

Supporting Information

Demonstration of Arithmetic Calculations by DNA

Tile-Based Algorithmic Self-Assembly

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Figure S1. Schematic of the binding domains of rule building blocks (*i.e.*, propagator, connector, and solution tiles) for addition and subtraction.

Table S1. Sticky-end binding domains with specific DNA base sequences in propagator, connector, and solution tiles for addition operation.

Table S2. Sticky-end binding domains with specific DNA base sequences in propagator, connector, and solution tiles for subtraction operation.

Figure S2. A sequence diagram with cartoon representation of the double-crossover (DX) tiles for the propagators used in the addition and subtraction operations.

Table S3. Sequence information of propagators for addition and subtraction.

Figure S3. A sequence diagram with cartoon representation of the connectors for addition.

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Table S5. Sequence information of connectors for subtraction.

Figure S5. A sequence diagram with cartoon representation of the solutions for addition.

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Figure S6. A sequence diagram with cartoon representation of the solutions for subtraction.

Table S7. Sequence information of solutions for subtraction.

a)

Definition	Dummy (d)	Input (I_L or I_R)	Carry _{Out} /2 $\pm C_o/2$	Input+(Carry _{In} /2) $(I_L \text{ or } I_R) \pm C_i/2$
Diagram				

b)

d			d'
0			0'
1			1'
$0_{c,l}$			$0_{c,l}'$
$-\frac{1}{2}_{c,l}$			$-\frac{1}{2}_{c,l}'$
$(0+0_{c,l})$			$(0+0_{c,l})'$
$(1+0_{c,l})$			$(1+0_{c,l})'$
$(0-\frac{1}{2}_{c,l})$			$(0-\frac{1}{2}_{c,l})'$
$(1-\frac{1}{2}_{c,l})$			$(1-\frac{1}{2}_{c,l})'$
$0_{c,r}$			$0_{c,r}'$
$-\frac{1}{2}_{c,r}$			$-\frac{1}{2}_{c,r}'$
$(0+0_{c,r})$			$(0+0_{c,r})'$
$(1+0_{c,r})$			$(1+0_{c,r})'$
$(0+\frac{1}{2}_{c,r})$			$(0+\frac{1}{2}_{c,r})'$
$(1+\frac{1}{2}_{c,r})$			$(1+\frac{1}{2}_{c,r})'$

c)

d			d'
0			0'
1			1'
$0_{c,l}$			$0_{c,l}'$
$-\frac{1}{2}_{c,l}$			$-\frac{1}{2}_{c,l}'$
$(0+0_{c,l})$			$(0+0_{c,l})'$
$(1+0_{c,l})$			$(1+0_{c,l})'$
$(0-\frac{1}{2}_{c,l})$			$(0-\frac{1}{2}_{c,l})'$
$(1-\frac{1}{2}_{c,l})$			$(1-\frac{1}{2}_{c,l})'$
$0_{c,r}$			$0_{c,r}'$
$-\frac{1}{2}_{c,r}$			$-\frac{1}{2}_{c,r}'$
$(0+0_{c,r})$			$(0+0_{c,r})'$
$(1+0_{c,r})$			$(1+0_{c,r})'$
$(0+\frac{1}{2}_{c,r})$			$(0+\frac{1}{2}_{c,r})'$
$(1+\frac{1}{2}_{c,r})$			$(1+\frac{1}{2}_{c,r})'$

Figure S1. Schematic of the binding domains of rule building blocks (*i.e.*, propagator, connector, and solution tiles) for addition and subtraction. (a) Binding domains of Dummy, Input, Carry_{Out}/2, and Input + (Carry_{In}/2) with corresponding geometrical representations. We have considered four types (*i.e.*, Dummy, Input, Carry_{Out}/2, and Input + (Carry_{In}/2)) of binding domains to demonstrate arithmetic calculation. Geometrical representations of the binding domains are also shown. **(b)** Name and corresponding geometrical representation of the binding domains for addition with specific colour coding. Pairs ($d-d'$), ($0-0'$ and $1-1'$), ($0_{c,l}-0_{c,r}$ ' and $\frac{1}{2}_{c,l}-\frac{1}{2}_{c,r}$ '), and [$(0+0_c)-(0+0_c)'$, $(1+0_c)-(1+0_c)'$, $(0+\frac{1}{2}_c)-(0+\frac{1}{2}_c)'$, and $(1+\frac{1}{2}_c)-(1+\frac{1}{2}_c)'$] belong to Dummy, Input, Carry_{Out}/2, and Input + (Carry_{In}/2) binding domains, respectively. Geometrical representation of unprimed and primed specific binding domains with identical colour coding (which were complementary to each other). **(c)** Name and corresponding geometrical representation of the binding domains for subtraction with specific colour coding. Binding domain pairs: ($d-d'$), ($0-0'$ and $1-1'$), ($0_{c,l}-0_{c,r}$ ' and $-\frac{1}{2}_{c,l}-\frac{1}{2}_{c,r}$ '), ($0_{c,r}-0_{c,l}$ ' and $-\frac{1}{2}_{c,r}-\frac{1}{2}_{c,l}$ '), [$(0+0_{c,l})-(0+0_{c,l})'$, $(1+0_{c,l})-(1+0_{c,l})'$, $(0-\frac{1}{2}_{c,l})-(0-\frac{1}{2}_{c,l})'$, and $(1-\frac{1}{2}_{c,l})-(1-\frac{1}{2}_{c,l})'$], and [$(0+0_{c,r})-(0+0_{c,r})'$, $(1+0_{c,r})-(1+0_{c,r})'$, $(0-\frac{1}{2}_{c,r})-(0-\frac{1}{2}_{c,r})'$, and $(1-\frac{1}{2}_{c,r})-(1-\frac{1}{2}_{c,r})'$] belong to Dummy, Input, Carry_{Out}/2 on left, Carry_{Out}/2 on right, Input + (Carry_{In}/2) on left, and Input + (Carry_{In}/2) on right, respectively

Sticky-end name	5' to 3'	5' to 3'	Sticky-end name
d	aactg	cagtt	d'
0	ctagt	actag	0'
1	agcat	atgct	1'
0 _c	gcgtt	tacgc	0 _c '
½ _c	agtcc	ggact	½ _c '
(0+0 _c)	tcgtc	gacga	(0+0 _c)'
(1+0 _c)	ctgag	ctcag	(1+0 _c)'
(0+½ _c)	gacga	tcgtc	(0+½ _c)'
(1+½ _c)	atcgg	ccgat	(1+½ _c)'

Table S1. Sticky-end binding domains with specific DNA base sequences in propagator, connector, and solution tiles for addition operation. The complementary sticky end pairs are indicated by unprimed and primed symbols (e.g. d and d').

Sticky-end name	5' to 3'	5' to 3'	Sticky-end name
d	aactg	cagtt	d'
0	ctagt	actag	0'
1	agcat	atgct	1'
0 _{c,l}	gcgtt	tacgc	0 _{c,l} '
-½ _{c,l}	agtcc	ggact	-½ _{c,l} '
(0+0 _{c,l})	tcgtc	gacga	(0+0 _{c,l})'
(1+0 _{c,l})	ctgag	ctcag	(1+0 _{c,l})'
(0-½ _{c,l})	gacga	tcgtc	(0-½ _{c,l})'
(1-½ _{c,l})	atcgg	ccgat	(1-½ _{c,l})'
0 _{c,r}	tgcct	tggca	0 _{c,r} '
-½ _{c,r}	cgtac	gtacg	-½ _{c,r} '
(0+0 _{c,r})	ggctt	agacc	(0+0 _{c,r})'
(1+0 _{c,r})	aggct	agcct	(1+0 _{c,r})'
(0-½ _{c,r})	gtacc	ggtag	(0-½ _{c,r})'
(1-½ _{c,r})	tagcc	gctta	(1-½ _{c,r})'

Table S2. Sticky-end binding domains with specific DNA base sequences in propagator, connector, and solution tiles for subtraction operation. The complementary sticky end pairs are indicated by unprimed and primed symbols (e.g. 0 and 0').

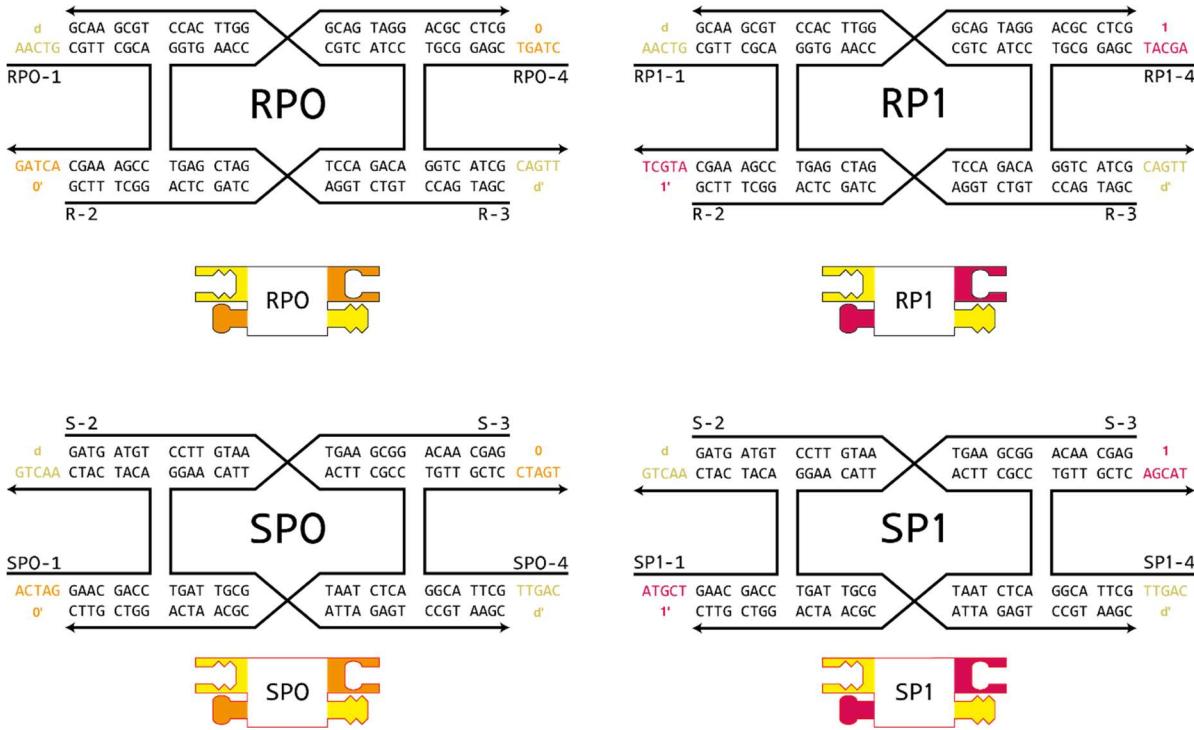


Figure S2. A sequence diagram with cartoon representation of the double-crossover (DX) tiles for the propagators used in the addition and subtraction operations. A propagator (four kinds, *i.e.*, RP0, RP1, SP0, and SP1 are available) delivers the initial input value placed on left side (I_L , which is either 0 or 1) through the next propagator (0' or 1') to the connector (0' or 1'). The propagator can also deliver the initial input value placed on right side (I_R) if it is rotated along the y-axis. Two inputs I_L and I_R are added to get the sum. For subtraction operation, I_R (subtrahend) is subtracted from I_L (minuend).

Tile	Strand	Length	Sequence (5' to 3')
RP0	RP0-1	26mers	aactgegttcgcacccgaaaggcactag
	R-2	48mers	gcttcggactcgatctcccgacacctactgcgggtcacctgcgaacg
	R-3	48mers	cgtatgcacctgtctggagatcgagttgttgcgaaccgcagtaggcgcctcg
	RP0-4	26mers	ctatgtcgaggcggtggtcatecgagg
RP1	RP1-1	26mers	aactgcgttcgcacccgaaaggcatgt
	R-2	48mers	gcttcggactcgatctcccgacacctactgcgggtcacctgcgaacg
	R-3	48mers	cgtatgcacctgtctggagatcgagttgttgcgaaccgcagtaggcgcctcg
	RP1-4	26mers	acatcgaggcggtggtcatcgagg
SP0	SP0-1	26mers	actaggaaacgaccacatcatcaactg
	S-2	48mers	gatgtatgtccctgttaacttcgcactctaattcgaaatcaggcggttc
	S-3	48mers	gagcaacacggcgaagttacaagggttgcgatttagagtccgtaaagc
	SP0-4	26mers	cagttgccttacgggtgtctcgatgt
SP1	SP1-1	26mers	atgcgtgaacgaccacatcatcaactg
	S-2	48mers	gatgtatgtccctgttaacttcgcactctaattcgaaatcaggcggttc
	S-3	48mers	gagcaacacggcgaagttacaagggttgcgatttagagtccgtaaagc
	SP1-4	26mers	cagttgccttacgggtgtctcgatgt

Table S3. Sequence information of propagators for addition and subtraction.

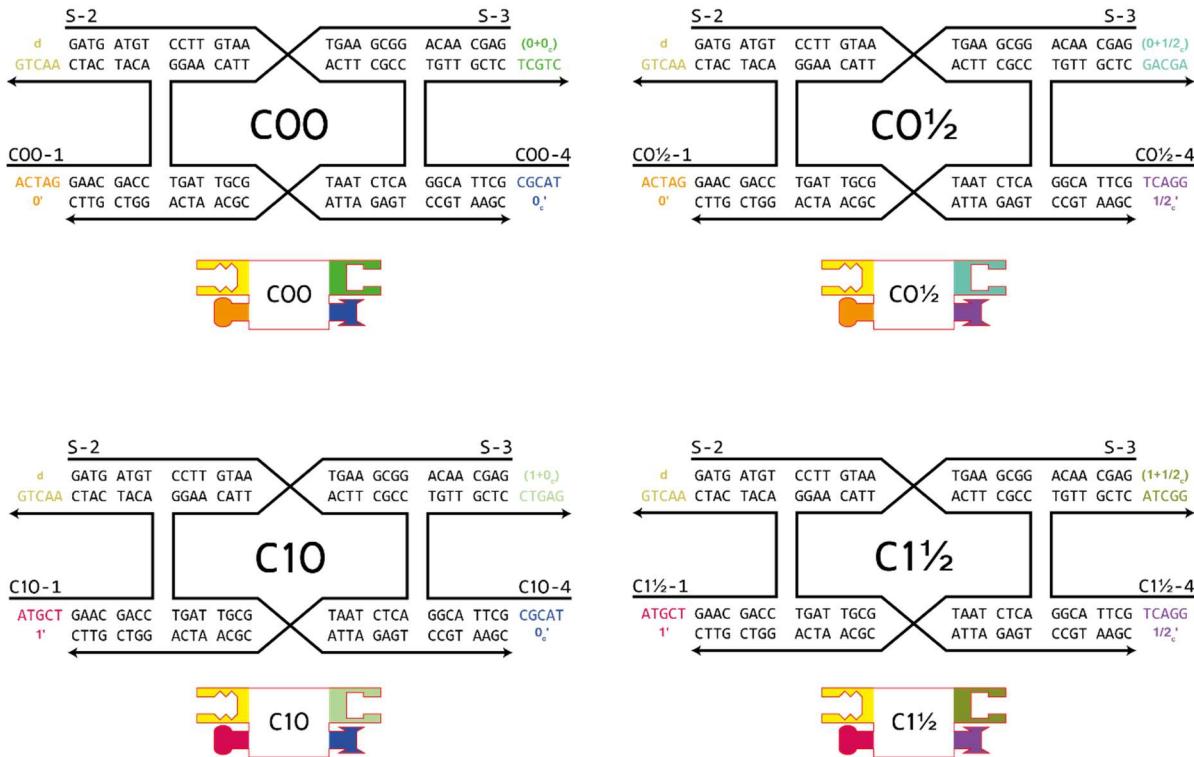


Figure S3. A sequence diagram with cartoon representation of the connectors for addition. A connector (C_{00} , C_0 , $C_{10\frac{1}{2}}$, and $C_{1\frac{1}{2}}$), which is placed between propagator and solution tiles, delivers bit information (0 or 1, which is obtained from propagator) to the solution tiles from both left and right sides.

Tile	Strand	Length	Sequence (5' to 3')
C00	C00-1	26mers	acttaggaacgaccacatcatcaactg
	S-2	48mers	gatgatgtccctgttaaacattcgccactctaattcgcaatcaggcggttc
	S-3	48mers	gagcaacaggcgaagtttacaagggtattgcattagatggcgtaagc
	C00-4	26mers	tacgcgcattacgggtgtgtctcg
C10	C10-1	26mers	atgctgaacgaccacatcatcaactg
	S-2	48mers	gatgatgtccctgttaaacattcgccactctaattcgcaatcaggcggttc
	S-3	48mers	gagcaacaggcgaagtttacaagggtattgcattagatggcgtaagc
	C10-4	26mers	tacgcgcattacgggtgtgtctcg
C0½	C0½-1	26mers	acttaggaacgaccacatcatcaactg
	S-2	48mers	gatgatgtccctgttaaacattcgccactctaattcgcaatcaggcggttc
	S-3	48mers	gagcaacaggcgaagtttacaagggtattgcattagatggcgtaagc
	C0½-4	26mers	ggactgcattacgggtgtgtctcg
C1½	C1½-1	26mers	atgctgaacgaccacatcatcaactg
	S-2	48mers	gatgatgtccctgttaaacattcgccactctaattcgcaatcaggcggttc
	S-3	48mers	gagcaacaggcgaagtttacaagggtattgcattagatggcgtaagc
	C1½-4	26mers	ggactgcattacgggtgtgtctcg

Table S4. Sequence information of connectors for addition.

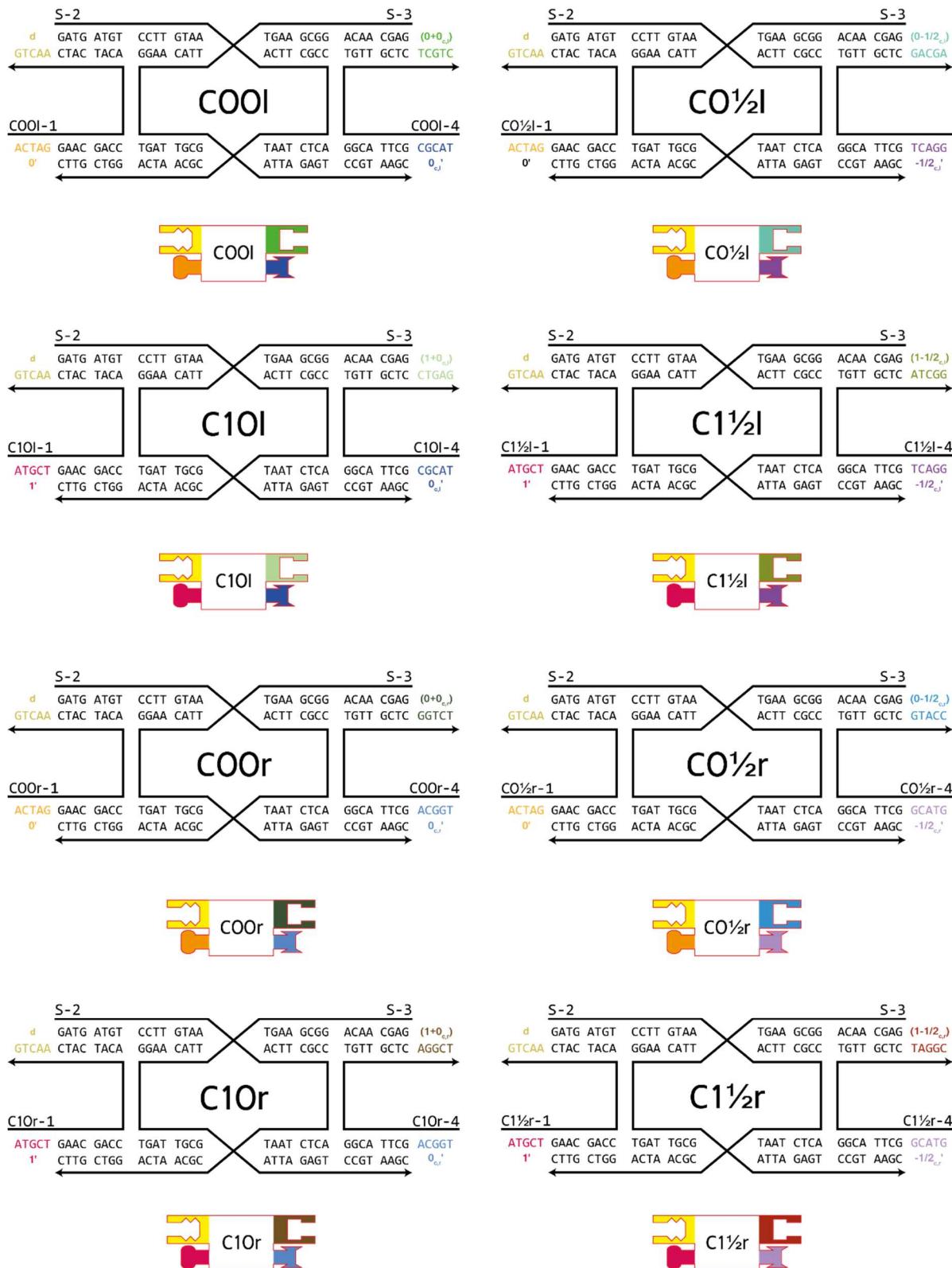


Figure S4. A sequence diagram with cartoon representation of the connectors for subtraction. A connector (COOI, CO1/2l, C10l, C11/2l, COOr, CO1/2r, C10r, and C11/2r), which is placed between propagator and solution tiles, delivers bit information (0 or 1, which is obtained from propagator) to the solution either from left (via COOI, CO1/2l, C10l, and C11/2l) or right (COOr, CO1/2r, C10r, and C11/2r).

Tile	Strand	Length	Sequence (5' to 3')
C00l	C00l-1	26mers	actaggaacgaccacatcatcaactg
	S-2	48mers	gatgatgcctgtaaacttcgcactctaattcgcaatcaggcggttc
	S-3	48mers	gagcaacaggcgaagtttacaagggttgcgatttagtccgtaagc
	C00l-4	26mers	tacgcgttacgggttgctctcgtc
C10l	C10l-1	26mers	atgctgaacgaccacatcatcaactg
	S-2	48mers	gatgatgcctgtaaacttcgcactctaattcgcaatcaggcggttc
	S-3	48mers	gagcaacaggcgaagtttacaagggttgcgatttagtccgtaagc
	C10l-4	26mers	tacgcgttacgggttgctctcgag
C0½l	C0½l-1	26mers	actaggaacgaccacatcatcaactg
	S-2	48mers	gatgatgcctgtaaacttcgcactctaattcgcaatcaggcggttc
	S-3	48mers	gagcaacaggcgaagtttacaagggttgcgatttagtccgtaagc
	C0½l-4	26mers	ggactgcgttacgggttgctcgacga
C1½l	C1½ -1	26mers	atgctgaacgaccacatcatcaactg
	S-2	48mers	gatgatgcctgtaaacttcgcactctaattcgcaatcaggcggttc
	S-3	48mers	gagcaacaggcgaagtttacaagggttgcgatttagtccgtaagc
	C1½l-4	26mers	ggactgcgttacgggttgctcatgg
C00r	C00r-1	26mers	actaggaacgaccacatcatcaactg
	S-2	48mers	gatgatgcctgtaaacttcgcactctaattcgcaatcaggcggttc
	S-3	48mers	gagcaacaggcgaagtttacaagggttgcgatttagtccgtaagc
	C00r-4	26mers	tggcagcttacgggttgctcggt
C10r	C10r-1	26mers	atgctgaacgaccacatcatcaactg
	S-2	48mers	gatgatgcctgtaaacttcgcactctaattcgcaatcaggcggttc
	S-3	48mers	gagcaacaggcgaagtttacaagggttgcgatttagtccgtaagc
	C10r-4	26mers	tggcagcttacgggttgctcaggct
C0½r	C0½r-1	26mers	actaggaacgaccacatcatcaactg
	S-2	48mers	gatgatgcctgtaaacttcgcactctaattcgcaatcaggcggttc
	S-3	48mers	gagcaacaggcgaagtttacaagggttgcgatttagtccgtaagc
	C0½r-4	26mers	gtacggcttacgggttgctcgacc
C1½r	C1½r-1	26mers	atgctgaacgaccacatcatcaactg
	S-2	48mers	gatgatgcctgtaaacttcgcactctaattcgcaatcaggcggttc
	S-3	48mers	gagcaacaggcgaagtttacaagggttgcgatttagtccgtaagc
	C1½r-4	26mers	gtacggcttacgggttgctctaggc

Table S5. Sequence information of connectors for subtraction.

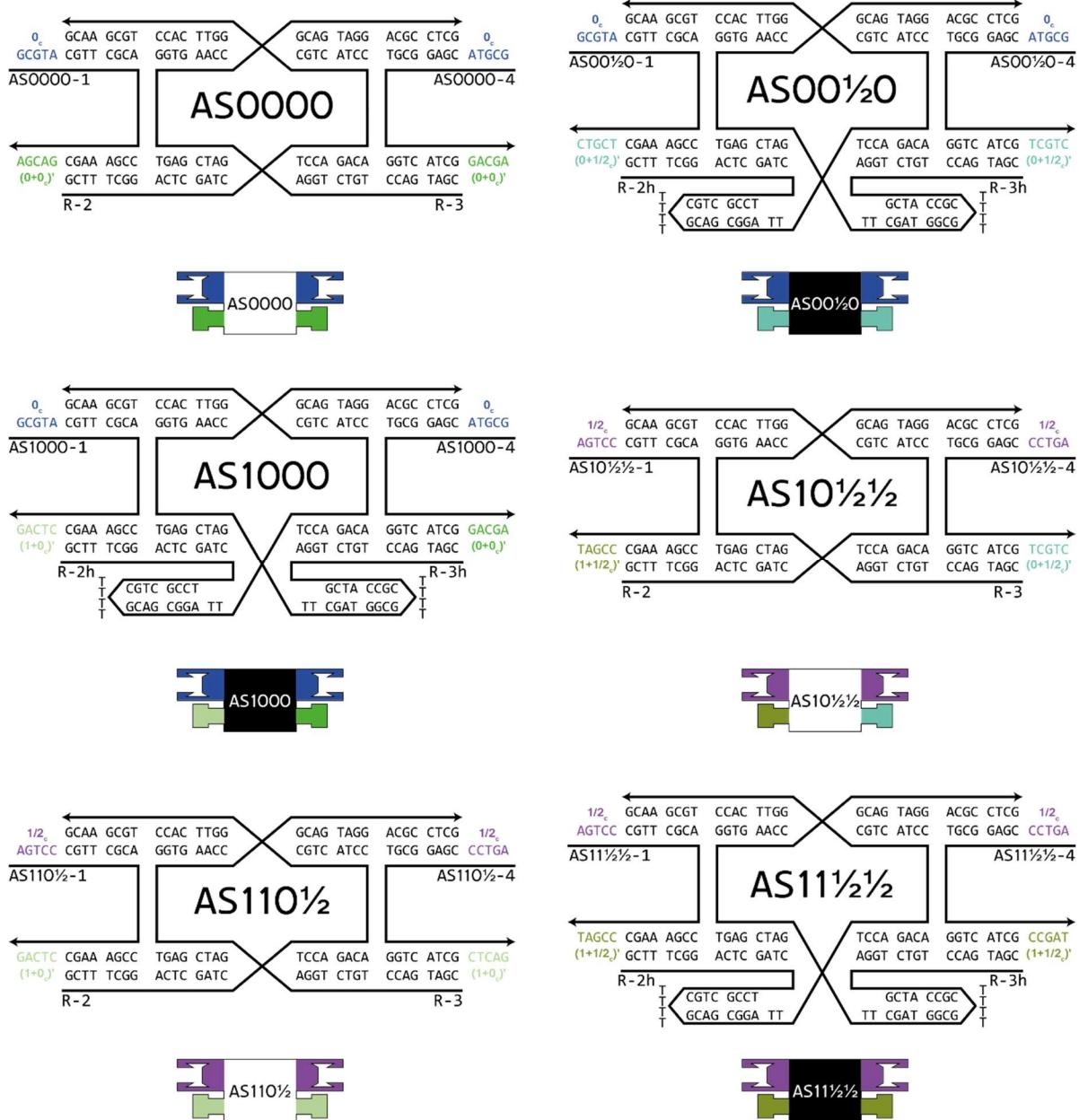


Figure S5. A sequence diagram with cartoon representation of the solutions for addition. A solution, *i.e.*, AS0000, AS00½0, AS1000, AS10½½, AS110½, and AS11½½, which is placed at the centre of crystal, reveals the result of summation based on the information obtained from left and right connectors.

Tile	Strand	Length	Sequence (5' to 3')
AS0000	AS0000-1	26mers	gcgtacgttcgcaccgaaaggcga
	R-2	48mers	gccttcggactcgatctccagacacctactcggttacactcgaa
	R-3	48mers	cgtacgttcgtggatcgagttggtaaccgcgttagggacgcctcg
AS1000	AS0000-4	26mers	gcgtacgaggcggtgtcatcgacga
	AS1000-1	26mers	gcgtacgttcgcaccgaaaggcctcg
	R-2h	70mers	gccttcggactcgatctccgtctttgcagcgattccagacacctactcggttacactcgaa
AS110½	R-3h	70mers	cgtacgttcgtggatcgagttggtaaccgcgttagggacgcctcg
	AS1000-4	26mers	gcgtacgaggcggtgtcatcgacga
	AS110½ -1	26mers	agtcccggttcgcaccgaaaggcctcg
AS00½0	R-2	48mers	gccttcggactcgatctccagacacctactcggttacactcgaa
	R-3	48mers	cgtacgttcgtggatcgagttggtaaccgcgttagggacgcctcg
	AS110½ -4	26mers	agtcccgaggcggtgtcatcgctcg
AS00½0	AS00½0-1	26mers	gcgtacgttcgcaccgaaaggcctcg
	R-2h	70mers	gccttcggactcgatctccgtctttgcagcgattccagacacctactcggttacactcgaa
	R-3h	70mers	cgtacgttcgtggatcgagttggtaaccgcgttagggacgcctcg
AS10½½	AS00½0-4	26mers	gcgtacgaggcggtgtcatcgctcg
	AS10½½ -1	26mers	agtcccggttcgcaccgaaaggcccgt
	R-2	48mers	gccttcggactcgatctccagacacctactcggttacactcgaa
AS11½½	R-3	48mers	cgtacgttcgtggatcgagttggtaaccgcgttagggacgcctcg
	AS10½½ -4	26mers	agtcccgaggcggtgtcatcgctcg
	AS11½½ -1	26mers	agtcccggttcgcaccgaaaggcccgt
AS11½½	R-2h	70mers	gccttcggactcgatctccgtctttgcagcgattccagacacctactcggttacactcgaa
	R-3h	70mers	cgtacgttcgtggatcgagttggtaaccgcgttagggacgcctcg
	AS11½½ -4	26mers	agtcccgaggcggtgtcatcgccgt

Table S6. Sequence information of solutions for addition.

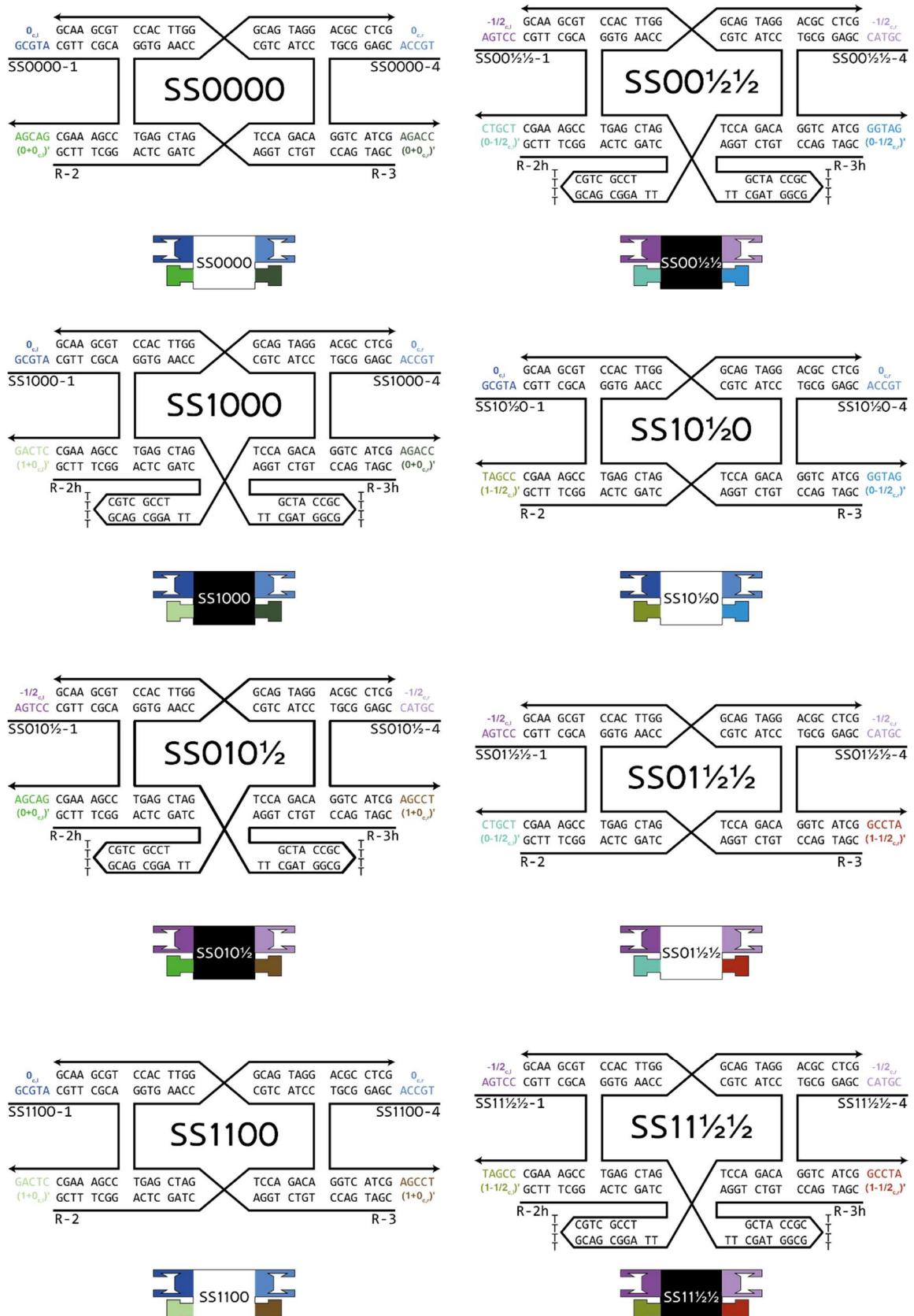


Figure S6. A sequence diagram with cartoon representation of the solutions for subtraction. A solution, *i.e.*, SS0000, SS00½½, SS1000, SS10½0, SS010½, SS01½½, SS1100, or SS11½½, which is placed at the centre of crystal, reveals the result of subtraction based on the information obtained from left and right connectors.

Tile	Strand	Length	Sequence (5' to 3')
SS0000	SS0000-1	26mers	gcgtacggtcgccaccgaaaggcgcga
	R-2	48mers	gcttcggactcgatctccagacacctactgcgggtcacctgcgaacg
	R-3	48mers	cgtacccgtctggagatcgagtggtaaccgcagtagggacgcctcg
	SS0000-4	26mers	tgcacgaggcggtggcatcgagacc
SS010½	SS010½ -1	26mers	agtcggactcgccaccgaaaggcgcga
	R-2h	70mers	gcttcggactcgatctccgtctttgcggattccagacacctactgcgggtcacctgcgaacg
	R-3h	70mers	cgtacccgtctggagatcgactttcggtagttgatcgagtggtaaccgcagtagggacgcctcg
	SS010½ -4	26mers	cgtaccgaggcggtggcatcgagcc
SS1000	SS1000-1	26mers	gcgtacggtcgccaccgaaaggcctcg
	R-2h	70mers	gcttcggactcgatctccgtctttgcggattccagacacctactgcgggtcacctgcgaacg
	R-3h	70mers	cgtacccgtctggagatcgactttcggtagttgatcgagtggtaaccgcagtagggacgcctcg
	SS1000-4	26mers	tgcacgaggcggtggcatcgagacc
SS1100	SS1100-1	26mers	gcgtacggtcgccaccgaaaggcctcg
	R-2	48mers	gcttcggactcgatctccagacacctactgcgggtcacctgegaacg
	R-3	48mers	cgtacccgtctggagatcgagtggtaaccgcagtagggacgcctcg
	SS1100-4	26mers	tgcacgaggcggtggcatcgagcc
SS00½	SS00½ -1	26mers	agtcggactcgccaccgaaaggcgtc
	R-2h	70mers	gcttcggactcgatctccgtctttgcggattccagacacctactgcgggtcacctgcgaacg
	½ R-3h	70mers	cgtacccgtctggagatcgactttcggtagttgatcgagtggtaaccgcagtagggacgcctcg
	S00½ -4	26mers	cgtaccgaggcggtggcatcggttag
SS01½	SS01½ -1	26mers	agtcggactcgccaccgaaagctcg
	R-2	48mers	gcttcggactcgatctccagacacctactgcgggtcacctgcgaacg
	½ R-3	48mers	cgtacccgtctggagatcgagtggtaaccgcagtagggacgcctcg
	SS01½ -4	26mers	cgtaccgaggcggtggcatcgcccta
SS10½0	SS10½0-1	26mers	gcgtacggtcgccaccgaaaggccccat
	R-2	48mers	gcttcggactcgatctccagacacctactgcgggtcacctgcgaacg
	R-3	48mers	cgtacccgtctggagatcgagtggtaaccgcagtagggacgcctcg
	SS10½0-4	26mers	tgcacgaggcggtggcatcggttag
SS11½	SS11½ -1	26mers	agtcggactcgccaccgaaaggcccgt
	R-2h	70mers	gcttcggactcgatctccgtctttgcggattccagacacctactgcgggtcacctgcgaacg
	½ R-3h	70mers	cgtacccgtctggagatcgactttcggtagttgatcgagtggtaaccgcagtagggacgcctcg
	SS11½ -4	26mers	cgtaccgaggcggtggcatcgcccta

Table S7. Sequence information of solutions for subtraction.