Quantitative Analysis of Attachment Time of Air Bubbles to Solid Surfaces in Water

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SUPPORTING INFORMATION

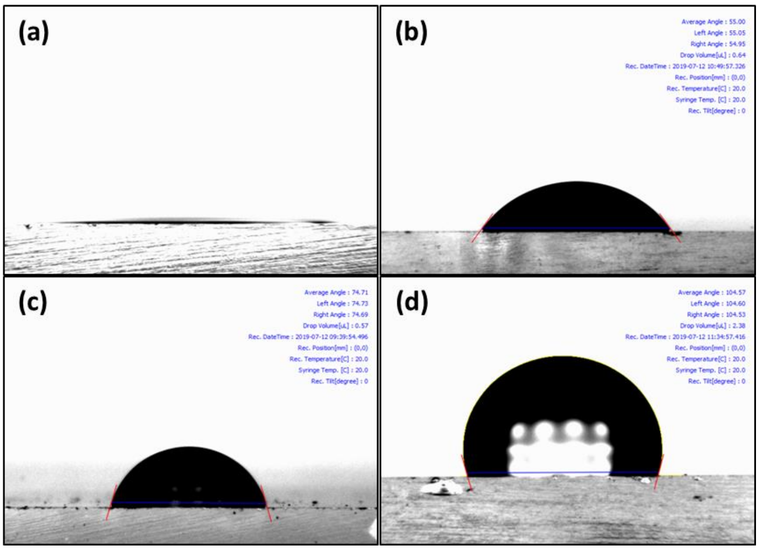

Figure S1. Images of the water droplet static contact angles (a) 0° (pristine glass slide), (b) 55° (0.01 mM OTS in toluene for 10 min), (c) 75° (0.1 mM OTS in toluene for 10 min), and (d) 105° (1 mM OTS in toluene for 10 min)

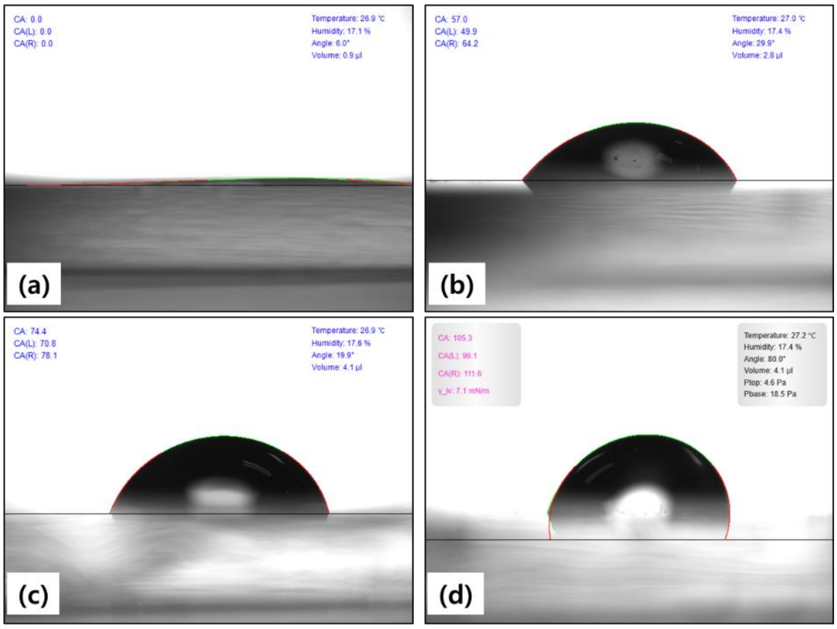

Figure S2. Images of the hysteresis of the water contact angles (a) 0° (pristine glass slide) (b) 55 ± 10° (0.01 mM OTS in toluene for 10 min), (c) 75 ± 5° (0.1 mM OTS in toluene for 10 min), and (d) 105 ± 5° (1 mM OTS in toluene for 10 min)

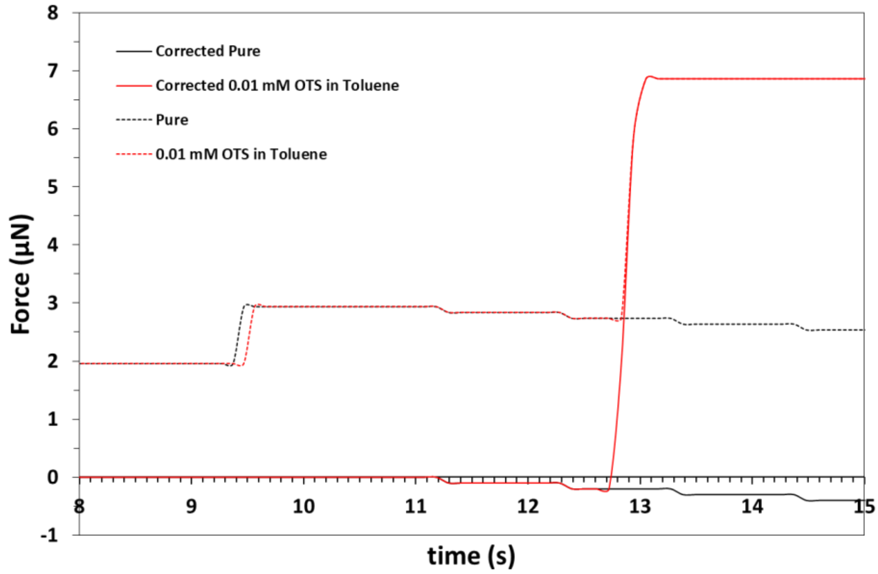

Figure S3. Corrected force curves incorporating the capillary effect of the needle produced by the water in the cell (solid line: after correction, dotted line: before correction)

Conversion to dimensionless forms

Capillary number: $Ca = \mu V / \sigma$

$$x = \frac{r}{R} Ca^{-1/4};\ \ y = \frac{h}{R} Ca^{-1/2};\ \ Y = \frac{H}{R} Ca^{-1/2};\ \ p = \frac{R}{\sigma} P;\ \ \tau = \frac{\sigma\sqrt{Ca}}{R\mu} t;\ \ \tau_0 = \frac{\sigma\sqrt{Ca}}{R\mu} t_0;$$

$$\wp = \frac{R}{\sigma}\Pi \quad ;\ \ \Phi = \frac{F}{2\pi R\sigma Ca^{1/2}};$$

$$\frac{\partial y}{\partial \tau} = \frac{1}{4x}\frac{\partial}{\partial x}\left\{xy^3 \frac{\partial p}{\partial x}\right\} + \frac{d^*}{x}\frac{\partial}{\partial x}\left\{xy^2 \frac{\partial p}{\partial x}\right\} = \frac{1}{4}\left\{\frac{y^3}{x}\frac{\partial p}{\partial x} + 3y^2 \frac{\partial y}{\partial x}\frac{\partial p}{\partial x} + y^3 \frac{\partial^2 p}{\partial x^2}\right\} + \ldots$$

$$p(y,x) = 2 - \frac{1}{x}\frac{\partial}{\partial x}\left\{x\frac{\partial y}{\partial x}\right\} - \wp(y) = 2 - \left\{\frac{\partial^2 y}{\partial x^2} + \frac{1}{x}\frac{\partial y}{\partial x}\right\} - \wp(y)$$

$$y(x,\tau = 0) = Y + x^2/2$$

$$p(x,y,\tau = 0) = 0$$

$$\frac{\partial y_N}{\partial \tau} = -1 - \frac{\partial \Phi}{\partial \tau}\left\{\log\frac{x_\infty Ca^{1/4}}{2} + B(\theta_o)\right\}$$

$$p(y, x \to \infty) = 0$$

$$\Phi = \int_0^{r_\infty}\left\{p(x,\tau) + \wp(x,\tau)\right\} x dx$$

At *x* = 0, we obtain the following limits:

$$\left(\frac{\partial Y}{\partial x}\right)_{x=0} = 0; \quad \left(\frac{\partial p}{\partial x}\right)_{x=0} = 0$$

$$p(Y,0) = 2 - 2\left(\frac{\partial^2 Y}{\partial x^2}\right)_{x=0} - \wp(Y)$$

$$\frac{\partial Y}{\partial \tau} = \left\{\frac{3}{2}Y^3 + d^* Y^2\right\}\left\{\frac{\partial^2 p}{\partial x^2}\right\}_{x=0}$$

Numerical Solution by the three-point Finite Difference Method

We divide the film radius into N-1 sections (stages) and consider N points on the 1D mesh. Applying the three-point FDM, we obtain the following ordinary differential equations (ODEs):

Section 1 (around the film symmetry): $Y = y_1;\ \Delta = \text{film radius} / \left(N-1\right)$

$$\frac{\partial y_1}{\partial \tau} = \left\{\frac{3}{2} y_1^{\,3} + d^* y_1^{\,2}\right\} \frac{-2p_1 + 2p_2}{\Delta^2} \quad ; \quad p_1 = 2 - \frac{-4y_1 + 4y_2}{\Delta^2} - \wp\left(y_1\right)$$

Section k = 2 to N-1 (intermediate sections):

$$\frac{\partial y_k}{\partial \tau} = \frac{m}{12}\left\{\frac{y_k^{\,3}}{x_k} \frac{p_{k+1} - p_{k-1}}{2\Delta} + 3y_k^{\,2} \frac{y_{k+1} - y_{k-1}}{2\Delta} \frac{p_{k+1} - p_{k-1}}{2\Delta} + y_k^{\,3} \frac{p_{k+1} - 2p_k + p_{k-1}}{\Delta^2}\right\} + \ldots$$

$$p_k = 2 - \left\{\frac{y_{k+1} - 2y_k + y_{k-1}}{\Delta^2} + \frac{y_{k+1} - y_{k-1}}{2x_k \Delta}\right\} - \wp\left(y_k\right)$$

Section N (at the film boundary):

$$\frac{\partial y_N}{\partial \tau} = -1 - \frac{\partial \Phi}{\partial \tau}\left\{\log \frac{x_\infty Ca^{1/4}}{2} + B\left(\theta_c\right)\right\} \quad ; \quad p_N = 0$$

This system of ODEs can be solved using the Matlab ODE solver “ODE15S.”

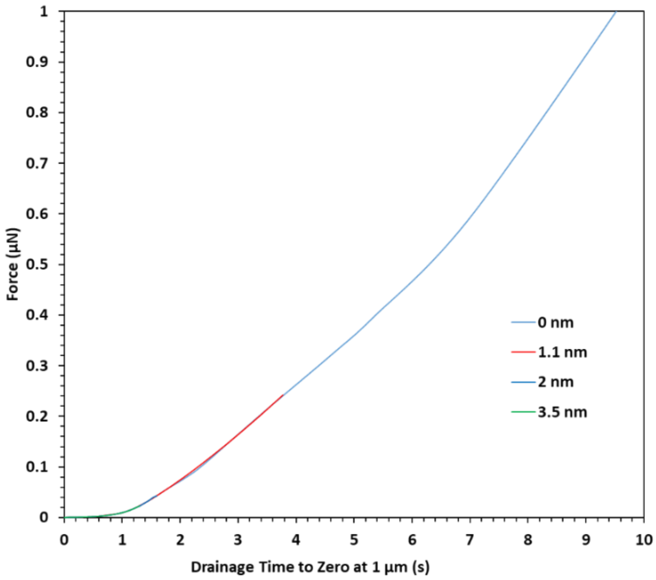

Figure S4. Force with respect to the drainage time calculated by Eq. 15 (before attachment)

The y-axis in Figure S3 is the force calculated by Eq. (15) in the manuscript:

$$F(t) = 2\pi \int_0^{r_\infty} \left\{ P(x,t) + \Pi(x,t) \right\} x dx \tag{15}$$

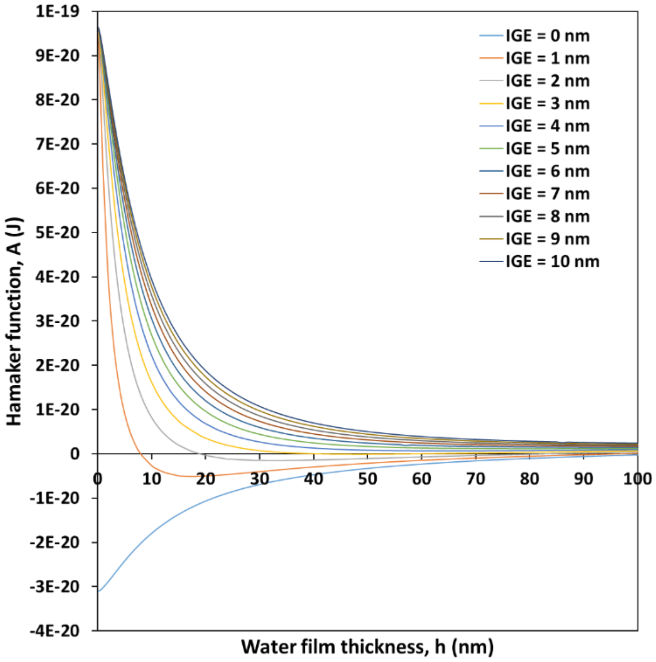

Figure S5. Hamaker function of silica showing increment with the IGE thickness.