

## **Supporting information**

# **Mathematical modeling of microalgal internal metabolic behaviors under heterotrophic condition and its application**

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## S1. Assumptions

In the proposed model takes below assumptions into account for established a mathematical model having acceptable complexity:

- Carbon, nitrogen, phosphorus, and sulfur is nutrient for algal growth. Micronutrients such as vitamins and minerals are neglected.
- The proposed model structural and equations are developed based on dominant phenomena.
- Only nitrogen and phosphate contribute to synthesis of nucleotides.
- Amino acid is synthesized by using carbon backbone (C pool), nitrogen (N pool), and sulfur (S pool).
- Chlroin family pigments are grouped as “pigment a”
- Carotenoid family pigments are grouped as “pigment b”
- Pigments, protein, polysaccharide, and membrane lipid are degraded under stress conditions.
- There is no substrate inhibition.

## S2. Parameter estimation results

The estimated model parameters of the proposed model and reduced models are enumerated below.

### *S2.1 Model parameters of the general model*

**Table S1:** Nominal parameter values for the proposed model obtained from experimental data in the literature <sup>1-</sup>

Index	Parameters	Value	Source	Index	Parameters	Value	Source
1	$q_{C,\max}$	30.54		40	$\pi_{pig1,\max}$	0.505	
2	$q_{N,\max}$	0.515		41	$\pi_{pig2,\max}$	0.861	
3	$q_{P,\max}$	0.345		42	$\pi_{ps,\max}$	2.598	
4	$q_{S,\max}$	0.269		43	$\pi_{ml,\max}$	8.512	
5	$q_{aa,\max}$	4.740	<sup>14</sup>	44	$\pi_{nl,\max}$	6.295	
6	$q_{nt,\max}$	7.110	<sup>14</sup>	45	$\lambda_{aa,\max}$	0.029	
7	$q_{ml,\max}$	5.925	<sup>14</sup>	46	$\lambda_{pro,\max}$	1.398	

8	$q_{nl,\max}$	19.75	14	47	$\lambda_{pig1,\max}$	0.094	
9	$q_{pig1,\max}$	1.750	14	48	$\lambda_{pig2,\max}$	0.509	
10	$q_{pig2,\max}$	1.750	14	49	$\lambda_{ps,\max}$	0.647	
11	$q_{pro,\max}$	25.68	14	50	$\lambda_{ml,\max}$	0.349	
12	$q_{ps,\max}$	11.07	14, 15	51	$\lambda_{nl,\max}$	0.185	
13	$q_{ms,\max}$	6.701	14, 15	52	$Y_{C/ms}$	2.631	
14	$q_{C,\min}$	0	Fixed	53	$Y_{C/pig1}$	6.409	
15	$q_{N,\min}$	0	Fixed	54	$Y_{C/pig2}$	6.185	
16	$q_{P,\min}$	0	Fixed	55	$Y_{C/aa}$	0.098	
17	$q_{S,\min}$	0	Fixed	56	$Y_{C/ml}$	0.158	
18	$q_{aa,\min}$	0	14	57	$Y_{aa/C}$	0.236	
19	$q_{nt,\min}$	1.185	14	58	$Y_{aa/N}$	1.880	
20	$q_{ml,\min}$	1.975	14	59	$Y_{aa/S}$	0.127	
21	$q_{nl,\min}$	3.950	14	60	$Y_{aa/pro}$	0.016	
22	$q_{pig1,\min}$	0.079	14	61	$Y_{aa/pig2}$	0.044	
23	$q_{pig2,\min}$	0.079	14	62	$Y_{nt/P}$	0.548	
24	$q_{pro,\min}$	11.85	14	63	$Y_{nt/aa}$	0.890	
25	$q_{ps,\min}$	1.230	14, 15	64	$Y_{pro/aa}$	4.601	
26	$q_{ms,\min}$	0.745	14, 15	65	$Y_{pig1/C}$	1.360	
27	$\mu_{\max}$	1.700	16	66	$Y_{pig2/C}$	1.316	
28	$k_d$	0.008	16	67	$Y_{pig2/aa}$	0.340	
29	$\rho_{C,\max}$	38.00		68	$Y_{ps/C}$	1.111	
30	$\rho_{N,\max}$	26.39		69	$Y_{ms/ps}$	0.177	
31	$\rho_{P,\max}$	5.011		70	$Y_{ml/C}$	0.223	
32	$\rho_{S,\max}$	7.355		71	$Y_{ml/P}$	1.332	
33	$K_{s_C}$	0.255		72	$Y_{nl/C}$	0.201	
34	$K_{s_N}$	0.159		73	$\sigma_{nt}$	0.657	
35	$K_{s_P}$	0.108		74	$\sigma_{pro}$	0.148	
36	$K_{s_S}$	0.085		75	$\sigma_{pig1}$	0.320	
37	$\pi_{aa,\max}$	9.924		76	$\sigma_{pig2}$	0.863	
38	$\pi_{nt,\max}$	1.647		77	$\sigma_{ps}$	0.655	
39	$\pi_{pro,\max}$	9.498		78	$\sigma_{ml}$	1.785	

## S2.2 Model parameters of the example 1

**Table S2:** Parameter refinement result for the model reduction example 1

Index	Parameters	Initial value	Converged value	95% confidence interval	
1	$q_{car,\max}$	63.60	163.9	147.8	188.1

2	$q_{car,\min}$	2.994	25.98	25.20	26.87
3	$q_{N,\max}$	0.515	1.547	1.519	1.581
4	$q_{N,\min}$	0	0	0	0
5	$q_{lip,\max}$	12.32	246.2	227.1	272.6
6	$q_{lip,\min}$	3.121	3.121	3.121	3.121
7	$q_{pro,\max}$	31.45	494.7	477.0	514.7
8	$q_{pro,\min}$	13.23	145.9	142.5	150.4
9	$\mu_{\max}$	1.700	22.04	21.28	24.93
10	$k_d$	0.008	0.008	0.004	0.018
11	$\rho_{C,\max}$	38.00	331.0	327.1	338.2
12	$\rho_{N,\max}$	26.39	105.0	82.89	137.2
13	$K_{s_C}$	0.255	0.192	0.131	0.317
14	$K_{s_N}$	0.159	0.159	0.159	0.159
15	$\pi_{ml,\max}$	7.795	7.795	7.795	7.795
16	$\pi_{nl,\max}$	7.212	7.212	7.212	7.212
17	$\pi_{pro,\max}$	13.31	78.74	76.62	81.84
18	$\lambda_{ml,\max}$	0.200	0.044	0.026	0.108
19	$\lambda_{pro,\max}$	2.728	3.556	3.091	5.001
20	$Y_{car/C}$	0.534	1.841	1.833	1.855
21	$Y_{car/ml}$	1.686	1.686	1.686	1.686
22	$Y_{car/pro}$	1.401	0.151	0.082	0.306
23	$Y_{ml/car}$	0.256	0.216	0.216	0.216
24	$Y_{nl/car}$	0.349	1.629	1.622	1.673
25	$Y_{pro/N}$	1.965	1.965	1.965	1.965
26	$Y_{pro/car}$	0.853	0.659	0.658	0.663
27	$\sigma_{car}$	0.457	0.457	0.457	0.457
28	$\sigma_{lip}$	0.186	0.186	0.186	0.186
29	$\sigma_{pro}$	0.765	0.765	0.765	0.765
30	$k_{nt}$	2.265	2.265	2.265	2.265
31	$k_{pig}$	0.066	0.066	0.066	0.066

### S2.3 Model parameters of the example 2

**Table S3:** Parameter refinement result for the model reduction example 2

Index	Parameters	Initial value	Converged value	95% confidence interval	
1	$q_{C,\max}$	30.54	46.03	35.46	61.83
2	$q_{C,\min}$	0	0	0	0
3	$q_{em,\max}$	19.75	46.87	41.09	58.82

4	$q_{em,\min}$	3.950	3.950	3.950	3.950
5	$q_{sm,\max}$	121.6	47.63	47.63	47.64
6	$q_{sm,\min}$	12.00	0.872	0.833	0.941
7	$K_{pH}$	0	$10.0^{-8.908}$	$10.0^{-8.908}$	$10.0^{-8.908}$
8	$K_{I,pH}$	$10.0^{-7.0}$	$10.0^{-5.284}$	$10.0^{-5.284}$	$10.0^{-5.284}$
9	$\mu_{\max}$	1.700	1.859	1.426	2.509
10	$k_d$	0.008	0.034	0.029	0.039
0.11	$\rho_{C,\max}$	38.00	69.21	55.36	98.49
12	$K_{s_C}$	0.255	4.443	2.924	7.316
13	$\pi_{em,\max}$	6.295	7.867	7.867	7.872
14	$\pi_{sm,\max}$	5.770	0.246	0.163	0.354
15	$\lambda_{sm,\max}$	0.039	28.27	11.83	47.80
16	$Y_{C/sm}$	2.064	0.723	0.723	0.723
17	$Y_{em/C}$	0.028	1.245	1.245	1.245
18	$Y_{sm/C}$	0.593	0.577	0.468	0.743
19	$\sigma_{sm}$	1.226	0.032	0.032	0.032
20	$k_{lutein}$	0.100	0.134	0.134	0.134

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