Supporting Information: Colorimetric and Near-Absolute Polarization Insensitive Refractive-Index Sensing in All-Dielectric Guided-Mode Resonance based Metasurface

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Theoretical Analysis of excitation of guided-mode resonances in the proposed 2D- grating

To shed some light on the physics behind this behavior, when the device is off-resonant, none of the diffracted wave orders from the grating can satisfy the conditions to become a guided-mode of the waveguide. So, the device essentially behaves as a multilayer film that mostly transmits the incoming wave, and some part is reflected due to Fresnel reflection. At resonance, however, a diffracted order can excite a guided-mode of the waveguide core. As the guided-mode propagates, the periodicity in the waveguide core causes it to continually radiate and leak energy to far-field. Accordingly, the phenomenon can be named as terms leaky-mode or quasi-guided mode resonance, as well.[1] The sharp increase in the reflection is due to the fact that the re-radiated waves interfere destructively with the directly transmitted waves, but constructively with the reflected waves. A more rigorous analysis is given in the Supporting Information.

To make the analysis more rigorous, the phase difference, Φ between directly transmitted waves, and secondary transmitted waves that after the guided-mode is re-diffracted is given by Eq.S1.[2] On the other hand, a waveguide mode is supported if it satisfies the self-consistency constraint, given in Eq.S2.[3? ?] Comparing these two equations shows that if Eq.S2 (properly exciting a guided-mode) is satisfied, then Φ needs to be an odd integer multiple of π by Eq.S1, so the transmitted waves to interfere destructively, and transmission is suppressed.

$$\Phi = 2ht_{Si_3N_4} + \Phi_{w,sub} + \Phi_{w,super} - \pi \tag{S1}$$

$$m2\pi = 2ht_{Si_3N_4} + \Phi_{w,sub} + \Phi_{w,super} \tag{S2}$$

where $h = k_0\sqrt{n_w^2 - n_{eff}^2}$ is the transverse component of the wavevector in the waveguide-grating, $k_0 = 2\pi/\lambda_0$ is free-space wavenumber, λ_0 is the free-space wavelength, n_w is the refractive index of the GWS, n_{eff} is the effective refractive index of the guided-mode in the core, and m is the order of the mode. The first term in both equations is due to the phase accumulated as guided-mode travels propagates inside the waveguide. The π term in Eq.S1 is because, each diffraction causes $\pi/2$ phase difference.[4] $\Phi_{w,sub}$ and $\Phi_{w,super}$ come from the total-internal reflection at the waveguide-substrate, and waveguide-superstrate boundaries, and the expressions for these are given in Eqns.S3 and S4, respectively.[2,5?]

$$\Phi_{w,sub} = -2\tan^{-1}[(\frac{n_w}{n_{sub}})^{2\rho} * \frac{p}{h}] \tag{S3}$$

$$\Phi_{w,super} = -2\tan^{-1}[(\frac{n_w}{n_{super}})^{2\rho} * \frac{q}{h}] \tag{S4}$$

where $p = k_0\sqrt{n_{eff}^2 - n_{sub}^2}$ $q = k_0\sqrt{n_{eff}^2 - n_{super}^2}$ and are the exponential decay constants in substrate (SiO_2) and superstrate regions, respectively. ρ is 0 for TE, 1 for TM modes; n_{super} and $n_{sub} = n_{SiO_2}$ are the refractive indices of the superstrate and substrate.

The effective index of refraction associated with the guided-mode inside the waveguide is not only controlled by Eq.S2, but also with the phase-matching condition. In other words, the real part of the complex wavevector component of the guided-mode in the direction of its propagation is equal to the components coming from the reciprocal lattice vector of grating and the incident wave.[2,6–10] Eq.S3 provides the phase-matching condition for the devices with 2D grating,[11] such as Device I in the main text, under normally incident light.

$$k_{x,g} = mK_x = n\frac{2\pi}{\Lambda_x} = n\frac{2\pi}{P\sqrt{3}}$$

$$k_{y,g} = mK_y = l\frac{2\pi}{\Lambda_y} = l\frac{2\pi}{P}$$

$$\beta = |k_{x,g}\hat{\mathbf{x}} + k_{y,g}\hat{\mathbf{y}}|$$

$$n_{eff} = \frac{\beta\lambda_0}{2\pi} \tag{S5}$$

where β is the propagation wavenumber along the guiding direction, Λ_x (K_x) and Λ_y (K_y) is the grating periods (reciprocal lattice vectors) along x and y-directions, respectively. n (l)

is the diffraction orders along x (y)-direction, and $k_{x,g}$ ($k_{y,g}$) is the wavevector component of the guided-mode along x (y)-direction.

Plugging-in Eqns. S3 and S4 inside Eq. S2 results in the well-known transcendental mode equations for TE and TM modes, given by Eqns. S6 and S7, respectively.

$$\tan(ht_{Si_3N_4}) = \frac{p+q}{h(1-\frac{pq}{h^2})} \tag{S6}$$

$$\tan(ht_{Si_3N_4}) = \frac{\overline{p}+\overline{q}}{h(1-\frac{\overline{p}\overline{q}}{h^2})} \tag{S7}$$

where $\bar{p}$ and $\bar{q}$ are defined as $p * n_w^2/n_{sub}^2$, $q * n_w^2/n_{super}^2$, respectively.

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