In-situ Measurement of 3D Crystal Size Distribution by Double-View Image Analysis with Case Study on L-glutamic Acid Crystallization

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Supporting Information

Derivation of Eq.(10)

From Fig.4a, the following two equations stand according to the common property of two similar triangles,

$$\frac{Z-f_{\mathrm{c}}}{Z}=\frac{b_1-a_1}{b_1} \tag{A1}$$

$$\frac{Z-f_{\mathrm{c}}}{Z}=\frac{b_2-a_2}{b_2} \tag{A2}$$

where $f_{\mathrm{c}}=f\cos\theta$ and $b_1+b_2=b$.

The above equations can be equivalently transformed into

$$\frac{b_1-a_1}{Z-f_{\mathrm{c}}}=\frac{b_1}{Z} \tag{A3}$$

$$\frac{b_2-a_2}{Z-f_{\mathrm{c}}}=\frac{b_2}{Z} \tag{A4}$$

Since $b_1+b_2=b$, it can be derived that

$$Z=\frac{bf_{\mathrm{c}}}{a_1+a_2} \tag{A5}$$

Fig.4b shows the geometric diagram in the left view of camera. According to the sine law, there follows

$$\frac{\sin(90+\tau_{\mathrm{l}})}{m_{\mathrm{l}}}=\frac{\sin(90-\theta-\tau_{\mathrm{l}})}{x_{\mathrm{l}}} \tag{A6}$$

where $n_{\mathrm{l}}=f\sin\theta$.

It can be derived from (A6) that

$$m_{\mathrm{l}}=\frac{x_{\mathrm{l}}\sin(90+\tau_{\mathrm{l}})}{\sin(90-\theta-\tau_{\mathrm{l}})} \tag{A7}$$

It can be seen from Fig.4b that

$$a_1=m_{\mathrm{l}}+n_{\mathrm{l}} \tag{A8}$$

Since Fig.4c shows the geometric diagram in the right view of camera, it follows from the sine law that

$$\frac{\sin(90-\tau_{\mathrm{r}})}{m_{\mathrm{r}}}=\frac{\sin(90-\theta+\tau_{\mathrm{r}})}{x_{\mathrm{r}}} \tag{A9}$$

where $n_{\mathrm{r}}=f\sin\theta-m_{\mathrm{r}}$.

It can be derived from (A9) that

$$m_{\mathrm{r}} = \frac{x_{\mathrm{r}} \sin(90-\tau_{\mathrm{r}})}{\sin(90-\theta-\tau_{\mathrm{r}})} \tag{A10}$$

It can be seen from Fig.4c that

$$a_2 = n_{\mathrm{r}} \tag{A11}$$

Therefore,

$$a_1 + a_2 = m_1 + n_1 + n_{\mathrm{r}} \tag{A12}$$

By substituting (A12) into (A5), it yields

$$Z = \frac{bf\cos\theta}{2f\sin\theta + \dfrac{x_1 \sin(90+\tau_1)}{\sin(90-\theta-\tau_1)} + \dfrac{x_{\mathrm{r}} \sin(90-\tau_{\mathrm{r}})}{\sin(90-\theta+\tau_{\mathrm{r}})}} \tag{A13}$$

which may be rewritten as

$$Z = \frac{bf\cos\theta}{2f\sin\theta + \dfrac{x_1 \cos\tau_1}{\cos(\theta+\tau_1)} - \dfrac{x_{\mathrm{r}} \cos\tau_{\mathrm{r}}}{\cos(\theta-\tau_{\mathrm{r}})}} \tag{A14}$$

where $\tan\tau_1 = x_1 / f$, $x_1 = \dfrac{\gamma}{\kappa}\left|u_1 - L/2\right|$, $\tan\tau_{\mathrm{r}} = x_{\mathrm{r}} / f$, and $x_{\mathrm{r}} = \dfrac{\gamma}{\kappa}\left|u_{\mathrm{r}} - L/2\right|$.