# Supporting information for: Improved accuracy and efficiency in quantum embedding through absolute localization

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#### 1 Choice of Embedding Strategy

In Figure 2 and Figure 3, the 3rd excited states of chlorodecane and decanol are not included due to the their nature of global excitatoins. Here, we show the embedding results for global excitations. When full-system basis is used the Huzinaga operator converge to the EOM-CCSD results on full system at the ninth carbon; while subsystem basis results are still not converged at the ninth carbon. This is a caveat that absolute localization should not be applied to global excitations and real charge-transfer states.



Figure S1: Natural transition orbitals (NTOs) of chlorodecane and decanol on the third singlet excited state. Shown on HONTOs and LUNTOs.



Figure S2: Unsigned errors of vertical excitation energy for the third excited state of chlorodecane and decanol using EOM-CCSD-in-M06. Results are compared to canonical EOM-CCSD.

molecule	1- decene			decanal			decanoic acid		
# of C	S1	S2	S3	S1	S2	S3	S1	S2	S3
0	8.843	9.047	9.138	4.222	8.620	10.063	6.326	8.603	9.452
1	8.405	8.600	8.965	4.542	8.717	10.013	6.311	8.811	9.467
2	8.530	8.694	8.892	4.534	8.547	9.440	6.220	8.703	9.415
3	8.475	8.698	8.871	4.561	8.538	9.268	6.243	8.735	9.500
4	8.486	8.727	8.902	4.566	8.548	9.270	6.229	8.743	9.471
5	8.504	8.738	8.908	4.572	8.572	9.331	6.233	8.756	9.479
6	8.512	8.751	8.923	4.572	8.568	9.326	6.228	8.757	9.466
7	8.529	8.754	8.927	4.574	8.579	9.326	6.229	8.764	9.470
8	8.531	8.764	8.939	4.574	8.578	9.327	6.228	8.766	9.466
9				4.576	8.582	9.329	6.229	8.774	9.462

Table S1: Excitation energies in eV using absolute localization for 1-decene, decanal, decanoic acid, 1-chlorodecane, 1-decanamine, and 1-decanol.

molecules	1-chlorodecane		1-decanaime			1-decanol			
# of C	S1	S2	S3	S1	S2	S3	S1	S2	S3
0	22.758	22.769	23.656	7.581	8.633	13.134	8.453	9.865	14.906
1	8.429	8.648	10.176	6.695	8.025	8.753	7.729	9.540	10.019
2	8.009	8.108	9.937	7.058	8.238	8.790	7.769	9.648	9.857
3	8.103	8.119	9.860	7.052	8.314	8.772	7.750	9.590	9.823
4	7.985	8.008	9.721	7.122	8.373	8.661	7.776	9.492	9.826
5	7.987	7.991	9.683	7.148	8.413	8.717	7.796	9.516	9.975
6	7.972	7.985	9.631	7.170	8.434	8.708	7.804	9.500	9.807
7	7.965	7.982	9.615	7.176	8.443	8.722	7.813	9.495	9.698
8	7.959	7.980	9.595	7.184	8.449	8.719	7.818	9.487	9.631
9	7.956	7.980	9.594	7.185	8.451	8.722	7.822	9.487	9.608
10	7.954	7.978	9.730	7.193	8.455	8.711	7.830	9.474	9.807

molecule	1- decene			decanal			decanoic acid		
# of C	S1	S2	S3	S1	S2	S3	S1	S2	S3
0	8.415	8.946	9.078	4.634	8.664	9.196	6.350	8.741	9.593
1	8.400	8.730	8.953	4.570	8.586	8.789	6.245	8.742	9.556
2	8.490	8.726	8.885	4.577	8.577	9.108	6.235	8.745	9.495
3	8.514	8.740	8.901	4.572	8.558	9.249	6.233	8.752	9.465
4	8.522	8.750	8.920	4.573	8.569	9.324	6.229	8.759	9.471
5	8.521	8.755	8.933	4.573	8.580	9.326	6.229	8.765	9.470
6	8.525	8.762	8.934	4.574	8.581	9.332	6.228	8.769	9.471
7	8.540	8.766	8.938	4.575	8.583	9.334	6.228	8.770	9.468
8	8.531	8.764	8.939	4.574	8.582	9.333	6.227	8.774	9.464
9				4.576	8.582	9.329	6.229	8.774	9.462

Table S2: Excitation energies in eV using Huzinaga operator with full-system basis for 1-decene, decanal, decanoic acid, 1-chlorodecane, 1-decanamine, and 1-decanol.

molecules	1-chlorodecane		1-decanaime			1-decanol			
# of C	S1	S2	S3	S1	S2	S3	S1	S2	S3
0	8.679	8.714	11.039	7.514	8.919	9.981	8.334	10.663	10.833
1	7.967	8.090	9.891	7.107	8.422	8.780	7.765	9.543	9.793
2	7.994	8.020	9.770	7.123	8.861	9.479	7.780	9.628	9.875
3	7.996	8.003	9.661	7.149	8.423	8.730	7.792	9.523	9.830
4	7.973	7.983	9.601	7.167	8.434	8.708	7.806	9.498	9.809
5	7.964	7.982	9.600	7.177	8.442	8.714	7.815	9.494	9.721
6	7.958	7.979	9.633	7.184	8.449	8.720	7.821	9.487	9.701
7	7.954	7.976	9.648	7.187	8.452	8.720	7.823	9.490	9.707
8	7.952	7.975	9.683	7.184	8.449	8.719	7.826	9.486	9.722
9	7.950	7.976	9.727	7.192	8.455	8.715	7.829	9.485	9.823
10	7.954	7.978	9.730	7.193	8.455	8.711	7.830	9.474	9.807

## 2 Eliminating Spurious Charge-Transfer States in TDDFT



Figure S3: The HONTOs and LUNTOs for embedded B3LYP/aug-cc-pVDZ calculations for acrolein in different amount of water molecules.

No.	n→	$n \rightarrow \pi^*$		π -	$\rightarrow \pi^*$	NTO
water	HONTO	LUNTO	weight	HONTO	LUNTO	weight
0	$\frac{S_1 = 3}{f = 0}$	53 eV 0002	1.00	$\frac{S_2=6}{f=0}$	.13 eV	0.74
5	$\bigcup_{i=0}^{n} \sum_{j=0}^{n} \sum_{i=0}^{n} \sum_{i=0}^{n} \sum_{i=0}^{n} \sum_{j=0}^{n} \sum_{i=0}^{n} \sum_{i$	77 eV 0002	1.00	$\sum_{k=1}^{\infty} \sum_{j=1}^{\infty} \sum_{j=1}^{\infty} \frac{S_{7}=6}{f=0}$	.10 eV	0.80
11	$\frac{S_1 = 3}{f = 0}$	77 eV 0003	1.00	$\frac{S_4=6}{f=0}$	24 eV 2434	0.66
19	$\frac{S_1 = 3}{f = 0}$	75 eV 0004	1.00	$S_{45} = 0$	5.96 eV 0.0964	0.71
24	$\frac{S_1 = 3}{f = 0}$	74 eV 0003	1.00	$S_{63} = f =$	6.03 eV 0.1811	0.78

Figure S4: The HONTOs and LUNTOs for B3LYP/aug-cc-pVDZ calculations for a crolein in different amount of water molecules.



Figure S5: The NTO pair with the largest weight for CAM-B3LYP/aug-cc-pVDZ calculations for acrolein in different amount of water molecules.

### 3 Excitation energies of acrolein + 2 water

Table S3: Excitatin energies in eV of TDHF-in-HF, TDDFT-in-DFT, EOM-CCSD-in-HF, EOM-CCSD-in-DFT compared to canonical TDHF, TDDFT, and EOM-CCSD using the non-relaxed (1) and relaxed (2) scheme, and corresponding correction (3, 4), respectively. All subsystems were described using the monomolecular basis.

Mathad	Error (kcal/mol)						
Method	1	2	3	4	full system		
TDHF					4.824		
TDHF-in-HF	4.774	4.745					
TDDFT					3.776		
TDDFT-in-DFT	3.759	3.729					
EOM-CCSD					4.065		
EOM-CCSD-in-HF	4.044	4.016	4.093	4.095			
EOM-CCSD-in-DFT	4.050	4.020	4.066	4.067			

#### 7HQ4

Table S4: Solvent-induced vertical excitation energy shifts  $\Delta \omega_{\pi \to \pi^*}$  in cm<sup>-1</sup> for the lowest  $\pi \to \pi^*$  transition in cis-7HQ chromophore. aug-cc-pVDZ basis set is used for all calculations. LDA is used for DFT and TDDFT calculations. Monomer basis is used for all embedding calculations. Complementary to Figure 5.

solvont	ground-state		state-a	verage	supermolecular		
sorvent	$(a)^{\dagger}$	$(b)^{\dagger}$	$(a)^{\dagger}$	$(b)^{\dagger}$	EOM-CCSD	TDLDA	
H <sub>2</sub> O	-458	-864	-492	-881	-525	-978	
$ m NH_3$	-647	-1136	-676	-1162	-784	-1380	
НСООН	-577	-1267	-586	-1305	-685	-1620	
$CH_{3}OH$	-263	-622	-248	-650	-342	-877	
$2H_2O$	-1079	-2003	-1089	-2053	-1273	-2564	
$\rm NH_3-H_2O-H_2O$	-1480	-2450	-1464	-2507	-1777	-3061	
$\rm NH_3$ - $\rm H_2O$ - $\rm NH_3$	-1384	-2326	-1381	-2380	-1624	-2823	
$\rm NH_3$ - $\rm NH_3$ - $\rm H_2O$	-1492	-2484	-1505	-2558	-1834	-3083	
Mean signed error <sup>§</sup>	183	-538	175	-582	0	-930	

<sup>†</sup> (a) refers to EOM-CCSD-in-LDA embedding; (b) refers to TDLDA-in-LDA embedding. <sup>§</sup> Mean signed errors are relative to the supermolecular EOM-CCSD results.

#### 5 GFP

Table S5: Excitation energy in eV of the brightest excited state in cluster TDDFT and embedded TDDFT-in-DFT calculations by successively adding the residues in the hydrogenbond chain from right to left and left to right.

right to	left	left to right			
embedded	cluster	embedded	cluster		
3.274	3.229	3.274	3.229		
3.263	3.225	3.242	3.374		
3.260	3.211	3.217	3.114		
3.234	3.240	3.193	3.216		
3.208	3.069	3.190	3.216		
3.175	3.069	3.175	3.069		

Table S6: Oscillator strength of the brightest excited state in cluster TDDFT and embedded TDDFT-in-DFT calculations by successively adding the residues in the hydrogen-bond chain from right to left and left to right.

right to	left	left to right			
embedded	cluster	embedded	cluster		
0.8373	0.5157	0.8373	0.5157		
0.8716	0.7177	0.7930	0.6187		
0.8712	0.7593	0.8411	0.7665		
0.7337	0.5605	0.7082	0.4837		
0.6121	0.8350	0.7102	0.4495		
0.7409	0.7345	0.7409	0.7345		