# Supporting Information for <br> Spatially-Decomposed Free Energy of Solvation Based on the Endpoint Density-Functional Method 

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[^0]Numerical values of $\Delta \mu(i) / v(i), u(i) / v(i), \Delta \mu(i)$, and $u(i)$ are provided with $\rho_{0} v(i)$, where $\rho_{0}$ is the number density of pure water. $\rho_{0} v(i)$ is equal to the average number of water molecules contained in the volume $v(i)$ at the pure-solvent state. Each value of $\Delta \mu(i) / v(i)$ and $u(i) / v(i)$ is listed in the unit of $10^{-3} \mathrm{kcal} / \mathrm{mol} / \AA^{3}$ and is rounded to a multiple of $10^{-5} \mathrm{kcal} / \mathrm{mol} / \AA^{3}$. The corresponding error is expressed at $95 \%$ confidence interval (twice the standard error), and is not shown when it is smaller than $10^{-5} \mathrm{kcal} / \mathrm{mol} / \AA^{3}$. The value is expressed in $\mathrm{kcal} / \mathrm{mol}$ to two decimal places for $\Delta \mu(i)$ and $u(i)$; the error is given at $95 \%$ confidence interval and is not written when it is smaller than $0.01 \mathrm{kcal} / \mathrm{mol}$.

The row of "sub-total" provides the sum of the contributions over the excluded-volume to the outer regions for each site; the "far-separated" contribution is not incorporated there. $\rho_{0} v(i)$ is not shown in the rows of "far-separated", "total (with spatial decomposition)", "total (without spatial decomposition)", and "total (exact)" since it grows with the system size in an unbounded region. The entries for $\Delta \mu(i) / v(i)$ and $u(i) / v(i)$ are left blank for the "far-separated" region since this region is unbounded. The long-range correction of the Lennard-Jones interaction is incorporated into $\Delta \mu(i)$ and $u(i)$ in the "far-separated" region.

The sums of $\Delta \mu(i)$ and $u(i)$ over all the regions are shown in the row of "total (with spatial decomposition)". In that row, there are no entries for $\Delta \mu(i) / v(i)$ and $u(i) / v(i)$ since they are for the densities of the solvation free energy and solute-solvent interaction energy in spatially decomposed regions, respectively. The row of "total (without spatial decomposition)" lists the values of the (total) solvation free energy and solute-solvent energy computed without introducing the spatial decomposition. The total free energy of solvation with and without spatial decomposition correspond to eqs 25 and 11 of the main text, respectively. In the method of energy representation, the free energy is approximately evaluated and solute-solvent energy is exact. The total value of the latter is thus the same between the rows of "total (with spatial decomposition)" and "total (without spatial decomposition)", while the former can be different between the two rows even when expressed in total. The difference is actually a measure of the performance of the approximate functional adopted in the energy-representation method, and is within $0.1 \mathrm{kcal} / \mathrm{mol}$ for the small solutes. Alanine dipeptide exhibits larger differences of $0.4-0.6 \mathrm{kcal} / \mathrm{mol}$, while the relative free energies among the four conformations agree within $0.2 \mathrm{kcal} / \mathrm{mol}$.

Equation 11 of the main text provides the solvation free energy in the (one-dimensional) energyrepresentation method, and is spatially decomposed with eqs 25 and 26 . Although these equations are exact, the present work adopted the approximation for the integration over the coupling parameter as described in Appendix A and in previous works. ${ }^{1-3}$ To assess the performance of the approximate scheme, the (total) solvation free energy was also computed with the method of Bennett acceptance ratio. ${ }^{4,5}$ This is an exact method for computing the free energy, and is conducted by introducing a set of intermediate states connecting the initial state without the solute and the final state with the solute at full coupling. The coupling parameters of $\lambda_{\mathrm{LJ}}$ and $\lambda_{\text {elec }}$ are employed to describe the process of solute insertion and control the Lennard-Jones (LJ) and electrostatic terms of solute-solvent potential, respectively $\left(0 \leq \lambda_{\mathrm{LJ}} \leq 1,0 \leq \lambda_{\text {elec }} \leq 1\right)$. In the present calculations, the soft-core potential was adopted for the site-site interaction in the form of

$$
\lambda_{\text {elec }} \frac{q_{u} q_{v}}{r}+4 \epsilon \lambda_{\mathrm{LJ}}\left(\frac{\sigma^{12}}{\left[\alpha \sigma^{6}\left(1-\lambda_{\mathrm{LJ}}\right)+r^{6}\right]^{2}}-\frac{\sigma^{6}}{\alpha \sigma^{6}\left(1-\lambda_{\mathrm{LJ}}\right)+r^{6}}\right)
$$

where $\epsilon$ and $\sigma$ are the LJ energy and length parameters, respectively, $q_{u}$ and $q_{v}$ are the charges on the solute and solvent sites, respectively, and $r$ is the radial distance between a pair of interaction sites of solute and solvent. ${ }^{6,7} \alpha$ is the parameter to specify the "softness" of the $\lambda_{\mathrm{LJ}}$ dependence of the LJ component and was set to 0.5 . The initial state $\left(\lambda_{\mathrm{LJ}}=0, \lambda_{\text {elec }}=0\right)$ corresponds to the pure solvent and the isolated solute, for which the solute and solvent are decoupled in the generation of the statistical ensemble. The final state $\left(\lambda_{\mathrm{LJ}}=1, \lambda_{\text {elec }}=1\right)$ refers to the solution system of interest with fully coupled solute-solvent interaction. The simulation was performed at $\lambda_{\mathrm{LJ}}=0,0.1,0.2, \ldots, 0.9$, and 1.0 with $\lambda_{\text {elec }}=0$ and at $\lambda_{\text {elec }}=0,0.2,0.4,0.6,0.8$, and 1.0 with $\lambda_{\mathrm{LJ}}=1$. The MD setups were identical to those described in the main text, including the restraining potential for alanine dipeptide, and the run length was 10 ns at each set of $\lambda_{\mathrm{LJ}}$ and $\lambda_{\text {elec }}$.

The exact value of the (total) solvation free energy is listed in the row of "total (exact)". The deviation of the approximate free energy from the exact one is within 1.1 and $1.6 \mathrm{kcal} / \mathrm{mol}$ for the small solutes and for the four conformations of alanine dipeptide, respectively, and the preference order of the solvation free energy agrees well between the approximate and exact values. It is further seen that the order of the conformational preference is not affected by introducing the approximation, as noted at the end of the subsection Alanine Dipeptide.

## REFERENCES

${ }^{1}$ N. Matubayasi and M. Nakahara, J. Chem. Phys. 117, 3605 (2002).
${ }^{2}$ N. Matubayasi, W. Shinoda, and M. Nakahara, J. Chem. Phys. 128, 195107 (2008).
${ }^{3}$ S. Sakuraba and N. Matubayasi, J. Comput. Chem. 35, 1592 (2014).
${ }^{4}$ D. Frenkel and B. Smit, Understanding Molecular Simulation: From Algorithms to Applications (Academic Press, London, UK, 1996).
${ }^{5}$ C. H. Bennett, J. Comput. Phys. 22, 245 (1976).
${ }^{6}$ T. C. Beutler, A. E. Mark, R. C. van Schaik, P. R. Gerber, and W. F. van Gunsteren, Chem. Phys. Lett. 222, 529 (1994).
${ }^{7}$ M. R. Shirts and V. S. Pande, J. Chem. Phys. 122, 134508 (2005).

## Small solutes

The definitions of the regions are given in Table 1 of the main text. C refers to the carbon atom in the methyl group of ethane, methylamine, methanol, and toluene, and Ph denotes the center-of-mass position of the six carbon atoms in the phenyl ring of toluene, aniline, and phenol. N is the amine nitrogen of methylamine and aniline, and O is the hydroxyl oxygen of methanol and phenol. The computed values for ethane were further averaged on the basis of symmetry.

Ethane

| site in the solute | region | $\rho_{0} v(i)$ | $\Delta \mu(i) / v(i)$ | $u(i) / v(i)$ | $\Delta \mu(i)$ | $u(i)$ |
| :--- | ---: | ---: | :--- | :--- | :--- | :--- |
|  | excluded volume | 2.11 | $43.86 \pm 0.03$ | 0.00 | 2.82 | 0.00 |
|  | first shell | 11.67 | $-2.20 \pm 0.01$ | $-5.86 \pm 0.02$ | -0.78 | $-2.09 \pm 0.01$ |
| C | second shell | 26.38 | -0.59 | -0.58 | -0.48 | -0.47 |
|  | outer region | 36.32 | -0.10 | -0.10 | -0.11 | -0.11 |
|  | sub-total | 76.48 |  | $1.45 \pm 0.01$ | $-2.67 \pm 0.01$ |  |
|  | far-separated |  |  | -0.24 | -0.25 |  |
| total (with spatial decomposition) |  |  | $2.66 \pm 0.02$ | $-5.58 \pm 0.02$ |  |  |
| total (without spatial decomposition) |  |  | $2.65 \pm 0.02$ | $-5.58 \pm 0.02$ |  |  |
| total (exact) |  |  |  |  |  |  |

Methylamine

| site in the solute | region | $\rho_{0} v(i)$ | $\Delta \mu(i) / v(i)$ | $u(i) / v(i)$ | $\Delta \mu(i)$ | $u(i)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | excluded volume | 2.08 | $41.93 \pm 0.03$ | 0.00 | 2.66 | 0.00 |
|  | first shell | 11.58 | $-3.11 \pm 0.07$ | $-7.32 \pm 0.16$ | $-1.10 \pm 0.03$ | $-2.59 \pm 0.06$ |
|  | second shell | 26.24 | $-0.63 \pm 0.01$ | $-0.67 \pm 0.03$ | $-0.51 \pm 0.01$ | $-0.54 \pm 0.02$ |
|  | outer region | 36.17 | -0.11 | $-0.12 \pm 0.01$ | $-0.12$ | $-0.13 \pm 0.01$ |
|  | sub-total | 76.07 |  |  | $0.93 \pm 0.03$ | $-3.26 \pm 0.07$ |
| N | excluded volume | 1.36 | $42.44 \pm 0.06$ | 0.00 | 1.76 | 0.00 |
|  | first shell | 2.18 | $-47.57 \pm 0.34$ | $-138.22 \pm 0.73$ | $-3.16 \pm 0.02$ | $-9.19 \pm 0.05$ |
|  | second shell | 13.95 | $-5.00 \pm 0.06$ | $-6.64 \pm 0.14$ | $-2.13 \pm 0.03$ | $-2.83 \pm 0.06$ |
|  | outer region 1 | 22.40 | $-0.56 \pm 0.02$ | $-0.75 \pm 0.04$ | $-0.39 \pm 0.01$ | $-0.51 \pm 0.03$ |
|  | outer region 2 | 36.17 | $-0.13 \pm 0.01$ | $-0.18 \pm 0.01$ | $-0.14 \pm 0.01$ | $-0.20 \pm 0.01$ |
|  | sub-total | 76.06 |  |  | $-4.06 \pm 0.03$ | $-12.73 \pm 0.06$ |
| far-separated |  |  |  |  | $-0.42 \pm 0.01$ | $-0.35 \pm 0.03$ |
| total (with spatial decomposition) |  |  |  |  | $-3.55 \pm 0.04$ | $-16.34 \pm 0.06$ |
| total (without spatial decomposition |  |  |  |  | $-3.66 \pm 0.03$ | $-16.34 \pm 0.06$ |
| total (exact) |  |  |  |  | $-3.71 \pm 0.07$ |  |

Methanol

| site in the solute | region | $\rho_{0} v(i)$ | $\Delta \mu(i) / v(i)$ | $u(i) / v(i)$ | $\Delta \mu(i)$ | $u(i)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| C | excluded volume | 2.06 | $41.72 \pm 0.02$ | 0.00 | 2.62 | 0.00 |
|  | first shell | 11.54 | $-4.04 \pm 0.10$ | $-9.03 \pm 0.22$ | $-1.42 \pm 0.04$ | $-3.18 \pm 0.08$ |
|  | second shell | 26.17 | $-0.66 \pm 0.02$ | $-0.79 \pm 0.03$ | $-0.53 \pm 0.01$ | $-0.63 \pm 0.03$ |
|  | outer region | 36.10 | $-0.12 \pm 0.01$ | $-0.15 \pm 0.02$ | $-0.13 \pm 0.01$ | $-0.16 \pm 0.02$ |
|  | sub-total | 75.87 |  |  | $0.54 \pm 0.04$ | $-3.97 \pm 0.08$ |
| O | excluded volume | 1.35 | $41.78 \pm 0.05$ | 0.00 | 1.72 | 0.00 |
|  | first shell | 2.17 | $-64.31 \pm 0.39$ | $-165.30 \pm 1.04$ | $-4.26 \pm 0.03$ | $-10.94 \pm 0.07$ |
|  | second shell | 13.90 | $-4.52 \pm 0.07$ | $-5.98 \pm 0.14$ | $-1.92 \pm 0.03$ | $-2.53 \pm 0.06$ |
|  | outer region 1 | 22.35 | $-0.56 \pm 0.02$ | $-0.79 \pm 0.05$ | $-0.38 \pm 0.02$ | $-0.54 \pm 0.03$ |
|  | outer region 2 | 36.10 | $-0.13 \pm 0.01$ | $-0.19 \pm 0.02$ | $-0.15 \pm 0.01$ | $-0.21 \pm 0.02$ |
|  | sub-total | 75.87 |  |  | $-4.98 \pm 0.05$ | $-14.23 \pm 0.11$ |
| far-separated |  |  |  |  | $-0.30 \pm 0.02$ | $-0.40 \pm 0.03$ |
| total (with spatial decomposition) |  |  |  |  | $-4.75 \pm 0.03$ | $-18.61 \pm 0.06$ |
| total (without spatial decomposition) |  |  |  |  | $-4.86 \pm 0.03$ | $-18.61 \pm 0.06$ |
| total (exact) |  |  |  |  | $-4.94 \pm 0.04$ |  |

Toluene

| site in the solute | region | $\rho_{0} v(i)$ | $\Delta \mu(i) / v(i)$ | $u(i) / v(i)$ | $\Delta \mu(i)$ | $u(i)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ph | excluded volume | 2.34 | $46.95 \pm 0.01$ | -0.01 | 3.35 | 0.00 |
|  | first shell | 20.66 | $-2.46 \pm 0.07$ | $-15.07 \pm 0.15$ | $-1.55 \pm 0.04$ | $-9.50 \pm 0.09$ |
|  | second shell | 29.86 | $-1.26 \pm 0.01$ | $-1.30 \pm 0.02$ | $-1.15 \pm 0.01$ | $-1.19 \pm 0.02$ |
|  | outer region | 30.64 | -0.28 | $-0.30 \pm 0.01$ | $-0.27$ | $-0.28 \pm 0.01$ |
|  | sub-total | 83.50 |  |  | $0.39 \pm 0.04$ | $-10.96 \pm 0.09$ |
| C | excluded volume | 2.57 | $44.77 \pm 0.02$ | 0.00 | 3.51 | 0.00 |
|  | first shell | 13.26 | $-4.74 \pm 0.07$ | $-11.43 \pm 0.15$ | $-1.92 \pm 0.03$ | $-4.62 \pm 0.06$ |
|  | second shell | 28.77 | $-1.12 \pm 0.01$ | $-1.15 \pm 0.02$ | $-0.98 \pm 0.01$ | $-1.01 \pm 0.02$ |
|  | outer region | 38.88 | -0.18 | $-0.19 \pm 0.01$ | $-0.21$ | $-0.22 \pm 0.01$ |
|  | sub-total | 83.48 |  |  | $0.40 \pm 0.02$ | $-5.86 \pm 0.05$ |
|  | far-separated |  |  |  | $-0.69 \pm 0.01$ | $-0.71 \pm 0.02$ |
| total (with spatial decomposition) |  |  |  |  | $0.10 \pm 0.04$ | $-17.53 \pm 0.07$ |
| total (without spatial decomposition) |  |  |  |  | $0.23 \pm 0.03$ | $-17.53 \pm 0.07$ |
| total (exact) |  |  |  |  | $-0.88 \pm 0.05$ |  |

Aniline

| site in the solute | region | $\rho_{0} v(i)$ | $\Delta \mu(i) / v(i)$ | $u(i) / v(i)$ | $\Delta \mu(i)$ | $u(i)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ph | excluded volume | 2.29 | $47.15 \pm 0.01$ | 0.00 | 3.29 | 0.00 |
|  | first shell | 20.37 | $-1.43 \pm 0.07$ | $-12.43 \pm 0.14$ | $-0.89 \pm 0.04$ | $-7.72 \pm 0.09$ |
|  | second shell | 29.57 | $-1.27 \pm 0.01$ | $-1.34 \pm 0.02$ | $-1.14 \pm 0.01$ | $-1.21 \pm 0.02$ |
|  | outer region | 30.39 | -0.29 | $-0.31 \pm 0.01$ | $-0.27$ | $-0.29 \pm 0.01$ |
|  | sub-total | 82.62 |  |  | $0.99 \pm 0.04$ | $-9.22 \pm 0.08$ |
| N | excluded volume | 1.67 | $44.78 \pm 0.03$ | 0.00 | 2.28 | 0.00 |
|  | first shell | 2.57 | $-23.66 \pm 0.28$ | $-93.42 \pm 0.88$ | $-1.85 \pm 0.02$ | $-7.32 \pm 0.07$ |
|  | second shell | 15.57 | $-6.26 \pm 0.05$ | $-8.74 \pm 0.10$ | $-2.97 \pm 0.02$ | $-4.15 \pm 0.05$ |
|  | outer region 1 | 24.26 | $-0.89 \pm 0.01$ | $-1.03 \pm 0.03$ | $-0.66 \pm 0.01$ | $-0.76 \pm 0.02$ |
|  | outer region 2 | 38.56 | $-0.19 \pm 0.01$ | $-0.22 \pm 0.01$ | $-0.22 \pm 0.01$ | $-0.25 \pm 0.02$ |
|  | sub-total | 82.63 |  |  | $-3.43 \pm 0.02$ | $-12.49 \pm 0.07$ |
|  | far-separated |  |  |  | $-0.73 \pm 0.01$ | $-0.74 \pm 0.01$ |
| total (with spatial decomposition) |  |  |  |  | $-3.17 \pm 0.02$ | $-22.45 \pm 0.05$ |
| total (without spatial decomposition) |  |  |  |  | $-3.21 \pm 0.02$ | $-22.45 \pm 0.05$ |
| total (exact) |  |  |  |  | $-3.73 \pm 0.04$ |  |

Phenol

| site in the solute | region | $\rho_{0} v(i)$ | $\Delta \mu(i) / v(i)$ | $u(i) / v(i)$ | $\Delta \mu(i)$ | $u(i)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Ph | excluded volume | 2.30 | $47.11 \pm 0.02$ | 0.00 | 3.30 | 0.00 |
|  | first shell | 20.41 | $-0.99 \pm 0.06$ | $-11.62 \pm 0.11$ | $-0.62 \pm 0.04$ | $-7.23 \pm 0.07$ |
|  | second shell | 29.61 | $-1.27 \pm 0.02$ | $-1.34 \pm 0.03$ | $-1.14 \pm 0.01$ | $-1.21 \pm 0.03$ |
|  | outer region | 30.42 | -0.29 | $-0.32 \pm 0.01$ | $-0.27$ | $-0.30 \pm 0.01$ |
|  | sub-total | 82.74 |  |  | $1.27 \pm 0.04$ | $-8.74 \pm 0.08$ |
| O | excluded volume | 1.68 | $43.83 \pm 0.03$ | 0.00 | 2.25 | 0.00 |
|  | first shell | 2.57 | $-43.15 \pm 0.29$ | $-130.75 \pm 0.68$ | $-3.38 \pm 0.02$ | $-10.25 \pm 0.05$ |
|  | second shell | 15.60 | $-6.31 \pm 0.04$ | $-9.05 \pm 0.08$ | $-3.00 \pm 0.02$ | $-4.31 \pm 0.04$ |
|  | outer region 1 | 24.29 | $-0.96 \pm 0.02$ | $-1.21 \pm 0.04$ | $-0.71 \pm 0.01$ | $-0.90 \pm 0.03$ |
|  | outer region 2 | 38.60 | $-0.21 \pm 0.01$ | $-0.27 \pm 0.03$ | $-0.25 \pm 0.02$ | $-0.32 \pm 0.03$ |
|  | sub-total | 82.74 |  |  | $-5.10 \pm 0.04$ | $-15.77 \pm 0.09$ |
| far-separated |  |  |  |  | $-0.73 \pm 0.02$ | $-0.84 \pm 0.03$ |
| total (with spatial decomposition) |  |  |  |  | $-4.56 \pm 0.04$ | $-25.35 \pm 0.08$ |
| total (without spatial decomposition) |  |  |  |  | $-4.63 \pm 0.04$ | $-25.35 \pm 0.08$ |
| total (exact) |  |  |  |  | $-5.29 \pm 0.06$ |  |

## Alanine dipeptide

Table 2 of the main text provides the definitions of the regions, and Figure 2 labels the $\mathrm{C}_{1}, \mathrm{~N}_{1}, \mathrm{C}_{2}$, $\mathrm{C}_{3}$, and $\mathrm{N}_{2}$ atoms in alanine dipeptide.

| site in the solute | region | $\rho_{0} v(i)$ | $\Delta \mu(i) / v(i)$ | $u(i) / v(i)$ | $\Delta \mu(i)$ | $u(i)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{C}_{1}$ | excluded volume | 1.90 | $46.52 \pm 0.03$ | 0.00 | 2.70 | 0.00 |
|  | first shell | 3.83 | $-13.19 \pm 0.68$ | $-67.24 \pm 1.46$ | $-1.54 \pm 0.08$ | $-7.85 \pm 0.17$ |
|  | second shell | 8.07 | $-10.47 \pm 0.22$ | $-14.89 \pm 0.47$ | $-2.58 \pm 0.05$ | $-3.67 \pm 0.12$ |
|  | outer region 1 | 16.22 | $-2.32 \pm 0.08$ | $-3.18 \pm 0.17$ | $-1.15 \pm 0.04$ | $-1.57 \pm 0.09$ |
|  | outer region 2 | 25.46 | $-0.48 \pm 0.02$ | $-0.66 \pm 0.04$ | $-0.37 \pm 0.02$ | $-0.51 \pm 0.03$ |
|  | sub-total | 55.48 |  |  | $-2.94 \pm 0.14$ | $-13.61 \pm 0.29$ |
| $\mathrm{N}_{1}$ | excluded volume | 0.83 | $45.57 \pm 0.03$ | 0.00 | 1.15 | 0.00 |
|  | first shell | 0.86 | $-59.94 \pm 0.99$ | $-172.52 \pm 2.64$ | $-1.57 \pm 0.03$ | $-4.53 \pm 0.07$ |
|  | second shell | 3.05 | $-17.40 \pm 0.56$ | $-26.85 \pm 1.09$ | $-1.62 \pm 0.05$ | $-2.50 \pm 0.10$ |
|  | outer region 1 | 7.18 | $-3.05 \pm 0.09$ | $-4.32 \pm 0.18$ | $-0.67 \pm 0.02$ | $-0.95 \pm 0.04$ |
|  | outer region 2 | 9.24 | $-0.54 \pm 0.04$ | $-0.79 \pm 0.08$ | $-0.15 \pm 0.01$ | $-0.22 \pm 0.02$ |
|  | sub-total | 21.16 |  |  | $-2.86 \pm 0.06$ | $-8.19 \pm 0.13$ |
| $\mathrm{C}_{2}$ | excluded volume | 1.25 | $47.16 \pm 0.02$ | 0.00 | 1.80 | 0.00 |
|  | first shell | 2.18 | $8.97 \pm 0.69$ | $-23.59 \pm 1.47$ | $0.60 \pm 0.05$ | $-1.57 \pm 0.10$ |
|  | second shell | 4.43 | $-9.73 \pm 0.37$ | $-13.95 \pm 0.75$ | $-1.31 \pm 0.05$ | $-1.88 \pm 0.10$ |
|  | outer region 1 | 8.71 | $-1.94 \pm 0.05$ | $-2.25 \pm 0.10$ | $-0.52 \pm 0.01$ | $-0.60 \pm 0.03$ |
|  | outer region 2 | 13.48 | $-0.40 \pm 0.04$ | $-0.48 \pm 0.07$ | $-0.17 \pm 0.01$ | $-0.20 \pm 0.03$ |
|  | sub-total | 30.05 |  |  | $0.40 \pm 0.09$ | $-4.25 \pm 0.19$ |
| $\mathrm{C}_{3}$ | excluded volume | 1.20 | $46.96 \pm 0.02$ | 0.00 | 1.72 | 0.00 |
|  | first shell | 2.06 | $-62.47 \pm 1.11$ | $-156.79 \pm 2.33$ | $-3.92 \pm 0.07$ | $-9.85 \pm 0.15$ |
|  | second shell | 6.96 | $-8.59 \pm 0.23$ | $-11.49 \pm 0.47$ | $-1.82 \pm 0.05$ | $-2.44 \pm 0.10$ |
|  | outer region 1 | 7.50 | $-1.44 \pm 0.07$ | $-1.97 \pm 0.15$ | $-0.33 \pm 0.02$ | $-0.45 \pm 0.03$ |
|  | outer region 2 | 10.46 | $-0.57 \pm 0.04$ | $-0.89 \pm 0.08$ | $-0.18 \pm 0.01$ | $-0.28 \pm 0.02$ |
|  | sub-total | 28.18 |  |  | $-4.54 \pm 0.10$ | $-13.02 \pm 0.20$ |
| $\mathrm{N}_{2}$ | excluded volume | 1.37 | $46.53 \pm 0.03$ | 0.00 | 1.94 | 0.00 |
|  | first shell | 1.80 | $2.08 \pm 0.41$ | $-60.57 \pm 1.22$ | $0.11 \pm 0.02$ | $-3.33 \pm 0.07$ |
|  | second shell | 6.99 | $-10.54 \pm 0.27$ | $-22.22 \pm 0.63$ | $-2.25 \pm 0.06$ | $-4.74 \pm 0.13$ |
|  | outer region 1 | 17.48 | $-2.66 \pm 0.09$ | $-3.34 \pm 0.19$ | $-1.42 \pm 0.05$ | $-1.78 \pm 0.10$ |
|  | outer region 2 | 23.26 | $-0.49 \pm 0.04$ | $-0.69 \pm 0.08$ | $-0.35 \pm 0.03$ | $-0.49 \pm 0.06$ |
|  | sub-total | 50.90 |  |  | $-1.96 \pm 0.11$ | $-10.33 \pm 0.26$ |
| $\frac{\text { far-separated }}{\text { total (with spatial decomposition) }}$ |  |  |  |  | $-1.51 \pm 0.06$ | $-2.00 \pm 0.13$ |
|  |  |  |  |  | $-13.43 \pm 0.06$ | $-51.43 \pm 0.13$ |
| total (without spatial decomposition) |  |  |  |  | $-13.85 \pm 0.05$ | $-51.43 \pm 0.13$ |
| total (exact) |  |  |  |  | $-14.99 \pm 0.09$ |  |


| site in the solute | region | $\rho_{0} v(i)$ | $\Delta \mu(i) / v(i)$ | $u(i) / v(i)$ | $\Delta \mu(i)$ | $u(i)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{C}_{1}$ | excluded volume | 1.93 | $46.68 \pm 0.04$ | 0.00 | 2.75 | 0.00 |
|  | first shell | 3.97 | $-14.85 \pm 0.36$ | $-70.08 \pm 0.86$ | $-1.80 \pm 0.04$ | $-8.49 \pm 0.10$ |
|  | second shell | 8.39 | $-9.34 \pm 0.19$ | $-12.48 \pm 0.40$ | $-2.39 \pm 0.05$ | $-3.19 \pm 0.10$ |
|  | outer region 1 | 16.88 | $-2.09 \pm 0.05$ | $-2.68 \pm 0.11$ | $-1.08 \pm 0.03$ | $-1.38 \pm 0.06$ |
|  | outer region 2 | 26.46 | $-0.42 \pm 0.02$ | $-0.53 \pm 0.05$ | $-0.34 \pm 0.02$ | $-0.43 \pm 0.04$ |
|  | sub-total | 57.63 |  |  | $-2.86 \pm 0.08$ | $-13.49 \pm 0.17$ |
| $\mathrm{N}_{1}$ | excluded volume | 0.87 | $45.64 \pm 0.04$ | 0.00 | 1.21 | 0.00 |
|  | first shell | 0.97 | $-46.62 \pm 0.87$ | $-145.31 \pm 2.03$ | $-1.38 \pm 0.03$ | $-4.30 \pm 0.06$ |
|  | second shell | 3.56 | $-16.11 \pm 0.40$ | $-25.37 \pm 0.83$ | $-1.75 \pm 0.04$ | $-2.75 \pm 0.09$ |
|  | outer region 1 | 8.57 | $-2.38 \pm 0.10$ | $-2.88 \pm 0.21$ | $-0.62 \pm 0.03$ | $-0.75 \pm 0.05$ |
|  | outer region 2 | 11.18 | $-0.39 \pm 0.03$ | $-0.45 \pm 0.06$ | $-0.13 \pm 0.01$ | $-0.16 \pm 0.02$ |
|  | sub-total | 25.15 |  |  | $-2.68 \pm 0.07$ | $-7.96 \pm 0.16$ |
| $\mathrm{C}_{2}$ | excluded volume | 1.21 | $46.98 \pm 0.02$ | 0.00 | 1.73 | 0.00 |
|  | first shell | 1.92 | $14.23 \pm 0.73$ | $-17.26 \pm 1.44$ | $0.83 \pm 0.04$ | $-1.01 \pm 0.08$ |
|  | second shell | 3.73 | $-9.88 \pm 0.33$ | $-14.59 \pm 0.68$ | $-1.12 \pm 0.04$ | $-1.66 \pm 0.08$ |
|  | outer region 1 | 7.11 | $-1.98 \pm 0.06$ | $-2.13 \pm 0.13$ | $-0.43 \pm 0.01$ | $-0.46 \pm 0.03$ |
|  | outer region 2 | 10.76 | $-0.42 \pm 0.04$ | $-0.46 \pm 0.09$ | $-0.14 \pm 0.01$ | $-0.15 \pm 0.03$ |
|  | sub-total | 24.73 |  |  | $0.88 \pm 0.07$ | $-3.28 \pm 0.15$ |
| $\mathrm{C}_{3}$ | excluded volume | 1.10 | $47.55 \pm 0.02$ | 0.00 | 1.60 | 0.00 |
|  | first shell | 1.61 | $-60.33 \pm 0.80$ | $-160.48 \pm 2.17$ | $-2.96 \pm 0.04$ | $-7.88 \pm 0.11$ |
|  | second shell | 4.97 | $-8.72 \pm 0.23$ | $-10.57 \pm 0.49$ | $-1.32 \pm 0.03$ | $-1.60 \pm 0.07$ |
|  | outer region 1 | 5.11 | $-1.38 \pm 0.08$ | $-1.61 \pm 0.18$ | $-0.22 \pm 0.01$ | $-0.25 \pm 0.03$ |
|  | outer region 2 | 6.95 | $-0.48 \pm 0.04$ | $-0.61 \pm 0.10$ | $-0.10 \pm 0.01$ | $-0.13 \pm 0.02$ |
|  | sub-total | 19.74 |  |  | $-3.01 \pm 0.07$ | $-9.86 \pm 0.16$ |
| $\mathrm{N}_{2}$ | excluded volume | 1.45 | $46.10 \pm 0.03$ | 0.00 | 2.04 | 0.00 |
|  | first shell | 2.08 | $-11.71 \pm 0.51$ | $-80.20 \pm 0.79$ | $-0.74 \pm 0.03$ | $-5.09 \pm 0.05$ |
|  | second shell | 8.27 | $-10.83 \pm 0.39$ | $-22.15 \pm 0.84$ | $-2.73 \pm 0.10$ | $-5.59 \pm 0.21$ |
|  | outer region 1 | 21.03 | $-2.31 \pm 0.04$ | $-2.83 \pm 0.08$ | $-1.48 \pm 0.03$ | $-1.81 \pm 0.05$ |
|  | outer region 2 | 28.19 | $-0.36 \pm 0.03$ | $-0.45 \pm 0.06$ | $-0.31 \pm 0.02$ | $-0.39 \pm 0.05$ |
|  | sub-total | 61.02 |  |  | $-3.23 \pm 0.10$ | $-12.88 \pm 0.19$ |
| far-separated |  |  |  |  | $-1.36 \pm 0.05$ | $-1.33 \pm 0.12$ |
| total (with spatial decomposition) |  |  |  |  | $-12.26 \pm 0.08$ | $-48.80 \pm 0.14$ |
| total (without spatial decomposition) |  |  |  |  | $-12.75 \pm 0.08$ | $-48.80 \pm 0.14$ |
| total (exact) |  |  |  |  | $-13.28 \pm 0.12$ |  |


| site in the solute | region | $\rho_{0} v(i)$ | $\Delta \mu(i) / v(i)$ | $u(i) / v(i)$ | $\Delta \mu(i)$ | $u(i)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{C}_{1}$ | excluded volume | 1.90 | $46.66 \pm 0.02$ | 0.00 | 2.70 | 0.00 |
|  | first shell | 3.83 | $-16.48 \pm 0.60$ | $-75.48 \pm 1.35$ | $-1.93 \pm 0.07$ | $-8.82 \pm 0.16$ |
|  | second shell | 8.07 | $-11.00 \pm 0.33$ | $-15.58 \pm 0.72$ | $-2.71 \pm 0.08$ | $-3.83 \pm 0.18$ |
|  | outer region 1 | 16.22 | $-2.55 \pm 0.07$ | $-3.58 \pm 0.14$ | $-1.26 \pm 0.0$ | $-1.77 \pm 0.07$ |
|  | outer region 2 | 25.45 | $-0.57 \pm 0.03$ | $-0.83 \pm 0.06$ | $-0.44 \pm 0.02$ | $-0.64 \pm 0.05$ |
|  | sub-total | 55.47 |  |  | $-3.63 \pm 0.1$ | $-15.06 \pm 0.27$ |
| $\mathrm{N}_{1}$ | excluded volume | 0.84 | $45.79 \pm 0.02$ | 0.00 | 1.17 | 0.00 |
|  | first shell | 0.88 | $-65.33 \pm 0.72$ | $-168.32 \pm 1.56$ | $-1.75 \pm 0.02$ | $-4.52 \pm 0.04$ |
|  | second shell | 3.14 | $-16.17 \pm 0.62$ | $-24.52 \pm 1.17$ | $-1.55 \pm 0.06$ | $-2.35 \pm 0.11$ |
|  | outer region 1 | 7.40 | $-2.87 \pm 0.12$ | $-4.19 \pm 0.24$ | $-0.65 \pm 0.03$ | $-0.95 \pm 0.05$ |
|  | outer region 2 | 9.54 | $-0.50 \pm 0.05$ | $-0.75 \pm 0.10$ | $-0.15 \pm 0.01$ | $-0.22 \pm 0.03$ |
|  | sub-total | 21.80 |  |  | $-2.92 \pm 0.09$ | $-8.03 \pm 0.16$ |
| $\mathrm{C}_{2}$ | excluded volume | 1.25 | $46.77 \pm 0.02$ | 0.00 | 1.78 | 0.00 |
|  | first shell | 2.17 | $11.10 \pm 0.70$ | $-19.42 \pm 1.40$ | $0.73 \pm 0.05$ | $-1.29 \pm 0.09$ |
|  | second shell | 4.41 | $-9.54 \pm 0.48$ | $-13.84 \pm 1.04$ | $-1.28 \pm 0.06$ | $-1.86 \pm 0.14$ |
|  | outer region 1 | 8.66 | $-2.06 \pm 0.11$ | $-2.49 \pm 0.23$ | $-0.54 \pm 0.03$ | $-0.66 \pm 0.06$ |
|  | outer region 2 | 13.40 | $-0.48 \pm 0.05$ | $-0.63 \pm 0.11$ | $-0.19 \pm 0.02$ | $-0.26 \pm 0.04$ |
|  | sub-total | 29.89 |  |  | $0.50 \pm 0.12$ | $-4.06 \pm 0.24$ |
| $\mathrm{C}_{3}$ | excluded volume | 1.17 | $47.24 \pm 0.02$ | 0.00 | 1.69 | 0.00 |
|  | first shell | 1.96 | $-76.17 \pm 0.58$ | $-186.29 \pm 1.44$ | $-4.55 \pm 0.03$ | $-11.14 \pm 0.09$ |
|  | second shell | 6.56 | $-9.85 \pm 0.29$ | $-13.69 \pm 0.57$ | $-1.97 \pm 0.06$ | $-2.74 \pm 0.11$ |
|  | outer region 1 | 7.03 | $-1.59 \pm 0.11$ | $-2.24 \pm 0.23$ | $-0.34 \pm 0.02$ | $-0.48 \pm 0.05$ |
|  | outer region 2 | 9.77 | $-0.67 \pm 0.08$ | $-1.06 \pm 0.16$ | $-0.20 \pm 0.02$ | $-0.32 \pm 0.05$ |
|  | sub-total | 26.49 |  |  | $-5.38 \pm 0.08$ | $-14.67 \pm 0.18$ |
| $\mathrm{N}_{2}$ | excluded volume | 1.39 | $46.32 \pm 0.03$ | 0.00 | 1.96 | 0.00 |
|  | first shell | 1.85 | $-4.80 \pm 0.29$ | $-68.24 \pm 0.74$ | $-0.27 \pm 0.02$ | $-3.85 \pm 0.04$ |
|  | second shell | 7.19 | $-10.15 \pm 0.50$ | $-21.23 \pm 1.04$ | $-2.23 \pm 0.11$ | $-4.66 \pm 0.23$ |
|  | outer region 1 | 18.02 | $-2.54 \pm 0.08$ | $-3.24 \pm 0.18$ | $-1.39 \pm 0.05$ | $-1.78 \pm 0.10$ |
|  | outer region 2 | 24.01 | $-0.42 \pm 0.03$ | $-0.55 \pm 0.06$ | $-0.31 \pm 0.02$ | $-0.40 \pm 0.05$ |
|  | sub-total | 52.46 |  |  | $-2.23 \pm 0.15$ | $-10.69 \pm 0.32$ |
|  | far-separated |  |  |  | $-1.58 \pm 0.12$ | $-2.15 \pm 0.24$ |
| total (with spatial decomposition) |  |  |  |  | $-15.28 \pm 0.06$ | $-54.69 \pm 0.10$ |
| total (without spatial decomposition) |  |  |  |  | $-15.71 \pm 0.05$ | $-54.69 \pm 0.10$ |
| total (exact) |  |  |  |  | $-16.91 \pm 0.08$ |  |


| site in the solute | region | $\rho_{0} v(i)$ | $\Delta \mu(i) / v(i)$ | $u(i) / v(i)$ | $\Delta \mu(i)$ | $u(i)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{C}_{1}$ | excluded volume | 1.87 | $46.93 \pm 0.02$ | 0.00 | 2.68 | 0.00 |
|  | first shell | 3.69 | $-4.95 \pm 0.40$ | $-53.01 \pm 0.90$ | $-0.56 \pm 0.05$ | $-5.97 \pm 0.10$ |
|  | second shell | 7.75 | $-9.21 \pm 0.16$ | $-11.69 \pm 0.33$ | $-2.18 \pm 0.04$ | $-2.76 \pm 0.08$ |
|  | outer region 1 | 15.53 | $-1.84 \pm 0.03$ | $-2.09 \pm 0.05$ | $-0.87 \pm 0.01$ | $-0.99 \pm 0.03$ |
|  | outer region 2 | 24.35 | $-0.36 \pm 0.01$ | $-0.41 \pm 0.02$ | $-0.27 \pm 0.01$ | $-0.30 \pm 0.01$ |
|  | sub-total | 53.19 |  |  | $-1.20 \pm 0.07$ | $-10.02 \pm 0.14$ |
| $\mathrm{N}_{1}$ | excluded volume | 0.87 | $45.41 \pm 0.02$ | 0.00 | 1.21 | 0.00 |
|  | first shell | 0.96 | $-56.92 \pm 0.93$ | $-147.32 \pm 1.99$ | $-1.67 \pm 0.03$ | $-4.31 \pm 0.06$ |
|  | second shell | 3.49 | $-14.13 \pm 0.34$ | $-20.08 \pm 0.61$ | $-1.50 \pm 0.04$ | $-2.14 \pm 0.07$ |
|  | outer region 1 | 8.38 | $-2.20 \pm 0.08$ | $-2.80 \pm 0.17$ | $-0.56 \pm 0.02$ | $-0.72 \pm 0.04$ |
|  | outer region 2 | 10.91 | $-0.37 \pm 0.03$ | $-0.47 \pm 0.06$ | $-0.12 \pm 0.01$ | $-0.16 \pm 0.02$ |
|  | sub-total | 24.61 |  |  | $-2.65 \pm 0.05$ | $-7.32 \pm 0.08$ |
| $\mathrm{C}_{2}$ | excluded volume | 1.24 | $47.05 \pm 0.02$ | 0.00 | 1.78 | 0.00 |
|  | first shell | 2.13 | $10.48 \pm 0.51$ | $-19.81 \pm 1.01$ | $0.68 \pm 0.03$ | $-1.29 \pm 0.07$ |
|  | second shell | 4.32 | $-8.78 \pm 0.12$ | $-12.05 \pm 0.24$ | $-1.16 \pm 0.02$ | $-1.59 \pm 0.03$ |
|  | outer region 1 | 8.46 | $-1.73 \pm 0.04$ | $-1.87 \pm 0.10$ | $-0.45 \pm 0.01$ | $-0.48 \pm 0.02$ |
|  | outer region 2 | 13.07 | $-0.38 \pm 0.02$ | $-0.44 \pm 0.04$ | $-0.15 \pm 0.01$ | $-0.18 \pm 0.02$ |
|  | sub-total | 29.22 |  |  | $0.71 \pm 0.05$ | $-3.53 \pm 0.10$ |
| $\mathrm{C}_{3}$ | excluded volume | 1.17 | $46.99 \pm 0.02$ | 0.00 | 1.68 | 0.00 |
|  | first shell | 2.00 | $-55.60 \pm 0.54$ | $-138.93 \pm 1.11$ | $-3.39 \pm 0.03$ | $-8.47 \pm 0.07$ |
|  | second shell | 6.73 | $-6.68 \pm 0.16$ | $-7.80 \pm 0.33$ | $-1.37 \pm 0.03$ | $-1.60 \pm 0.07$ |
|  | outer region 1 | 7.25 | $-1.02 \pm 0.05$ | $-1.18 \pm 0.11$ | $-0.23 \pm 0.01$ | $-0.26 \pm 0.02$ |
|  | outer region 2 | 10.11 | $-0.37 \pm 0.02$ | $-0.48 \pm 0.04$ | $-0.11 \pm 0.01$ | $-0.15 \pm 0.01$ |
|  | sub-total | 27.26 |  |  | $-3.43 \pm 0.05$ | $-10.48 \pm 0.11$ |
| $\mathrm{N}_{2}$ | excluded volume | 1.39 | $47.58 \pm 0.03$ | 0.00 | 2.02 | 0.00 |
|  | first shell | 1.83 | $17.83 \pm 0.55$ | $-33.59 \pm 1.56$ | $1.00 \pm 0.03$ | $-1.87 \pm 0.09$ |
|  | second shell | 7.09 | $-9.90 \pm 0.21$ | $-21.76 \pm 0.50$ | $-2.14 \pm 0.05$ | $-4.70 \pm 0.11$ |
|  | outer region 1 | 17.72 | $-2.27 \pm 0.04$ | $-2.45 \pm 0.08$ | $-1.22 \pm 0.02$ | $-1.32 \pm 0.04$ |
|  | outer region 2 | 23.57 | $-0.34 \pm 0.01$ | $-0.36 \pm 0.03$ | $-0.24 \pm 0.01$ | $-0.26 \pm 0.02$ |
|  | sub-total | 51.60 |  |  | $-0.60 \pm 0.07$ | $-8.16 \pm 0.17$ |
|  | far-separated |  |  |  | $-1.11 \pm 0.04$ | $-1.22 \pm 0.09$ |
| total (with spatial decomposition) |  |  |  |  | $-8.28 \pm 0.05$ | $-40.75 \pm 0.10$ |
| total (without spatial decomposition) |  |  |  |  | $-8.87 \pm 0.05$ | $-40.75 \pm 0.10$ |
| total (exact) |  |  |  |  | $-9.85 \pm 0.07$ |  |

Volumes $v(i)$ of the excluded-volume, first-shell, and second-shell regions


The lines connecting the data are drawn for eye guide, and are dashed, solid, and dotted for the excluded volume, first shell, and second shell, respectively. C, N, and O refer to the methyl carbon, amine nitrogen, and hydroxyl oxygen, respectively, and Ph is the center of the phenyl group. For ethane, the data are plotted both at "C" and "C, N, O" and the lines connecting them are horizontal. The data and line overlap between methylamine and methanol and between aniline and phenol.

Correlation plots between $\Delta \mu(i) / v(i)$ and $u(i) / v(i)$ in the first and second shells


In the second shell, $\Delta \mu(i) / v(i)$ and $u(i) / v(i)$ are multiplied by factors of 10 and 5 for the small solutes and for alanine dipeptide, respectively.


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