

## **Supporting Information.**

### Nonlinear metasurfaces governed by bound states in the continuum

Kirill Koshelev<sup>1,2</sup>, Yutao Tang<sup>3</sup>, Kingfai Li<sup>3</sup>, Duk-Yong Choi<sup>4,5</sup>,  
Guixin Li<sup>3</sup>, and Yuri Kivshar<sup>1,2</sup>

<sup>1</sup>Nonlinear Physics Center, Australian National University, Canberra ACT 2601, Australia

<sup>2</sup>ITMO University, St. Petersburg 197101, Russia

<sup>3</sup>Department of Material Science and Engineering, Shenzhen Institute for Quantum Science and Engineering, Southern University of Science and Technology, Shenzhen 518055, China

<sup>4</sup>Laser Physics Center, Australian National University, Canberra ACT 2601, Australia

<sup>5</sup>College of Information Science and Technology, Jinan University,  
Guangdong 510632, China

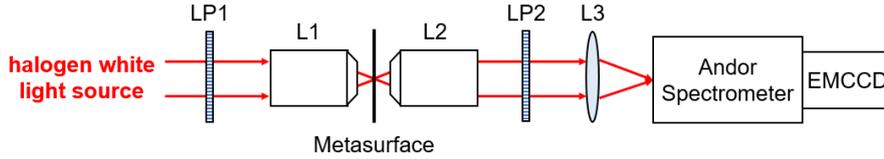
### **Contents**

I. Experimental setup for linear and nonlinear measurements

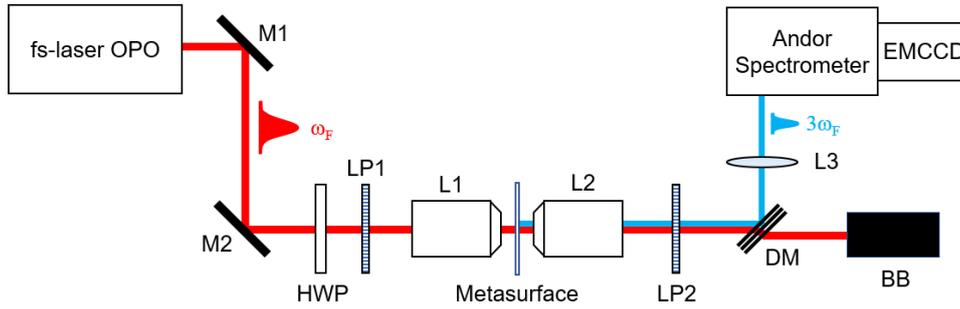
II. Details of numerical simulations

## I. Experimental setup for linear and nonlinear measurements

The schematic of the experimental setup for observation of the linear and third-harmonic generation from a silicon metasurface, described in the methods section, is shown in Figs. S1 and S2.



**Figure S1. Schematic of the experimental setup for linear optical measurements.** LP1/LP2, linear polarizers; L1/L2, objective lenses; L3, achromatic spherical lens.

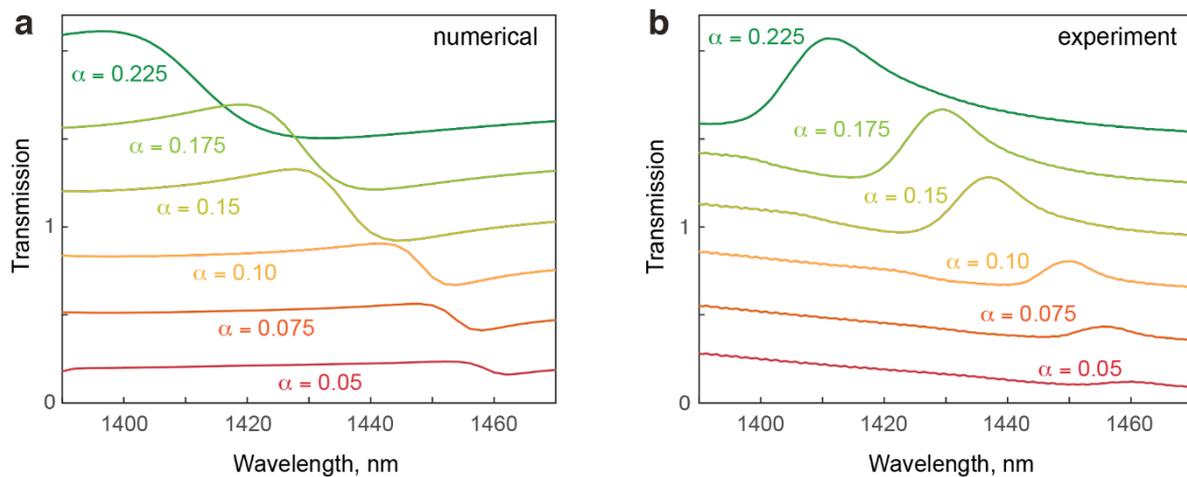


**Figure S2. Schematic of the experimental setup for THG measurements.** M1/M2, mirrors; HWP, half-wave plate; LP1/LP2, linear polarizers; L1/L2, objective lenses; DM, dichroic mirror; BB, beam blocker; L3, achromatic spherical lens. Bold lines indicate laser paths, red for fundamental wave and blue for THG signal wave.

## II. Details of numerical simulations.

For linear transmission calculations, shown in Fig. 2d of the main text, we limit the non-radiative losses to the real material losses. For nonlinear calculations, we introduce the additional non-radiative losses corresponding to  $Q_{nr} = 175$ . The origin of real experimental non-radiative losses is surface scattering, sample finiteness and disorder. For the sake of simplicity, we do not consider these complicated mechanisms of losses and introduce non-

radiative losses artificially as material absorption via a correction to the extinction coefficient  $\Delta k = n/(2Q_{nr})$ , where  $n$  is the refractive index. Such absorption losses give the correct values for  $Q_{nr}$  and the field enhancement inside the structure, however, predict different shape of peaks in linear transmittance compared to experimental results as shown in Fig. S3. Since the TH response is governed by the value of mode Q factor (see Eq. 4 of the main text), such approach gives a good approximation for description of the nonlinear response, as confirmed by Figs. 3b,d of the main text.



**Figure S3. Comparison of simulated and measured linear spectra.** Evolution of the (a) simulated and (b) measured transmission spectra vs. meta-atom asymmetry  $\alpha$ . Spectra are shifted relatively by 0.3 units.