}(\cosh\alpha)\{\frac{\sin\theta}{2(\cosh\alpha+\cos\theta)}f(k_{\mathrm{R}},\tau)-\tau g(k_{\mathrm{R}},\tau)\}d\tau \tag{19}$$

The mass flux across the gas-liquid interface

$$(J(\alpha))_{\beta=3\pi-\theta}=\frac{D}{R}(\cosh\alpha+\cos\theta)\frac{\partial C(\alpha,\beta)}{\partial\beta}\Big|_{\beta=3\pi-\theta}$$
$$=\frac{(\cosh\alpha+\cos\theta)^{3/2}}{R/D}\times$$
$$\int_0^{\infty}E_{\mathrm{C}}^{*}(\tau)P_{i\tau-0.5}(\cosh\alpha)\{\frac{\cosh[(\theta-\pi)\tau]\sin\theta}{2(\cosh\alpha+\cos\theta)}-\tau\sinh[(\theta-\pi)\tau]\}d\tau \tag{20}$$

According to $(q(\alpha))_{\beta=\pi-\theta}=L(J(\alpha))_{\beta=3\pi-\theta}$, the following can be obtained:

$$\sin\theta\int_0^{\infty}\frac{P_{i\tau-0.5}(\cosh\alpha)}{\cosh\alpha+\cos\theta}\frac{k_{\mathrm{L}}E_{\mathrm{S}}^{*}(\tau)f(k_{\mathrm{R}},\tau)-LDE_{\mathrm{C}}^{*}(\tau)\cosh[(\theta-\pi)\tau]}{2}d\tau$$
$$=\int_0^{\infty}P_{i\tau-0.5}(\cosh\alpha)\{k_{\mathrm{L}}E_{\mathrm{S}}^{*}(\tau)\tau g(k_{\mathrm{R}},\tau)-LDE_{\mathrm{C}}^{*}(\tau)\tau\sinh[(\theta-\pi)\tau]\}d\tau \tag{21}$$

Together with Eqs. (9), (10), it can be obtained as

$$E_{\mathrm{S}}^{*}(\tau)\{\frac{k_{\mathrm{L}} f(k_{\mathrm{R}},\tau)}{3}\frac{\partial F(\theta,\tau)}{\partial\theta}-F(\theta,\tau)k_{\mathrm{L}}\tau g(k_{\mathrm{R}},\tau)\}$$
$$=E_{\mathrm{C}}^{*}(\tau)\{\frac{\partial F(\theta,\tau)}{\partial\theta}\frac{LD\cosh[(\theta-\pi)\tau]}{3}-LD\tau F(\tau,\theta)\sinh[(\theta-\pi)\tau]\} \tag{22}$$

It is assumed that the saturated vapor concentration varies linearly with temperature along the droplet surface,

$$C(\alpha,3\pi-\theta)=a+bT_{\mathrm{L}}(\alpha,\pi-\theta),$$

Together with Eqs. (2), (15), it can be obtained as

$$C_{\infty}+\sqrt{\cosh\alpha+\cos\theta}\int_0^{\infty}E_{\mathrm{C}}^{*}(\tau)P_{i\tau-0.5}(\cosh\alpha)\cosh[(\theta-\pi)\tau]d\tau$$
$$=a+bT_{\mathrm{W}}+b\sqrt{\cosh\alpha+\cos\theta}\int_0^{\infty}P_{i\tau-0.5}(\cosh\alpha)E_{\mathrm{S}}^{*}(\tau)f(k_{\mathrm{R}},\tau)d\tau \tag{23}$$

Hence

$$\int_0^{\infty}P_{i\tau-0.5}(\cosh\alpha)\{E_{\mathrm{C}}^{*}(\tau)\cosh[(\theta-\pi)\tau]-bE_{\mathrm{S}}^{*}(\tau)f(k_{\mathrm{R}},\tau)\}d\tau=\frac{a+bT_{\mathrm{W}}-C_{\infty}}{\sqrt{\cosh\alpha+\cos\theta}} \tag{24}$$

According to the Mehler-Fock integral transform

$$\int_0^{\infty}\frac{\cosh\tau\theta}{\cosh\tau\pi}P_{i\tau-0.5}(\cosh\alpha)d\tau=\frac{1}{\sqrt{2\cosh\alpha+2\cos\theta}} \tag{25}$$

Eq.(24) becomes

$$E_{\mathrm{C}}^{*}(\tau)\cosh[(\theta-\pi)\tau]-bE_{\mathrm{S}}^{*}(\tau)f(k_{\mathrm{R}},\tau)=\sqrt{2}(a+bT_{\mathrm{W}}-C_{\infty})\frac{\cosh(\tau\theta)}{\cosh(\tau\pi)} \tag{26}$$

Together with Eqs. (22), (26), the following can be obtained

$$E_{\mathrm{S}}^{*}(\tau)=\frac{\sqrt{2}(a+bT_{\mathrm{W}}-C_{\infty})E_0\cosh(\tau\theta)}{b\cosh(\pi\tau)}\times$$
$$\frac{\frac{\partial F(\theta,\tau)}{3\partial\theta}-\tau F(\theta,\tau)\tanh[(\theta-\pi)\tau]}{\{\frac{f(k_{\mathrm{R}},\tau)}{3}\frac{\partial F(\theta,\tau)}{\partial\theta}-F(\theta,\tau)\tau g(k_{\mathrm{R}},\tau)\}-E_0\{\frac{\partial F(\theta,\tau)}{3\partial\theta}-\tau F(\theta,\tau)\tanh[(\theta-\pi)\tau]\}f(k_{\mathrm{R}},\tau)} \tag{27}$$

$$E_{\mathrm{C}}^{*}(\tau)=\frac{\sqrt{2}(a+bT_{\mathrm{W}}-C_{\infty})\cosh(\tau\theta)}{\cosh(\pi\tau)}\times$$

$$\frac{\operatorname{sech}[(\theta-\pi)\tau](\frac{f(k_{\mathrm{R}},\tau)}{3}\frac{\partial F(\theta,\tau)}{\partial\theta}-\tau F(\theta,\tau)g(k_{\mathrm{R}},\tau))}{\{\frac{f(k_{\mathrm{R}},\tau)}{3}\frac{\partial F(\theta,\tau)}{\partial\theta}-\tau F(\theta,\tau)g(k_{\mathrm{R}},\tau)\}-E_0\{\frac{\partial F(\theta,\tau)}{3\partial\theta}-\tau F(\theta,\tau)\tanh[(\theta-\pi)\tau]\}f(k_{\mathrm{R}},\tau)} \tag{28}$$

where $E_0=bLD/k_{\mathrm{L}}$ is evaporative cooling number.

hence the non-dimension concentration around droplet is

$$\tilde{C}(\alpha,\beta)=\frac{C(\alpha,\beta)-C_{\infty}}{C_{\mathrm{e}}-C_{\infty}}=\sqrt{2\cosh\alpha-2\cos\beta}\int_0^{\infty}E_{\mathrm{C}}(\tau)P_{i\tau-0.5}(\cosh\alpha)\cosh[(2\pi-\beta)\tau]d\tau \tag{29}$$

non-dimensional temperature within droplet is

$$\tilde{T}_L(\alpha,\beta)=\frac{T_{\mathrm{L}}(\alpha,\beta)-T_{\mathrm{W}}}{T_{\mathrm{W}}-T_{\infty}}=\sqrt{2\cosh\alpha-2\cos\beta}$$
$$\int_0^{\infty}P_{i\tau-0.5}(\cosh\alpha)E_{\mathrm{S}}(\tau)\{-H(\theta_{\mathrm{Sub}},\tau)\sinh[(\pi-\theta_{\mathrm{Sub}}-\beta)\tau]+\sinh(\theta_{\mathrm{Sub}}\tau)\cosh[(\pi-\theta_{\mathrm{Sub}}-\beta)\tau]\}d\tau \tag{30}$$

Non-dimensional temperature within the substrate is

$$\tilde{T}_{\mathrm{S}}(\alpha,\beta)=\frac{T_{\mathrm{S}}(\alpha,\beta)-T_{\mathrm{W}}}{T_{\mathrm{W}}-T_{\infty}}=\sqrt{2\cosh\alpha-2\cos\beta}\int_0^{\infty}E_{\mathrm{S}}(\tau)P_{i\tau-0.5}(\cosh\alpha)\sinh[(\pi-\beta)\tau]d\tau \tag{31}$$

Where

$$E_{\mathrm{S}}(\tau)=\frac{E_0\cosh(\tau\theta)}{\cosh(\pi\tau)}\times$$

$$\frac{\frac{\partial F(\theta,\tau)}{3\partial\theta}-\tau F(\theta,\tau)\tanh[(\theta-\pi)\tau]}{\{\frac{f(k_{\mathrm{R}},\tau)}{3}\frac{\partial F(\theta,\tau)}{\partial\theta}-F(\theta,\tau)\tau g(k_{\mathrm{R}},\tau)\}-E_0\{\frac{\partial F(\theta,\tau)}{3\partial\theta}-\tau F(\theta,\tau)\tanh[(\theta-\pi)\tau]\}f(k_{\mathrm{R}},\tau)} \tag{32}$$

$$E_{\mathrm{C}}(\tau)=\frac{\cosh(\tau\theta)}{\cosh(\pi\tau)}\times$$

$$\frac{\operatorname{sech}[(\theta-\pi)\tau](\frac{f(k_{\mathrm{R}},\tau)}{3}\frac{\partial F(\theta,\tau)}{\partial\theta}-\tau F(\theta,\tau)g(k_{\mathrm{R}},\tau))}{\{\frac{f(k_{\mathrm{R}},\tau)}{3}\frac{\partial F(\theta,\tau)}{\partial\theta}-\tau F(\theta,\tau)g(k_{\mathrm{R}},\tau)\}-E_0\{\frac{\partial F(\theta,\tau)}{3\partial\theta}-\tau F(\theta,\tau)\tanh[(\theta-\pi)\tau]\}f(k_{\mathrm{R}},\tau)} \tag{33}$$

4. The lifetime of droplet

After integrating the evaporative flux over the gas-liquid interface in toroidal coordinates, the evaporation rate can be obtained:

$$\frac{dm(\theta)}{dt}=\rho_{\mathrm{L}}\frac{dV(\theta)}{dt}=-\int_0^{\infty}2\pi R^2 J(\alpha,\theta)\frac{\sinh\alpha}{(\cosh\alpha+\cos\theta)^2}d\alpha$$
$$=-D(C_{\mathrm{e}}-C_{\infty})2\sqrt{2}\pi R\times$$
$$\int_0^{\infty}d\tau\int_0^{\infty}(\cosh\alpha+\cos\theta)^{-0.5}\sinh\alpha\{E_{\mathrm{C}}(\tau)P_{i\tau-0.5}(\cosh\alpha)[\frac{\cosh[(\theta-\pi)\tau]\sin\theta}{2(\cosh\alpha+\cos\theta)}-\tau\sinh[(\theta-\pi)\tau]]\}d\alpha \tag{34}$$

In the above equation, the double integral term is a function of θ, E_0 and k_R, it can be defined as

$$\varphi(\theta,k_R,E_0)=$$
$$\int_0^{\infty}d\tau\int_0^{\infty}(\cosh\alpha+\cos\theta)^{-0.5}\sinh\alpha\{E_{\mathrm{C}}(\tau)P_{i\tau-0.5}(\cosh\alpha)[\frac{\cosh[(\theta-\pi)\tau]\sin\theta}{2(\cosh\alpha+\cos\theta)}-\tau\sinh[(\theta-\pi)\tau]]\}d\alpha \tag{35}$$

Thus

$$\frac{dm(\theta)}{dt}=\rho_{\mathrm{L}}\frac{dV(\theta)}{dt}=-D(C_{\mathrm{e}}-C_{\infty})2\sqrt{2}\pi R\varphi(\theta,k_R,E_0) \tag{36}$$

The volume of droplet can be obtained:

$$V=\frac{\pi R^3}{3g(\theta)}-\frac{\pi R^3}{3g(\theta_{\mathrm{Sub}})} \tag{37}$$

where $g(\theta)=\frac{\sin^3\theta}{(1-\cos\theta)^2(2+\cos\theta)}$,

During the evaporation the droplet is assumed to be pinned on the substrate, i.e. it is in the Constant Contact Radius (CCR) mode, the base radius is kept constant and the contact angle of droplet decreases with time, so together with Eqs. (34), (37), it can be obtained as

$$\frac{d\theta}{dt}=-\frac{D(C_{\mathrm{e}}-C_{\infty})}{\rho_{\mathrm{L}}R^2}2\sqrt{2}\varphi(\theta,k_R,E_0)(1+\cos\theta)^2 \tag{38}$$

The lifetime of evaporation of droplet can be obtained

$$t_{\mathrm{CCR}}=\frac{\rho_{\mathrm{L}}R_0^{\ 2}}{2\sqrt{2}D(C_{\mathrm{e}}-C_{\infty})}\int_{\theta_{sub}}^{\theta_0}\frac{1}{\varphi(\theta,k_{\mathrm{R}},E_0)(1+\cos\theta)^2}d\theta \tag{39}$$

where the θ_0 and R_0 are the initial tangential angle of the droplet surface with the plane at the edge and initial base radius respectively.