Supporting Information

Heat Recovery Assisted by Thermo-magnetic-electric Conversion

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Section 1. The deformation in PVDF film and induced electricity during the movement of NiCoMnIn ribbon.

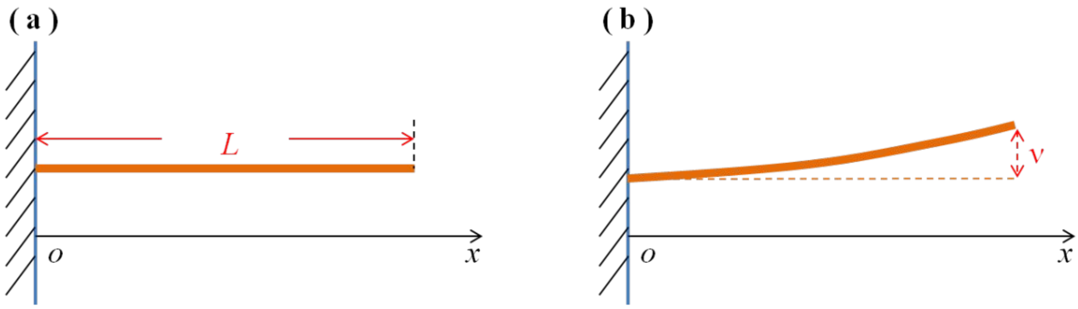

Figure S1. Schematic of the bending of beam caused by the upward movement of NiCoMnIn ribbon from initial (a) to final (b) states.

Upon heating the NiCoMnIn ribbon, the magnetic force makes the beam deform from its initial (Figure S1a) to final (Figure S1b) states. One end of this beam (x=0) is fixed and another end (x=L) is free. According to Material Mechanics, the curvature (κ) of this cantilever beam at the final state is given by

$$\kappa(x) = \frac{3\nu}{L^3}(L - x) \tag{S1}$$

where ν is the distance between permanent magnet and heat source. According to neutral axis theory [S1], the average strain is therefore expressed as

$$\overline{\text{strain}} = \kappa(x)\frac{Y_2 + Y_1}{2} \tag{S2}$$

where Y_2 and Y_1 are the maximum and minimum distance from the neutral axis. The position of neutral axis can be easily obtained based on Ref. S1. The generated open-circuit voltage (V) and electric current (I) can be calculated by

$$V = \frac{Q}{C} = \frac{DA}{C} = \frac{\int_0^L d_{31} \cdot \overline{strain} \cdot E \cdot w dx}{\varepsilon_0 \cdot \varepsilon_r \cdot \frac{A}{d}} = \frac{3 \cdot v \cdot d_{31} \cdot E \cdot (Y_2 + Y_1) \cdot d}{4 \cdot L^2 \cdot \varepsilon_0 \cdot \varepsilon_r}$$

$$I = \frac{V}{R' + R_{Load}} \tag{S3}$$

where Q is the generated charge, C is capacitance, D is charge displacement, A is area of electrode (A=2 cm^2), R' and R_{Load} are the resistance of PVDF film and external load, ε_0 is electric constant, ε_r, E, d_{31}, w and d are the relative permittivity, Young's modulus, piezoelectric coefficient, width and thickness of the PVDF film used in this experiment (ε_r=12.4, E=2×10^9 Pa, d_{31}=23 pC/N, w=4 mm, d=110 μm). Figure S2 shows the calculated open-circuit voltage and short-circuit current as a function of v. In the case of v=10 mm, the open-circuit voltage and short-circuit current induced by the movement of NiCoMnIn ribbon reach 17.53 V and 0.35 μA, respectively.

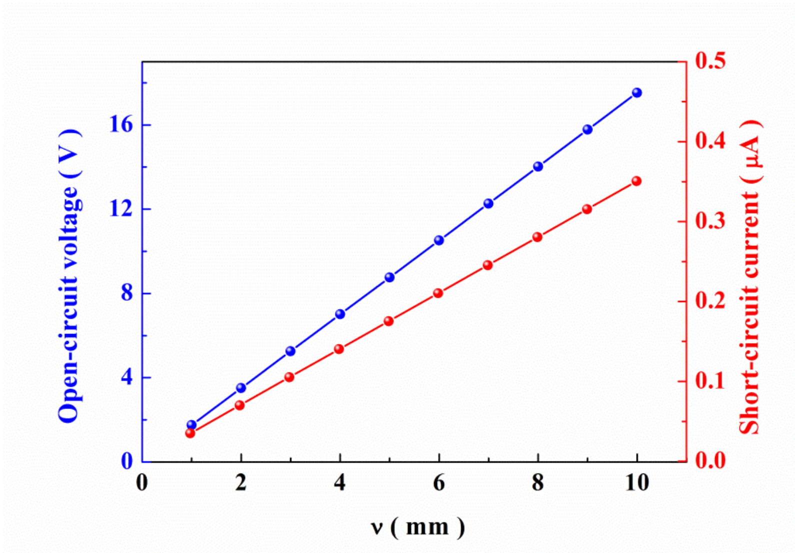

Figure S2. Calculated open-circuit voltage and short-circuit current as a function of the distance between heat source and permanent magnet

Equation S3 can also be used to calculate the generated electricity during the downward movement of NiCoMnIn ribbon. In that case, Figure S1b is the initial state and Figure S1a is changed to be the final state. The resulted V and I have the same values as that during upward movement of NiCoMnIn ribbon but with an opposite direction.

Section 2. The deformation in PVDF and induced electricity during the oscillation of beam.

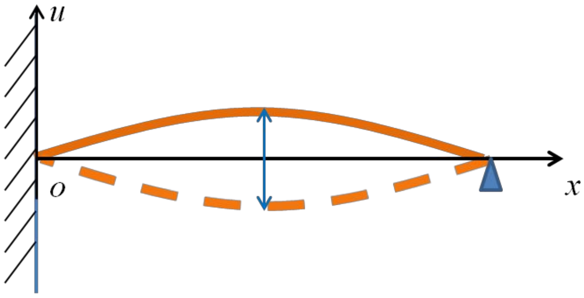

Figure S3. The oscillation of beam after the NiCoMnIn ribbon strikes permanent magnet/heat source.

After the NiCoMnIn ribbon strikes the permanent magnet/heat source with an initial velocity, the beam begins to freely oscillate as shown in Figure S3. The governing equation of lateral displacement (*μ*) can be written as

$$\frac{\partial}{\partial x^2}\left(EJ\frac{\partial u}{\partial x^2}\right)+\rho S\frac{\partial^2 u}{\partial t^2}+c\frac{\partial u}{\partial t}=0 \quad \text{(S4)}$$

where *J* is the inertia moment relative to neutral axis, *ρ* is density and *S* is sectional area. The third term on the left side is due to damping effect. The *u*(*x*,*t*) can be represented by an absolutely and uniformly convergent series of the eigenfunctions as

$$u(x,t)=\sum_{i=1}^{\infty}U_i(x)\eta_i(t) \quad \text{(S5)}$$

where *i* means the *i* th vibration mode. During the oscillation, the beam can be perceived as simply supported beam since the lateral displacement at *x*=*L* is constrained by the permanent magnet/heat source. Therefore, $U_{\mathrm{i}}(x)$ is expressed as

$$U_i(x)=\sqrt{\frac{2}{\rho SL}}sin\frac{i\pi x}{L}, i=1,2,3\ldots \quad \text{(S6)}$$

The expression of $\eta_{\mathrm{i}}(t)$ can be obtained by solving

$$\frac{\partial^2\eta_i(t)}{\partial t^2}+2\zeta\omega_i\frac{\partial\eta_i(t)}{\partial t}+\omega_i^2\eta_i(t)=0 \tag{S7}$$

where ζ is damping coefficients (ζ=0.15) and ω_i is the undamped natural frequency of the i th mode given by

$$\omega_i=(\frac{i\pi}{L})\sqrt{\frac{EJ}{\rho S}},\ i=1,2,3... \tag{S8}$$

The general solution for Equation S7 is

$$\eta_i(t)=e^{-\zeta\omega_i t}(\eta_{i0}cos\omega_{id}t+\frac{\zeta\omega_i\eta_{i0}+\dot{\eta}_{i0}}{\omega_{id}}sin\omega_{id}t)\omega_{id}=\omega_i\sqrt{1-\xi^2} \tag{S9}$$

η_{i0} and $\dot{\eta}_{\mathrm{i0}}$ can be obtained according to the initial conditions

$$u(x{,}0)=f_1(x)\,,\left.\frac{\partial u(x{,}t)}{\partial t}\right|_{t=0}=f_2(x)$$

and

$$\begin{cases}\eta_{\mathrm{i0}}=\int_0^L\rho Sf_1(\mathrm{x})U_{\mathrm{i}}(x)dx\\ \dot{\eta}_{\mathrm{i0}}=\int_0^L\rho Sf_2(\mathrm{x})U_{\mathrm{i}}(x)dx\end{cases} \tag{S10}$$

Here, we assume that t=0 is the moment at which the beam is unstrained and the lateral velocity of beam at t=0 is $\dot{\theta}x$, where $\dot{\theta}$ is angular velocity. In that case,

$$\begin{cases}f_1(\mathrm{x})=0\\ f_2(x)=\dot{\theta}x\end{cases}$$

and $u(x{,}t)$ is given by

$$u(x,t) = \sum_{i=1}^{\infty} \frac{2\dot{\theta}L}{\omega_{id}}[-\frac{(-1)^i}{i\pi}]sin\,(\frac{i\pi x}{L})sin(\omega_{id}t)e^{-\zeta\omega_i t} \tag{S11}$$

Angular velocity is confirmed based on the conservation law of energy. During the upward movement of NiCoMnIn ribbon, the change of magneto-static potential energy stored in NiCoMnIn ribbon is equal to the increase of kinetic and potential energies of the beam.

$$m' \cdot M \cdot |\triangle B| = \frac{mgv}{2} + \frac{1}{2}I\dot{\theta}^2 \tag{S12}$$

where m' and m is the mass of NiCoMnIn ribbon and beam (m'=7.5 mg and m=0.16g), M is the saturated magnetization of NiCoMnIn ribbon at Austenite (M=55 emu/g), I is rotational inertia of the beam and B is the magnetic induction intensity of the external magnetic field at different position. In this experiment, B is measured using a commercial gaussmeter (Lakeshore). The value of B at the surface of permanent magnet is 0.5 T and decreases to 0.13 T when the gaussmeter is 10 mm away. Since t=0 is assumed to be the moment at which the beam is unstrained, the elastic energy stored in beam doesn't include in Equation S12. According to Equation S12, $\dot{\theta}$ during upward movement of NiCoMnIn ribbon can be confirmed.

During the downward movement of NiCoMnIn ribbon, the elastic energy stored in beam and the potential energy of the beam is changed to the kinetic energy. According to Mechanics of Materials, the elastic energy plus potential energy is equal to $\frac{3EJv^2}{L^3}$. And therefore, $\dot{\theta}$ can be calculated by

$$\frac{3EJv^2}{L^3} = \frac{1}{2}I\dot{\theta}^2 \tag{S13}$$

Here, we assume that the value of M is zero because NiCoMnIn ribbon is in weak-magnetizaiton martensite and thus the change of magneto-static potential energy doesn't include in Equation S13. According to Equation S13, $\dot{\theta}$ during downward movement of NiCoMnIn ribbon can be confirmed.

According to Equation S11, the curvature of beam can be obtained by

$$\kappa(x) = \frac{\ddot{u}(x,t)}{\left(1 + (\dot{u}(x,t))^2\right)^{3/2}} \tag{S14}$$

Based on Equations S2, S3 and S14, the deformation in PVDF film and the generated electricity can be calculated.

The induced open-circuit voltage in the case of v=10 mm is shown in Figure S4. The largest open-circuit voltage is 25.4 V and the short-circuit current is 0.51 μA. The negative values in Figure S4 means the direction of voltage/current is opposite to that during the movement of NiCoMnIn ribbons.

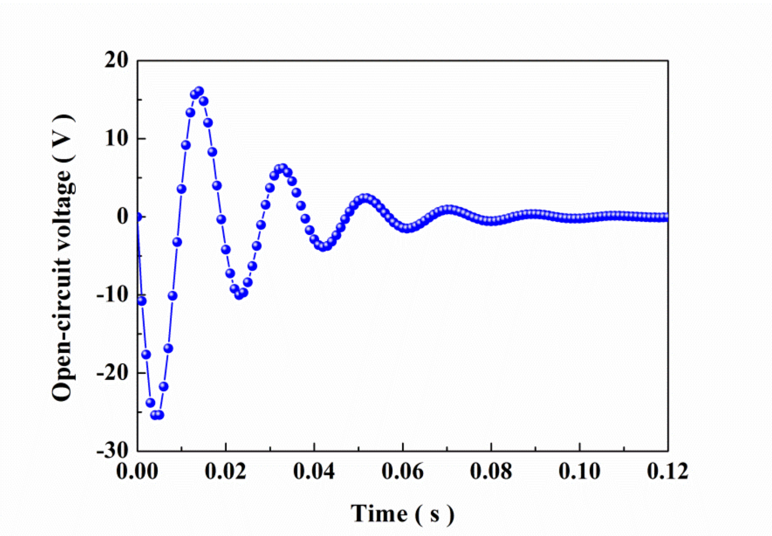

Figure S4. The generated open-circuit voltage during oscillation.

References

[S1] Lee, S. M.; Kwon, J. Y.; Yoon, D.; Cho, H. D.; You, J. H.; Kang, Y. T.; Choi, D.; Hwang, W. B. Bendability optimization of flexible optical nanoelectronics via neutral axis engineering. *Nanoscale Res. Lett*. **2012**, *7*, 256.