## Supporting Information for

# Atomically Layered and Ordered Rare-Earth $i$-MAX Phases: A New Class of Magnetic Quaternary Compounds 

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## Section S1. Structure analysis by STEM and XRD

$\mathbf{a}\left(\mathrm{Mo}_{2 / 3} \mathrm{Pr}_{1 / 3}\right)_{2} \mathrm{AIC} \mathrm{C} 2 / m$


b $\left(\mathrm{Mo}_{2 / 3} \mathrm{Nd}_{1 / 3}\right)_{2} \mathrm{AIC} \mathrm{C} 2 / \mathrm{c}$

c $\left(\mathrm{Mo}_{2 / 3} \mathrm{Sm}_{1 / 3}\right)_{2} \mathrm{AlC} \mathrm{\quad C} \quad \mathrm{C} / \mathrm{c}$

d $\left(\mathrm{Mo}_{2 / 3} \mathrm{Sm}_{1 / 3}\right)_{2} \mathrm{AIC} \mathrm{C2/m}$


## e $\left(\mathrm{Mo}_{2 / 3} \mathrm{Gd}_{1 / 3}\right)_{2} \mathrm{AlC} \mathrm{C} 2 / \mathrm{c}$


f $\left(\mathrm{Mo}_{2 / 3} \mathrm{Dy}_{1 / 3}\right)_{2} \mathrm{AlC}$ C2/c



Fig. S1. Crystal structure of RE i-MAX. STEM images of RE=Ce, Pr, Nd, Sm, Gd, Dy, Ho, Er, Tm, Lu.



Fig. S2. a-f Rietveld refinement of XRD for RE=Nd, Sm, Gd, Dy, Ho, Er, Tm, Lu.
The XRD patterns from the samples containing Ce and Pr showed them to be of too low phase purity to motivate a detailed refinement. Furthermore, these $i$-MAX phases were accompanied with the ferromagnetic $\mathrm{RE}_{4} \mathrm{Mo}_{4} \mathrm{Al}_{7} \mathrm{C}_{3}$ and the nonmagnetic $\mathrm{Mo}_{2} \mathrm{C}$ and $\mathrm{REAlO}_{3}$ phases, hindering detailed magnetization analysis.

Table S1. Cell parameters and atom coordinates obtained from XRD Rietveld refinement at RT.

| Space group | C2/c (\#15) | C2/c (\#15) | C2/c (\#15) | C2/C (\#15) | C2/C (\#15) |
| :---: | :---: | :---: | :---: | :---: | :---: |
| RE | Nd | Sm | Gd | Tb | Dy |
| $x^{2}$ | 29.2 | 12.4 | 6.6 | 7.7 | 6.4 |
| $a(\dot{A})$ | a 9.66570 (10) | a 9.61514 (8) | a 9.57673 (12) | a 9.54627 (8) | a 9.52628 (10) |
| $b(A)$ | b 5.59353 (6) | b 5.56482 (5) | b 5.54259 (6) | b 5.52473(5) | $b 5.51408$ (5) |
| $c(A)$ | c 14.17243 (13) | c 14.12072 (11) | C 14.09418 (16) | c 14.06829 (12) | c 14.04969 (14) |
| a | 90 | 90 | 90 | 90 | 90 |
| $\beta$ | 103.48465(69) | 103.52045(53) | 103.53428(82) | 103.53555(62) | 103.53767(69) |
| Y | 90 | 90 | 90 | 90 | 90 |
| RE | $8 f(0.95833(29)$ $0.41825(63)$ $0.12101(14))$ | $\begin{aligned} & \hline 8 f(0.96127(15) \\ & 0.42102(34) \\ & 0.12001(7)) \end{aligned}$ | $\begin{aligned} & 8 f(0.96189(28) \\ & 0.41552(67) \\ & 0.12100(13), \end{aligned}$ | 8 f (0.95948(21), $0.41824(53)$, $0.11744(11))$ | $8 f(0.96183(23)$ $0.42445(58)$ $0.11833(12))$ |
| Mo | $8 f$ <br> (0.26981 (45) <br> $0.43116(83)$ <br> $0.07611(19))$ <br> 8 f <br> (0.60799 ( 39 ) <br> $0.41324(94)$ <br> $0.07398(18))$ | $\begin{aligned} & 8 f(0.60793(21) \\ & 0.40602(49) \\ & 0.07686(9)) \\ & 8 f(0.27080(25) \\ & 0.42406(52) \\ & 0.07669(9)) \end{aligned}$ | $\begin{aligned} & 8 f(0.27093(43) \\ & 0.42028(92) \\ & 0.07794(17)) \\ & 8 f(0.60976(39) \\ & 0.41053(88) \\ & 0.07992(15)) \end{aligned}$ | $8 f$ <br> (0.27021(32), <br> $0.41601(74)$, <br> $0.07950(15))$ <br> $8 f$ <br> (0.60740(28), <br> 0.40992(67), <br> $0.07954(14))$ | $\begin{aligned} & 8 f(0.27173(37) \\ & 0.42346(82) \\ & 0.07922(17)) \\ & 8 f(0.60919(33) \\ & 0.41425(75) \\ & 0.07949(16)) \end{aligned}$ |
| AI | $8 f$ <br> (0.74535(136) <br> 0.14219 (264) <br> $0.25375(90)$ ) <br> 4 e <br> (0.00000( 0) <br> 0.89498 (350) <br> 0.25000 ( 0 ) | $\begin{aligned} & 8 f(0.75084(77) \\ & 0.15562(145) \\ & 0.25598(48)) \\ & 4 e(0.00000(0) \\ & 0.91541(204) \\ & 0.25000(0)) \end{aligned}$ | $\begin{aligned} & 8 f(0.73895(135) \\ & 0.15809(272) \\ & 0.25744(81)) \\ & 4 \mathrm{e}(0.00000(0) \\ & 0.92446(355) \\ & 0.25000(0)) \end{aligned}$ | $8 f$ <br> (0.74760(116), <br> $0.16653(224)$, <br> $0.25031(66))$ <br> 4e (0.00000, <br> 0.91889(293), <br> 0.25000 ) | $\begin{aligned} & 8 f(0.75544(110) \\ & 0.14299(200) \\ & 0.24995(71)) \\ & 4 \mathrm{e}(0.00000(0) \\ & 0.92469(309) \\ & 0.25000(0)) \end{aligned}$ |
| C | $\begin{aligned} & 8 \mathrm{f}(0.40600 \\ & 0.26952 \\ & 0.00000) \\ & 4 \mathrm{~d}(0.25,0.25, \\ & 0.50) \end{aligned}$ | 8f ( 0.40600 <br> $0.269520 .00000)$ <br> 4d (0.25, 0.25, <br> 0.50 ) | $\begin{aligned} & 8 f(0.41481, \\ & 0.23022,0.0000) \\ & 4 \mathrm{~d}(0.25,0.25, \\ & 0.50) \end{aligned}$ | $\begin{aligned} & 8 \mathrm{f}(0.41481 \text {, } \\ & 0.23022, \\ & 0.0000) \\ & 4 \mathrm{~d}(0.25,0.25, \\ & 0.50) \end{aligned}$ | $\begin{aligned} & 8 \mathrm{f}(0.41481, \\ & 0.23022,0.0000) \\ & 4 \mathrm{~d}(0.25,0.25, \\ & 0.50) \end{aligned}$ |


| C2/c (\#15) | C2/c (\#15) | C2/c (\#15) | C2/c (\#15) |
| :---: | :---: | :---: | :---: |
| Ho | Er | Tm | Lu |
| 23.1 | 19.0 | 85.3 | 23.5 |
| a 9.51198 (9) | a 9.49271 (11) | a 9.47697 (12) | a 9.44912 (23) |
| b 5.50274 (5) | b 5.49282 (6) | b 5.48572 (7) | b 5.47033 (13) |
| c 14.03831 (13) | c 14.01770 (15) | c 14.00152 (17) | C 13.97305 (28) |
| 90 | 90 | 90 | 90 |
| 103.54111(69) | 103.53001(66) | 103.51039(92) | 103.48195(166) |
| 90 | 90 | 90 | 90 |
| $\begin{aligned} & 8 f(0.95804(22) \\ & 0.41472(54) \\ & 0.11384(11)) \end{aligned}$ | $\begin{aligned} & 8 f(0.95802(20) \\ & 0.42234(47) \\ & 0.11485(9)) \end{aligned}$ | $\begin{aligned} & \hline 8 f(0.95779(27) \\ & 0.42251(62) \\ & 0.11326(14)) \end{aligned}$ | $\begin{aligned} & \hline 8 f(0.95760(33) \\ & 0.42200(75) \\ & 0.111609(14)) \end{aligned}$ |
| $\begin{aligned} & 8 f(0.27422(32) \\ & 0.42187(72) \\ & 0.07969(15)) \end{aligned}$ | $\begin{aligned} & 8 f(0.27571(29) \\ & 0.41851(73) \\ & 0.08114(14)) \end{aligned}$ | $\begin{aligned} & 8 f(0.27500(45) \\ & 0.42032(97) \\ & 0.07995(21)) \end{aligned}$ | $\begin{aligned} & 8 f(0.27592(59) \\ & 0.41943(125) \\ & 0.08670(106)) \end{aligned}$ |
| $\begin{aligned} & 8 f(0.60923(30) \\ & 0.40790(71) \\ & 0.07878(14)) \end{aligned}$ | $\begin{aligned} & 8 f(0.60960(27) \\ & 0.41194(65) \\ & 0.08095(13)) \end{aligned}$ | $\begin{aligned} & 8 f(0.61040(41) \\ & 0.40852(88) \\ & 0.08070(20)) \end{aligned}$ | $\begin{aligned} & \text { 8f (0.60313( 175) } \\ & 0.39619(346) \\ & 0.08390(94)) \end{aligned}$ |
| $\begin{aligned} & 8 f(0.73641(105) \\ & 0.19293(178) \\ & 0.24889(64)) \end{aligned}$ | $\begin{aligned} & 8 f(0.73793(106) \\ & 0.15368(224) \\ & 0.25207(59)) \end{aligned}$ | $\begin{aligned} & 8 f(0.75967(129) \\ & 0.13616(247) \\ & 0.25761(83)) \end{aligned}$ | $\begin{aligned} & 8 f(0.75230(482) \\ & 0.11147(1074) \\ & 0.25366(350)) \end{aligned}$ |
| $\begin{aligned} & 4 \mathrm{e}(0.00000(0) \\ & 0.91073(297) \\ & 0.25000(0)) \end{aligned}$ | $\begin{aligned} & 4 \mathrm{e}(0.00000(0) \\ & 0.91088(258) \\ & 0.25000(0)) \end{aligned}$ | $\begin{aligned} & 4 \mathrm{e}(0.00000(0) \\ & 0.92483(381) \\ & 0.25000(0)) \end{aligned}$ | $\begin{aligned} & 4 \mathrm{e}(0.00000(0) \\ & 0.96327(1570) \\ & 0.25000(\quad 0)) \end{aligned}$ |
| $\begin{aligned} & 8 f(0.41481, \\ & 0.23022,0.0000) \end{aligned}$ | $\begin{aligned} & 8 f(0.41481, \\ & 0.23022,0.0000) \end{aligned}$ | $\begin{aligned} & 8 f(0.41481, \\ & 0.23022,0.0000) \end{aligned}$ | $\begin{aligned} & 8 f(0.41481 \text {, } \\ & 0.23022,0.0000) \end{aligned}$ |
| $\begin{aligned} & \text { 4d }(0.25,0.25, \\ & 0.50) \end{aligned}$ | $\begin{aligned} & \text { 4d }(0.25,0.25, \\ & 0.50) \end{aligned}$ | $\begin{aligned} & \text { 4d }(0.25,0.25, \\ & 0.50) \end{aligned}$ | 4d (0.25, 0.25, 0.50) |

Table S2. Phase purity from Rietveld refinement in weight \%.

| RE i-MAX | $\mathrm{Mo}_{2} \mathrm{C}$ | $\mathrm{Mo}_{3} \mathrm{Al}_{2} \mathrm{C}$ | $\mathrm{REAlO}_{3}$ | $\mathrm{REAl}_{3} \mathrm{C}_{3}$ | Others |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Nd 88.5 | 6.8 | $\mathrm{~N} / \mathrm{A}$ | 4.7 | $\mathrm{~N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ |
| Sm94.5 | 4.5 | 1.0 | $\mathrm{~N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ |
| Gd 94.4 | $\mathrm{N} / \mathrm{A}$ | 4.4 | 1.2 | $\mathrm{~N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ |
| Tb 98.1 | 1.9 | $\mathrm{~N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ |
| Dy 95.1 | $\mathrm{N} / \mathrm{A}$ | 4.0 | 0.9 | $\mathrm{~N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ |
| Ho 83.0 | 6.8 | 2.4 | 3.8 | 4.04 | $\mathrm{~N} / \mathrm{A}$ |
| Er 91.7 | 2.0 | 4.5 | 1.8 | $\mathrm{~N} / \mathrm{A}$ | $1.79 \mathrm{Er}_{2} \mathrm{O}_{3}$ |
| Tm 88.5 | 1.3 | 6.0 | $\mathrm{~N} / \mathrm{A}$ | 2.6 | $1.55 \mathrm{TmAl}_{2}$ |
| Lu 70.9 | 2.6 | 7.9 | $\mathrm{~N} / \mathrm{A}$ | $\mathrm{N} / \mathrm{A}$ | $8.3 \quad \mathrm{Mo}_{3} \mathrm{Al}$ |
|  |  |  |  |  | $10.3 \mathrm{Lu}_{2} \mathrm{O}_{3}$ |

The Mo containing impurities observed in most samples, $\mathrm{Mo}_{2} \mathrm{C}$ and $\mathrm{Mo}_{3} \mathrm{Al}_{2} \mathrm{C}$, are paramagnetic. $\mathrm{NdAlO}_{3}$ is antiferromagnetic below $0.93 \mathrm{~K}^{1}$, and the magnetic transition observed at 7.6 K is therefore attributed to $\left(\mathrm{Mo}_{2 / 3} \mathrm{Nd}_{1 / 3}\right)_{2} \mathrm{AlC}$.

For the $\left(\mathrm{Mo}_{2 / 3} \mathrm{Sm}_{1 / 3}\right)_{2} \mathrm{AlC}$ sample, the magnetization is very weak and no magnetic transition can be observed down to 2 K . For the $\left(\mathrm{Mo}_{2 / 3} \mathrm{Gd}_{1 / 3}\right)_{2} \mathrm{AlC}$ sample, $\mathrm{GdAlO}_{3}$ orders below $3.69 \mathrm{~K}^{2}$, and the magnetic transition observed at 26 K is therefore attributed to the $i$-MAX phase.

For $\left(\mathrm{Mo}_{2 / 3} \mathrm{~Tb}_{1 / 3}\right)_{2} \mathrm{AlC}$, only $\sim 2 \%$ of $\mathrm{Mo}_{2} \mathrm{C}$ can be found, and the two magnetic transitions can therefore be attributed to the $i$-MAX phase. We also clarified the mechanism of the two magnetic transitions by NPD.
$\mathrm{DyAlO}_{3}$ orders at $3.52 \mathrm{~K}^{3}$, and can therefore be excluded as origin to the observed magnetic characteristics. The behavior of $\left(\mathrm{Mo}_{2 / 3} \mathrm{Dy}_{1 / 3}\right)_{2} \mathrm{AlC}$ is also very similar to that of $\left(\mathrm{Mo}_{2 / 3} \mathrm{~Tb}_{1 / 3}\right)_{2} \mathrm{AlC}$.
$\mathrm{HoAlO}_{3}$ is antiferromagnetic below $0.16 \mathrm{~K}^{4}$. The magnetism of $\mathrm{HoAl}_{3} \mathrm{C}_{3}$ is unknown. $\mathrm{Er}_{2} \mathrm{O}_{3}$ orders at $3.36 \mathrm{~K}^{5}$. The magnetic contribution of $\mathrm{Er}_{2} \mathrm{O}_{3}$ to the NPD was considered following ref. 5.

No magnetic transition can be observed down to 2 K in the $\left(\mathrm{Mo}_{2 / 3} \mathrm{Tm}_{1 / 3}\right)_{2} \mathrm{AlC}$ sample. The $\left(\mathrm{Mo}_{2 / 3} \mathrm{Lu}_{1 / 3}\right)_{2} \mathrm{AlC} \mathrm{Lu}$ sample is not analyzed with respect to magnetism.

## Section S2. Relationship between three polymorphs

The three polymorphs identified possess the same $\left(\mathrm{Mo}_{2 / 3} \mathrm{RE}_{1 / 3}\right)_{2} \mathrm{C}$ layers as building blocks (Fig. 1b), with the difference between them being the layer stacking along the $c$ axis. In the $C 2 / c$ structure the layers are stacked in an ABAB sequence, and are rotated $60^{\circ}$ with respect to each other, see Fig. S3 for a schematic. In the Cmcm structure, on the other hand, the layers are stacked in an ACAC sequence, and are rotated $180^{\circ}$ with respect to each other. In the $C 2 / \mathrm{m}$ structure, the layers are stacked exactly on top of each other, viz. the stacking is AAAA.


Fig. S3. Schematic illustration of different stacking sequences for REMAX in space group $C 2 / m$ (12), C2/c (15), and Cmcm (63). The structure consists of alternating layers of $\left(\mathrm{Mo}_{2 / 3} \mathrm{RE}_{1 / 3}\right)_{2} \mathrm{C}$ and Al building blocks where $\left(\mathrm{Mo}_{2 / 3} \mathrm{RE}_{1 / 3}\right)_{2} \mathrm{C}$ can have three different orientations; A, B (rotated $+60^{\circ}$ relative A), and C (rotated $+180^{\circ}$ relative A). Different stacking sequences of $\left(\mathrm{Mo}_{2 / 3} \mathrm{RE}_{1 / 3}\right)_{2} \mathrm{C}$ results in $\mathrm{C} 2 / \mathrm{c}$ structure with AAAA stacking, $\mathrm{C} 2 / m$ structure with ABAB stacking, and Cmcm structure with ACAC stacking.

## Section S3. Theoretical modeling



Fig. S4. Schematic representation of six collinear spin configurations considered for $\left(\mathrm{Mo}_{2 / 3} \mathrm{RE}_{1 / 3}\right)_{2} \mathrm{AlC}$ where the spin direction at each RE site is represented by a black arrow.


Fig. S5. Local magnetic moment of $R E$ for $\left(\mathrm{Mo}_{2 / 3} R E_{1 / 3}\right)_{2} A I C$ with $U=9 \mathrm{eV}$ and $\mathrm{J}=2 \mathrm{eV}$.

Section S4. Curie-Weiss fitting in the paramagnetic state








Fig S6. a-i H/M vs. T for RE = Pr, Nd, Sm, Gd, Tb, Dy, Er, Ho, and Tm, and their fitting to the Curie-Weiss law.

Supplementary Fig. 6 plots the inverse susceptibility $1 / \chi$ (deduced from Arrott plots) or $\mathrm{H} / \mathrm{M}$ vs. T plots for 9 RE phases. In all cases, but Sm, a Curie-Weiss, C-W, law of the form

$$
\begin{equation*}
\chi=C /(T-\theta), \tag{1}
\end{equation*}
$$

where $C$ is the Curie constant and T and $\theta$ are the temperature in K and the Curie-Weiss temperature, respectively. The resulting parameters are listed in Fig. 4e. The effective moment $\mu_{e f f}$ per RE element is estimated from

$$
\begin{equation*}
C=N \mu_{B}^{2} \mu_{e f f}^{2} / 3 k_{B}, \tag{2}
\end{equation*}
$$

where $N$ is the number of magnetic atoms per unit volume, $\mu_{\mathrm{B}}$ the Bohr magneton and $k_{B}$, Boltzmann's constant. All phases, except $\mathrm{RE}=\mathrm{Sm}$, fit well to Eq. 1 in the 100 K~300 K temperature interval.

## Section S5. Properties of $\left(\mathrm{Mo}_{2 / 3} \mathrm{Gd}_{1 / 3}\right)_{2} \mathrm{AIC}$ and $\left(\mathrm{Mo}_{2 / 3} \mathrm{~Tb}_{1 / 3}\right)_{2} \mathrm{AIC}$



Fig. S7. a. $\mathrm{M} / \mathrm{H}$ of $\left(\mathrm{Mo}_{2 / 3} \mathrm{Gd}_{1 / 3}\right)_{2} \mathrm{AIC}$. A weak bump is observed at $\mathrm{T}_{\mathrm{N}}=26 \mathrm{~K}$. b. Resistance vs. Temperature. c. Arrott plots above the transition temperature. d. Arrott plots below transition temperature.


Fig S8. a. Heat capacity of $\mathrm{RE}=\mathrm{Lu}, \mathrm{Tb}, \mathrm{Nd}$, and Er. b. Thermal variation of magnetic entropy. The phonon contribution is subtracted by the non-magnetic analog RE=Lu. R In2 is indicated by dashed line.


Fig S9. a. M-H at different temperatures for $\left(\mathrm{Mo}_{2 / 3} \mathrm{~Tb}_{1 / 3}\right)_{2} \mathrm{AlC}$. b. $\mathrm{dM} / \mathrm{dH}$ plot for $\left(\mathrm{Mo}_{2 / 3} \mathrm{~Tb}_{1 / 3}\right)_{2} \mathrm{AlC}$ at 2 K . Arrows indicate field induced transitions.

## Section S6. Detailed magnetic structure analysis of the NPD data

$\left(\mathbf{M o}_{2 / 3} \mathbf{T b}_{1 / 3}\right)_{2} \mathbf{A I C}$
Using the k -search program, the propagation vector for the structure below 20 K is indexed to be $\mathbf{k}=(0,1 / 2,0)$. The magnetic unit cell is twice the size of the nuclear cell. The symmetry analysis is performed by using the program BasIreps. The list of the irreducible representations is given in the table below. Note that the tables are only referred in this section, so they are numbered in Roman numbers. The magnetic representation can be decomposed:

$$
\Gamma=6 \Gamma_{1} \oplus 6 \Gamma_{2}
$$

Table I . Irreducible representations. p=1/2.

| Irreps | \{1\} | \{2y $\mid 00 \mathrm{p}$ \} | \{-1\} | \{ $\mathrm{m}_{\mathrm{y}} \mid 00 \mathrm{p}$ \} |
| :---: | :---: | :---: | :---: | :---: |
| $\Gamma_{1}$ | $\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}$ | $\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}$ | $\begin{array}{cc}1 & 0 \\ 0 & -1\end{array}$ | $\begin{array}{cc}1 & 0 \\ 0 & -1\end{array}$ |
| $\Gamma_{2}$ | $\begin{array}{ll}1 & 0 \\ 0 & 1\end{array}$ | $\begin{array}{cc}-1 & 0 \\ 0 & -1\end{array}$ | $\begin{array}{cc}1 & 0 \\ 0 & -1\end{array}$ | $\begin{array}{cc} \hline-1 & 0 \\ 0 & 1 \end{array}$ |

Table ㅍ. Basis vectors of Tb atoms

| Irrep. | Basis vector | Tb1 <br> $(\mathrm{x}, \mathrm{y}, \mathrm{z})$ | Tb2 <br> $(-\mathrm{x}, \mathrm{y},-\mathrm{z}+1 / 2)$ | Tb3 <br> $(-\mathrm{x},-\mathrm{y},-\mathrm{z})$ | Tb4 <br> $(\mathrm{x},-\mathrm{y}, \mathrm{z}+1 / 2)$ |
| :--- | :--- | ---: | ---: | ---: | :---: |
| $\Gamma_{1}$ | $\mathrm{~V}_{1}$ | $[100]$ | $[-100]$ | $[100]$ | $[-100]$ |
|  | $\mathrm{V}_{2}$ | $[010]$ | $[010]$ | $[010]$ | $[010]$ |
|  | $\mathrm{V}_{3}$ | $[001]$ | $[00-1]$ | $[001]$ | $[00-1]$ |
|  | $\mathrm{V}_{4}$ | $[100]$ | $[-100]$ | $[-100]$ | $[100]$ |
|  | $\mathrm{V}_{5}$ | $[010]$ | $[010]$ | $[0-10]$ | $[0-10]$ |
|  | $\mathrm{V}_{6}$ | $[001]$ | $[00-1]$ | $[00-1]$ | $[001]$ |
|  | $\mathrm{V}_{1}$ | $[100]$ | $[100]$ | $[100]$ | $[100]$ |
|  | $\mathrm{V}_{2}$ | $[010]$ | $[0-10]$ | $[010]$ | $[0-10]$ |
|  | $\mathrm{V}_{3}$ | $[001]$ | $[001]$ | $[001]$ | $[001]$ |
|  | $\mathrm{V}_{4}$ | $[100]$ | $[100]$ | $[-100]$ | $[-100]$ |
|  | $\mathrm{V}_{5}$ | $[010]$ | $[0-10]$ | $[0-10]$ | $[010]$ |
|  | $\mathrm{V}_{6}$ | $[001]$ | $[001]$ | $[00-1]$ | $[00-1]$ |

A trial and error method is used to determine the correct magnetic model. The irreps $\Gamma_{2}$ appears to be the correct solution. We have tried all possible magnetic models. The $V_{1}$ and $V_{3}$ appears to be the dominant vectors. In Table III, we summarized parameters obtained from refinement for the 8 K data by considering different magnetic models. As can be seen, the best agreement is achieved by considering $\mathrm{V}_{1}, \mathrm{~V}_{2}$, and $\mathrm{V}_{3}$. The $\mathrm{R}_{\text {mag }}=6.23 \%$ is slightly better than $\mathrm{R}_{\mathrm{mag}}=7.18 \%$ considering $\mathrm{V}_{1}$ and $\mathrm{V}_{3}$. The refined magnetic moment is close to $5.0(2) \mu_{\mathrm{B}}$.

Table III. Parameters from refinement of 8 K data by considering different models of $\Gamma_{2}$.

| V1 | V2 | V3 | V4 | V5 | V6 | R-mag <br> $(\%)$ | M <br> $(\mu \mathrm{B})$ | $\mathrm{M}(\mu \mathrm{B})$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathbf{- 3 . 4 4 8}$ | $\mathbf{0 . 5 5 1}$ | $\mathbf{2 . 8 3 9}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{0}$ | $\mathbf{6 . 2 3}$ | $\mathbf{4 . 9}$ |  |
| -3.438 | 0 | 2.875 | 0.407 | 0 | 0 | 6.99 | 4.641 | 5.313 |
| -3.432 | 0 | 2.885 | 0 | 0.379 | 0 | 6.92 | 4.988 |  |
| -3.443 | 0 | 2.867 | 0 | 0 | 0.417 | 7.01 | 4.67 | 5.285 |
| -3.44 | 0 | 2.884 | 0 | 0 | 0 | 7.18 | 4.98 |  |
| -3.442 | 0.529 | 2.83 | 0.279 | 0.363 | 0.334 | 6.26 | 5.049 | 4.945 |

Similarly, we indexed the propagation vector for the structure at 22 K by k-search. No commensurate solution was found. The best solution is $\mathbf{k}=(0, \sim 0.6,0)$. The symmetry for $\mathbf{k}=(0$, $\sim 0.6,0)$ is the same as $\mathbf{k}=(0,0.5,0)$. Similarly, a trial and error is used to determine the correct solution. Irreps $\Gamma_{2}$ is the correct solution.

Table IV. Parameters from refinement of 22 K data by considering different models of $\Gamma_{2}$. M corresponds to the peak moment of the modulated structure.

| V1 | V2 | V3 | V4 | V5 | V6 | R-mag \% | $\mathrm{M}(\mu \mathrm{B})$ | $\boldsymbol{k}=(0, \delta, 0)$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\begin{aligned} & 2.80 \\ & 7 \end{aligned}$ | -0.462 | $2.599$ | 0 | 0 | 0 | 12.37 | 4.27 | 0.6053 |
| $\begin{aligned} & 2.74 \\ & 1 \end{aligned}$ | 0 | $2.669$ | 0.877 | 0 | 0 | 12.16 | 4.973 | 0.6054 |
| $\begin{aligned} & 2.73 \\ & 7 \\ & \hline \end{aligned}$ | 0 | $2.646$ | 0 | -0.979 | 0 | 12.17 | 4.341 | 0.6053 |
| $\begin{aligned} & 2.78 \\ & 7 \\ & \hline \end{aligned}$ | 0 | $2.614$ | 0 | 0 | 0.705 | 12.20 | 4.03 | 0.6054 |
| 2.77 | 0 | -2.67 | 0 | 0 | 0 | 11.18 | 4.27 | 0.6053 |
| $\begin{aligned} & 2.75 \\ & 5 \\ & \hline \end{aligned}$ | -0.323 | $2.576$ | 0.851 | -0.977 | 0.601 | 12.38 | 4.683 | 0.6054 |

Similarly with the 8 K structure, the $\mathrm{V}_{1}$ and $\mathrm{V}_{3}$ are the dominant vectors contributing to the 22 K structure. Only the $4^{\text {th }}$ vector or $6^{\text {th }}$ vector contribute to the peak at $\sim 15$ degree, so it's necessary to include either $4^{\text {th }}$ or $6^{\text {th }}$ vector even though $\mathrm{V}_{1}+\mathrm{V}_{3}$ gives the best agreement. The solution considering $\mathrm{V}_{1}, \mathrm{~V}_{3}$, and $\mathrm{V}_{4}$ gives comparable agreement factor $\mathrm{R}_{\mathrm{mag}}=12.16 \%$ to the solution considering $\mathrm{V}_{1}, \mathrm{~V}_{3}$ and $\mathrm{V}_{6}, \mathrm{R}_{\text {mag }}=12.20 \%$. For solution involving the $4^{\text {th }}$ vector, the magnetic moment oscillates between $\sim 1 \mu_{\mathrm{B}}$ to $\sim 5 \mu_{\mathrm{B}}$. The propagation vector is refined to be $\boldsymbol{k}=(0, \delta, 0)$, $\delta=0.60543$.

## $\left(\mathrm{Mo}_{2 / 3} \mathrm{Er}_{1 / 3}\right)_{2} \mathrm{AlC}$

The NPD pattern of $\left(\mathrm{Mo}_{2 / 3} \mathrm{Er}_{1 / 3}\right)_{2} \mathrm{AlC}$ at 1.5 K showed additional magnetic reflections emerge compared to the room temperature and 22 K patterns. The angular position of these magnetic reflections was obtained by subtracting the 22 K profile from the 1.5 K profile. Attempts to index the magnetic structure using special symmetry points revealed that the reflection at $\mathrm{Q} \sim 0.37 \AA^{-1}$ can be indexed using the $\mathbf{k}_{1}=(1 / 2,0,1 / 2)$ propagation vector, however, a successful indexing of the remaining magnetic reflections using " $k$-search" turned out to be unsuccessful. This is due to the low resolution of the E6 instrument and the low symmetry of the i-MAX unit cell, which resulted in the overlap of many magnetic reflections. Therefore, a systematic Brillouin zone scan combined with a simulated annealing magnetic refinement was performed as implemented in the SARAh package. For each tested propagation vector the magnetic structure was refined using Monte Carlo optimization with 100 steps per each vector. The refinement parameters were the magnitude and directions of the four Er atoms not related by translational symmetry within the 8 f Wyckoff site. The automated scan yielded the $\mathbf{k}_{\mathbf{2}}=(0,0.68,0)$ propagation vector as a possible indexing of the magnetic unit cell. Using $\mathbf{k}_{1}$ and $\mathbf{k}_{2}$ as starting points, the irreducible representations of the little group were calculated using the BasIreps package. The results for $\mathbf{k}_{2}$ are the same as with the Tb case. After trying the possible representations, we find that $\Gamma_{2}$ is the correct representation. Refinement of the coefficients of the basis vectors $\mathrm{V}_{1}, \ldots, \mathrm{~V}_{6}$ shows that $\mathrm{V}_{4}$ and $\mathrm{V}_{6}$ are the dominant vectors. Refining both $\mathrm{V}_{1}$ and $\mathrm{V}_{3}$ in addition to the two dominant vectors results in a slightly better agreement. The full results of the refinement are summarized in Table V . The best fit results in a maximal magnetic moment of 5.6(2) $\mu_{\mathrm{B}} / E r$. The value of the propagation vector $\mathbf{k}_{\mathbf{2}}=(0, \delta, 0)$ was also refined, resulting in with $\delta=0.675(1)$ (Table V$)$. It is worth mentioning that the magnetic scattering of $\mathrm{Er}_{2} \mathrm{O}_{3}(\sim 2 \mathrm{wt} . \%)$ was considered.

Table V. Refinement results for different magnetic configurations considered.

| V1 | V2 | V3 | V4 | V5 | V6 | R-mag <br> $(\%)$ | M <br> $(\mu \mathrm{B})$ | $\boldsymbol{k}=(0, \delta, 0)$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 0 | 0 | 0 | 2.9920 | 0 | 5.3357 | 11.5 | 5.4723 | 0.67622 |
| 1.7121 | 0 | 0 | 2.9339 | 0 | 5.1005 | 10.5 | 6.0418 | 0.67449 |
| 0 | 0.0063 | 0 | 3.0070 | 0 | 5.3484 | 11.4 | 5.4878 | 0.67592 |
| 0 | 0 | 1.4503 | 2.9017 | 0 | 5.0939 | 10.4 | 5.4106 | 0.67384 |
| $\mathbf{0 . 4 9 8 8}$ | $\mathbf{0}$ | $\mathbf{1 . 5 2 2 3}$ | $\mathbf{2 . 9 0 2 8}$ | $\mathbf{0}$ | $\mathbf{5 . 0 9 3 8}$ | $\mathbf{1 0 . 3}$ | $\mathbf{5 . 6 0 0 6}$ | $\mathbf{0 . 6 7 3 8 5}$ |
| 0 | 0 | 0 | 2.9920 | 1.4557 | 5.3513 | 11.2 | 5.6754 | 0.67593 |

Symmetry analysis of $\mathbf{k}_{\mathbf{1}}=(1 / 2,0,1 / 2)$ yields two irreducible representations $\Delta_{1}$ and $\Delta_{2}$ (Table VI). The 4 Er atoms in the 8 f site, that are not related by the base translation C , split into two orbits. For each orbit the decomposition of the magnetic group is $\Delta_{m}=3 \Delta_{1} \oplus 3 \Delta_{2}$. Similarly to the case of $\mathbf{k}_{2}$, we combine the two orbits in a symmetric and antisymmetric way, leading to a magnetic representation of $\Delta_{m}=6 \Delta_{1} \oplus 6 \Delta_{2}$. Each representation has 6 basis vectors labeled by $\mathrm{W}_{1}, \ldots, \mathrm{~W}_{6}$.

Table VI. The irreducible representations of the little group $G_{k}, \mathbf{k}=(1 / 2,0,1 / 2)$.

| Irreps | $\{1\}$ | $\left\{\mathrm{m}_{\mathrm{y}} \mid 00 \mathrm{p}\right\}$ |
| :---: | :---: | :---: |
| $\Delta_{1}$ | 1 | -i |
| $\Delta_{2}$ | 1 | i |

The magnetic configurations that correspond to each basis vector are given in Table VII. Here the relevant Er atoms are chosen to remain inside the basic unit cell, and therefore use unconventional symmetry operators.

Table VII. The magnetic configurations corresponding to each basis vector of $\Delta_{1}$ and $\Delta_{2}$.

| Irrep. | Basis vector | $\begin{aligned} & \text { Er1 } \\ & (\mathrm{x}, \mathrm{y}, \mathrm{z}) \end{aligned}$ | $\begin{aligned} & \text { Er8 } \\ & (x-1 / 2,-y+1 / 2, z+1 / 2) \end{aligned}$ | $\begin{aligned} & \text { Er2 } \\ & (-\mathrm{x}+1, \mathrm{y},-\mathrm{z}+1 / 2) \end{aligned}$ | $\begin{aligned} & \text { Er3 } \\ & (-x+1,-y+1,-z+1) \end{aligned}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\Delta_{1}$ | $\mathrm{W}_{1}$ | [100] | [100] | [100] | [-i00] |
|  | $\mathrm{W}_{2}$ | [010] | [0-10] | [010] | [0i0] |
|  | $\mathrm{W}_{3}$ | [001] | [001] | [001] | [00-i] |
|  | $\mathrm{W}_{4}$ | [100] | [100] | [-100] | [i00] |
|  | $\mathrm{W}_{5}$ | [010] | [0-10] | [0-10] | [0-i0] |
|  | $\mathrm{W}_{6}$ | [001] | [001] | [00-1] | [00i] |
| $\Delta_{2}$ | $\mathrm{W}_{1}$ | [100] | [-100] | [100] | [i00] |
|  | $\mathrm{W}_{2}$ | [010] | [010] | [010] | [0-i0] |
|  | $\mathrm{W}_{3}$ | [001] | [00-1] | [001] | [00i] |
|  | $\mathrm{W}_{4}$ | [100] | [-100] | [-100] | [-i00] |
|  | $\mathrm{W}_{5}$ | [010] | [010] | [0-10] | [0i0] |
|  | $\mathrm{W}_{6}$ | [001] | [00-1] | [00-1] | [00-i] |

A trial and error search has been performed on all basis vectors. Two possible solutions, which agree with the observed data, were found. These are $\mathrm{W}_{5}$ of representation $\Delta_{1}$ and $\mathrm{W}_{6}$ of representation $\Delta_{2}$. The results of these two configurations are given in Table VIII. The best fit configuration gives a magnetic moment of $0.80(3) \mu_{\mathrm{B}} / \mathrm{Er}$ atom.

Table VIII: The refinement results for the possible magnetic configurations of the $\mathrm{k}_{2}$ propagation vector.

| Configuration | $\mathrm{M}\left(\mu_{\mathrm{B}}\right)$ | R-mag (\%) |
| :--- | :--- | :--- |
| $\Delta_{\mathbf{1}}, \mathrm{W}_{5}$ | 0.796 | 11.5 |
| $\boldsymbol{\Delta}_{\mathbf{2}}, \mathbf{W}_{\mathbf{6}}$ | $\mathbf{0 . 9 8 8 9}$ | $\mathbf{1 1 . 2}$ |

## Section S7. Structural parameters from NPD

Table S3. Structural refinement from NPD. The global residuals for the refinement of $R E=T b, R p=9.60 \%, R w p=14.4 \%, \operatorname{Re}=1.21 \%$, $\operatorname{Rnd} x^{2}=141$. For $R E=E r, R p=1.87 \%$, $R w p=2.47 \%, \operatorname{Re}=0.66 \%$, and $x^{2}=13.8$.

| $\begin{aligned} & \left(\mathrm{Mo}_{2 / 3} \mathrm{~Tb}_{1 / 3}\right)_{2} \mathrm{AlC} \\ & @, 100 \mathrm{~K} \\ & \hline \end{aligned}$ | $\begin{array}{\|l} \hline \text { Bragg R = } \\ 21.6 \\ \hline \end{array}$ | RF $=22.3$ |  |  |
| :---: | :---: | :---: | :---: | :---: |
| C 2/c | $a=9.52821$ <br> (44) | $\begin{array}{\|l\|} \hline b=5.51591 \\ (22) \\ \hline \end{array}$ | $\begin{aligned} & c=14.04858 \\ & (86) \\ & \hline \end{aligned}$ | $\begin{aligned} & \beta=103.57702 \\ & (282) \end{aligned}$ |
|  | Wyckoff | x | y | z |
| Mo | 8 f | $0.27038(209)$ | $0.44714(354)$ | 0.07563( 116) |
| Mo | 8 f | 0.60505( 212) | 0.39864( 324) | 0.08213(107) ) |
| Tb | 8 f | 0.95567(172) | 0.41863 ( 328) | 0.11074(100) |
| AI | 8 f | 0.75950( 323) | 0.15239(624) | 0.24360( 183) |
| AI | 4e | 0 | 0.92257(924) | 0.25 |
| C | 8 f | 0.42383( 213) | 0.26362(356) | -0.00809(122) |
| C | 4d | 0.25 | 0.25 | 0.5 |


| $\left(\mathrm{Mo}_{2 / 3} \mathrm{Er}_{1 / 3}\right)_{2} \mathrm{AlC}$ <br> $@ 320 \mathrm{~K}$ | $\mathrm{Bragg} \mathrm{R}=$ <br> 21.6 | $\mathrm{RF}=22.3$ |  |  |
| :--- | :--- | :--- | :--- | :--- |
| $\mathrm{C} \mathrm{2/c}$ | $a=9.37679$ <br> $(64)$ | $b=5.45083$ <br> $(29)$ | $c=13.88838$ <br> $(125)$ | $\beta=103.49207$ <br> $(612)$ |
|  | Wyckoff | x | y | z |
| Mo | 8 f | $0.27287(177)$ | $0.43060(354)$ | $0.08314(72)$ |
| Mo | 8 f | $0.61766(169)$ | $0.41526(358)$ | $0.08109(66)$ |
| Er | 8 f | $0.95352(148)$ | $0.42417(347)$ | $0.11260(62)$ |
| Al | 8 f | $0.74449(297)$ | $0.15634(533)$ | $0.24482(177)$ |
| Al | 4 e | 0 | $0.91914(773)$ | 0.25 |
| C | 8 f | $0.42158(153)$ | $0.23696(292)$ | $-0.01378(69)$ |
| C | 4 d | 0.25 | 0.25 | 0.5 |

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