Appendix

1. Fault tree Calculations (for t=420s)

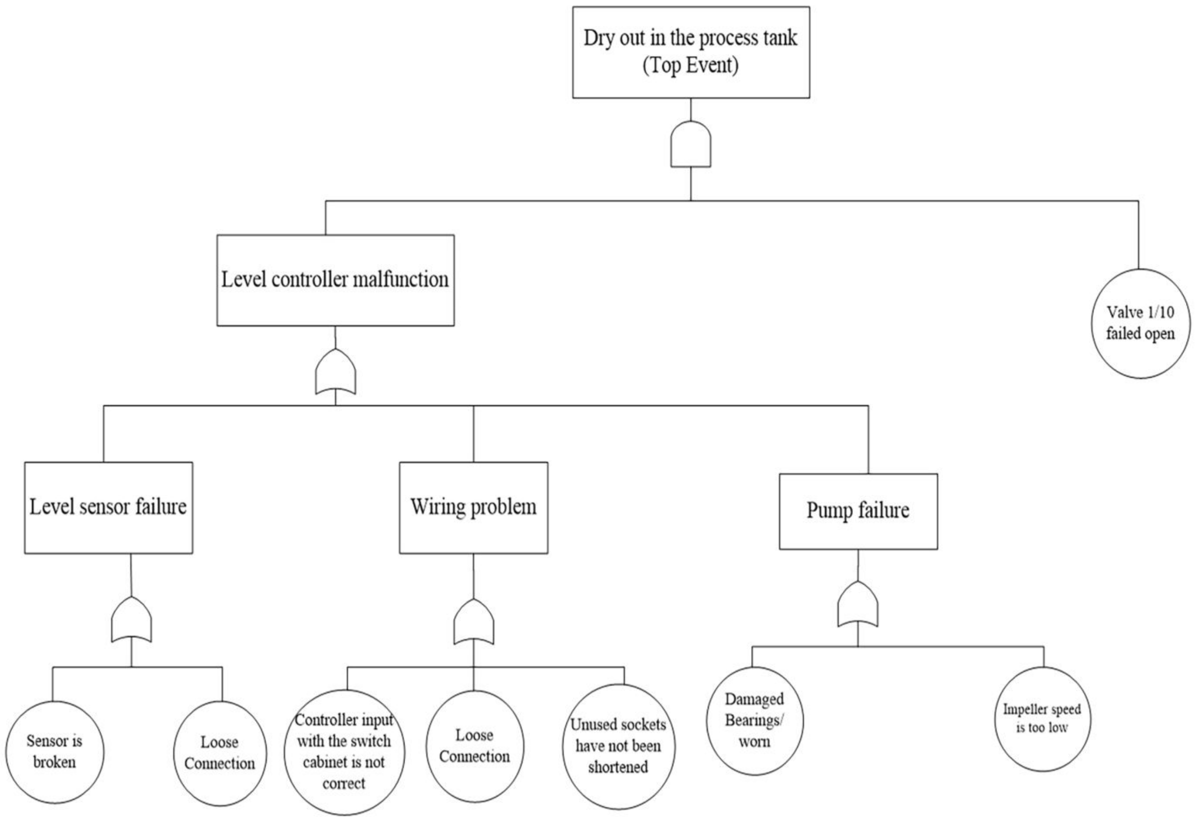


Figure A: Fault tree

Analysis

Table A shows the failure rates of the basic events used in the fault tree (Figure A) and dynamic fault tree (Figure B) and their corresponding probability for 420s.

Table A: Calculations of Probability of Basic Events for Static Fault Tree [27]

Events	failure rates (failure/s)	Probability, P t=420s
Controller input with switch cabinet is not correct (BE_1)	6.33×10^{-12}	2.66×10^{-09}
Loose Connection (BE_2)	2.00×10^{-10}	8.40×10^{-08}
Unused Sockets have not been shorted (BE_3)	6.33×10^{-13}	2.66×10^{-10}
Sensor is broken (BE_4)	9.11×10^{-09}	3.83×10^{-06}
Damaged bearings/ worn (BE_5)	8.00×10^{-16}	3.36×10^{-13}
Impeller speed is too high (BE_6)	2.00×10^{-09}	8.40×10^{-07}
Valve failed open (BE_7)	8.25×10^{-09}	3.46×10^{-06}

Sample Calculations

A sample calculation for an intermediate event (Level sensor failure) is shown here:

- P (Level Sensor failure) = P (BE_4 U BE_2)

$$= P(BE_4) + P(BE_2) - P(BE_4 \cap BE_2)$$

$$= (3.83\times10^{-06} + 8.40\times10^{-08}) - (3.83\times10^{-06} \times 8.40\times10^{-08})$$

So, P (Level Sensor failure) = 3.91×10^{-06}

Table B represents the intermediate events and top event probabilities for fault tree.

Table B: Calculated probabilities of intermediate events and top event from fault tree

Events	Probability, P t=420s
Wiring Problem	8.69×10^{-08}
Level Sensor Failure	3.91×10^{-06}
Pump P1 failure	8.4×10^{-07}
Level Controller Malfunction	4.83×10^{-06}
Valve Failed Open	3.46×10^{-06}
Dry out in the process tank (Top event)	1.67×10^{-11}

- For Specific Case: "Sensor is Broken"

Fault Tree Result:

P (Sensor is broken) = 1 [as it is already broken]

So, P (Level sensor failure) = P (Level controller malfunction) =1

P (valve failed open) = 3.46×10^{-06}

So, P (Dry out in the Process tank) = $1 \times (3.46\times10^{-06}) = 3.46\times10^{-06}$ (3.46×10^{-04} %)

2. <u>Dynamic Fault Tree Calculations (for t=420s)</u>

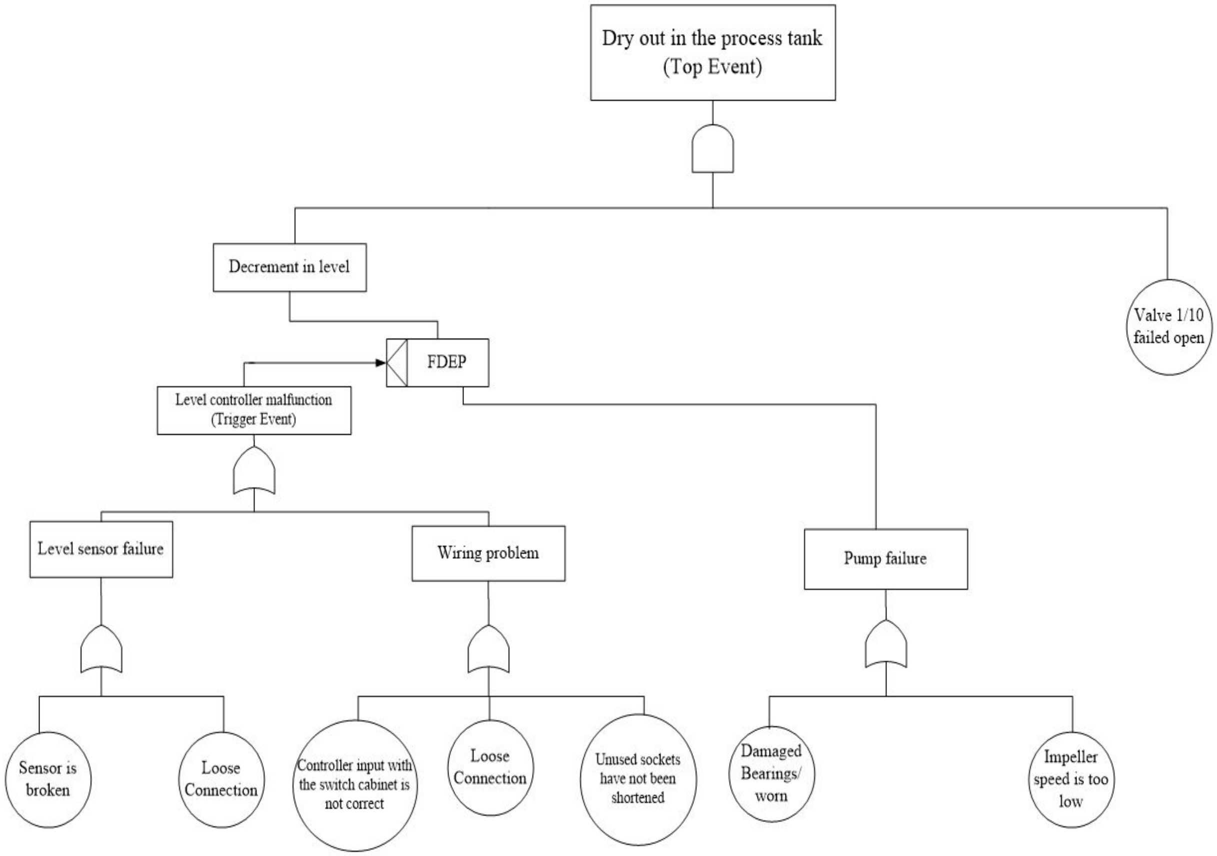
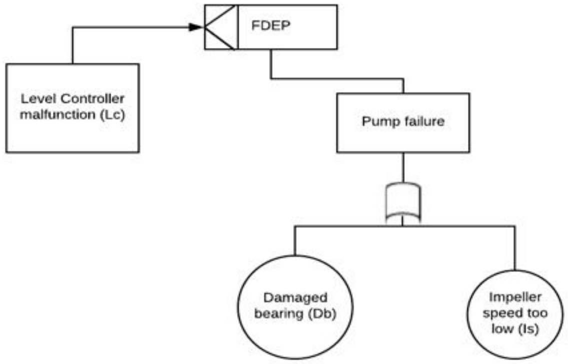


Figure B: Dynamic Fault tree

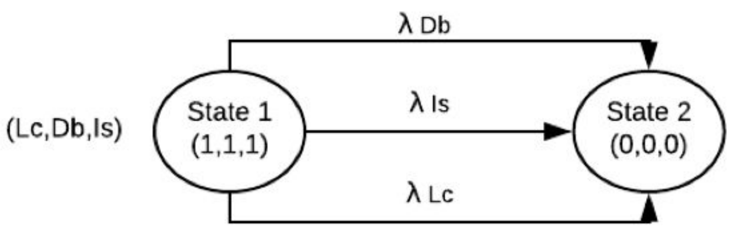
Sample Calculations

Sample calculations for Pump Failure and Decrement in level using Markov Chain are shown here:

- <u>Probability of pump failure (using Markov Chain):</u>



<u>State diagram for Pump failure (P):</u>



Now state1 probability P1,

$$\frac{dP1}{dt} = -(\lambda Lc + \lambda Db + \lambda Is)P1$$

$$So, \frac{dP1}{P1} = -(\lambda Lc + \lambda Db + \lambda Is)dt \text{..............} \text{ (i)}$$

Integrating equation (i),

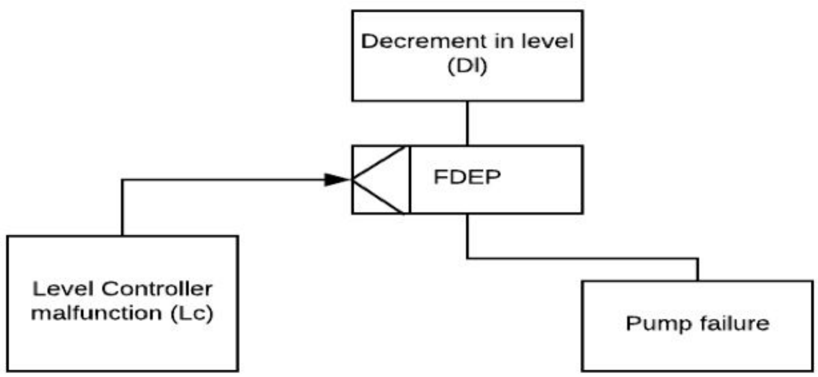
$\ln(P1) = -(\lambda Lc + \lambda Db + \lambda Is)t + C$ [at t=0, P1=1 so C=0]

P1= $e^{-(\lambda Lc + \lambda Db + \lambda Is)t}$ = 999.994×10^{-03}

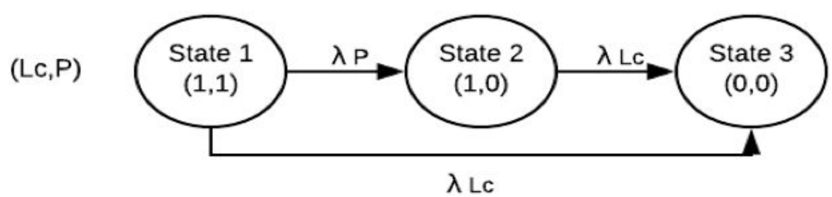
So, State 2 probability, P2= 1-P1 = 5.67×10^{-06}

The probability of Pump failure, P_P = 5.67×10^{-06} (λ_P = 1.35×10^{-08})

- <u>Probability of 'Decrement in level' (using Markov Chain):</u>



The state diagram for the gate is:



Now, State 1 probability, P1

$$\frac{dP1}{dt} = -(\lambda Lc + \lambda p)P1\ldots\ldots\ (ii)$$

$$\ln(P1) = -(\lambda Lc + \lambda p)t \quad \text{[integrating equation (ii)]}$$

$$P1 = e^{-(\lambda Lc + \lambda p)t} = 999.989 \times 10^{-03}$$

State 2 Probability, P2

$$\frac{dP2}{dt} = \lambda p \times P1 - \lambda Lc \times P2$$

$$\frac{dP2}{dt} + \lambda Lc \times P2 = \lambda p \times P1$$

Integrating factor= $e^{\lambda Lc \times t}$

$$e^{\lambda Lc \times t}\left[\frac{dP2}{dt} + \lambda Lc \times P2\right] = e^{\lambda Lc \times t}[\lambda p \times P1] = e^{\lambda Lc \times t}[\lambda p \times e^{-(\lambda Lc + \lambda p)t}] \quad \text{..........} \ (iii)$$

Integrating equation (iii),

$$e^{\lambda Lc \times t}P2 = -e^{-\lambda p \times t} + C$$

$$P2 = -e^{-(\lambda p + \lambda Lc) \times t} + Ce^{-\lambda Lc \times t}$$

$$P2 = -e^{-(\lambda Lc + \lambda p) \times t} + e^{-\lambda Lc \times t} \quad \text{[at t=0, P2=0 so, C=1]}$$

$$P2 = 6.1 \times 10^{-06}$$

So, State 3 probability, P3= 1-P1-P2 = 4.8×10^{-06}

So, the probability of Decrement in level, $P_{Dl} = 4.8 \times 10^{-06}$

The calculated probabilities for the intermediate events and top event are represented in table C.

Table C: Calculated Probabilities of the intermediate events and top events from DFT

Events	Probability, P t=420s
Wiring Problem	8.69×10^{-08}
Level Sensor Failure	3.91×10^{-06}
Level Controller Malfunction	4.83×10^{-06}
Pump Failure	5.67×10^{-06}
Decrement in Level	4.8×10^{-06}
Valve Failed Open	3.46×10^{-06}
Dry out in the Process Tank (Top Event)	1.66×10^{-11}

<u>For Specific Case: “Sensor is Broken”</u>

Dynamic Fault Tree:

P (Sensor is broken) = 1 [as it is already broken]

P (Level sensor failure) = P (level controller malfunction) = P (Pump failure) =1

P (Decrement in level) = 1

P (Valve failed open) = 3.46×10^{-06}

P (Dry out in the process tank) = $1 \times (3.46\times10^{-06}) = 3.46\times10^{-06}$ (3.46×10^{-04} %)

3. Table D shows the calculated dry out probabilities from the cumulative distribution graphs (figure 8 and figure 9)

Table D: Probabilities of flow rate and level less than the assumed threshold from CDFs

Experimental data sets	Flow rate probability less than 100 l/h	Level probability less than 10% in the tank
T1 (0 to 420s)	0.13	0.07
T2 (0 to 720s)	0.15	0.11
T3 (0 to 1020s)	0.17	0.15
T4 (0 to 1320s)	0.21	0.20

4. **Cumulative Gain Chart Construction:**

The construction of cumulative gain charts for the experimental data sets is described here. First, for fault tree, the assumption for dry out is flow rate of water less than 100 l/h. So, data which falls into this category is given the probability of 1 and data which does not fall into this category is given the probability of 0. Then after ranking, the data set was categorized into ten groups (deciles). The cumulative positive responses were determined afterwards. For dynamic fault tree, the assumption of dry out is flow rate less than 100 l/h and level in the process tank less than 10%. So, similarly, the data which falls into this class is given the probability of 1 and data which does not fall into this group is given the probability of 0. Ranking the data set and categorizing them into ten groups the cumulative positive responses were determined. Likewise, for Bayesian network assumption, dry out depends 70% on flow rate and 30% on level in the process tank threshold, the ranking and grouping of data set was done. Then cumulative positive responses were determined. The base model is the linear model which considers the same amount of positive responses as the decile of data, meaning, 10% of data will give 10% positive responses, 20% of data will give 20% positive responses and so on. For experimental data set T4, the results are in the following table E:

Table E: Cumulative positive responses for top event (dry out) considering three models for T4 (0-1320s) data set

Deciles	Cumulative positive responses for FT	Cumulative positive responses for DFT	Cumulative positive responses for BN
1	28.77%	31.63%	32.55%
2	57.77%	63.52%	65.35%
3	86.77%	95.41%	98.16%
4	100%	100%	100%
5	100%	100%	100%
6	100%	100%	100%
7	100%	100%	100%
8	100%	100%	100%
9	100%	100%	100%
10	100%	100%	100%

Now, putting them in graph provides the following Figure C:

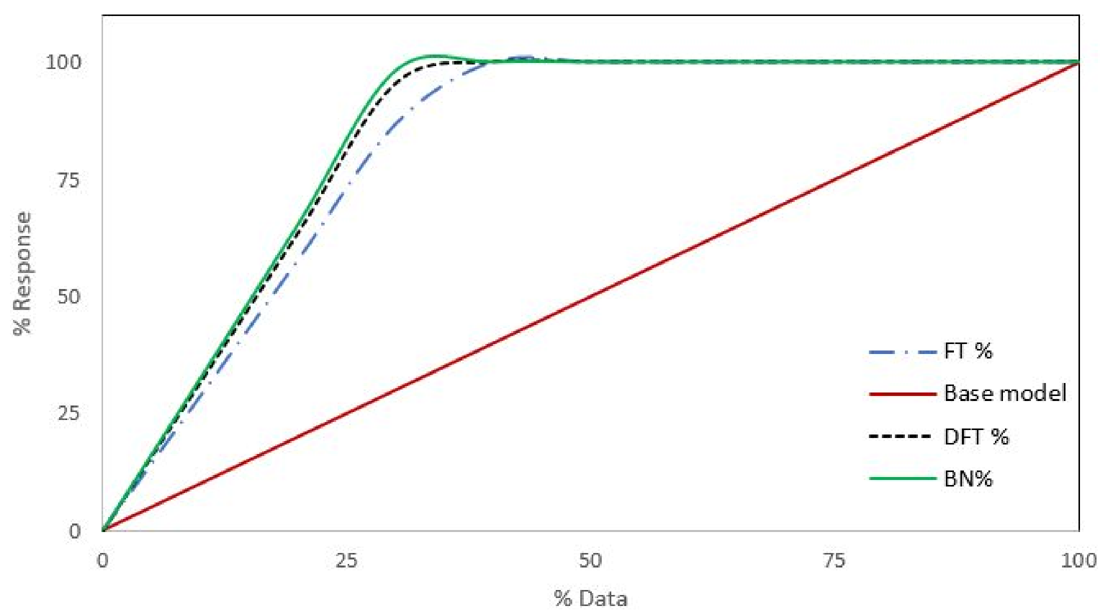


Figure C: Cumulative gain chart for T4 data set

The steeper the model curve is, the better the model is performing. From the graph, we can see that Bayesian network is performing the best among the three models. Dynamic fault tree is performing better than fault tree. Similarly, cumulative gain charts for T1, T2 and T3 data set were constructed.