Supporting information for

Classification and Prediction of Dripping Drop Size for a Wide Range of Nozzles by Wetting Diameter

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Analysis of flowrate effect on each term of the drop size prediction model:

Scheele and Meister proposed a correlation for predicting drop volume V_{drop} in a liquid–liquid system, for drop formation with different fluids from nozzle diameters of 0.813 mm $\leq D_{nozzle} \leq$ 6.88 mm, in a low feeding velocity range (< 0.352 m/s)[1]. The gravitational/buoyancy force, kinetic force owing to the liquid flow, interfacial tension force, and drag force by the ambient liquid are all considered, and the correction equation becomes

$$V_{drop}\left(=\frac{\pi d^3}{6}\right)=F\left[\underbrace{\frac{\pi\sigma D_{nozzle}}{g\Delta\rho}}_{[1]}+\underbrace{\frac{20\mu Q D_{nozzle}}{d^2 g\Delta\rho}}_{[2]}-\underbrace{\frac{16\rho Q^2}{3\pi D_{nozzle}^2 g\Delta\rho}}_{[3]}+\underbrace{4.5\left(\frac{Q^2 D_{nozzle}^2\rho\sigma}{(g\Delta\rho)^2}\right)^{\frac{1}{3}}}_{[4]}\right], \qquad \text{(S1)}$$

where F is the Harkins-Brown correction factor, which can be obtained through the third-order polynomial from the regression analysis of the data in Fig. 3 of Scheele and Meister (1968), and is shown as $F = -0.0787x^3 + 0.4102x^2 - 0.7077x + 1.0037$, where $x = D_{nozzle}(F/V_{drop})^{1/3}$. Q and μ are the liquid feeding rates at the exit of the nozzle and viscosity of the ambient liquid, respectively. $\Delta\rho$ is the density difference between the liquid drop (of density ρ) and ambient liquid (with density ρ_a). These four terms on the right-hand side of (S1) represent the [1] static term dominated by interfacial tension force, [2] drag term by the drag force of ambient fluid, [3] inertial term by the kinetic force of the feeding flow, and [4] dynamic term by the additional volumetric flow into the pendant drop during necking, respectively. The application range of (S1) has covered various experiments with 683 kg/m^3 $< \rho <$ 986 kg/m^3, 990 kg/m^3 $< \rho_a <$ 1254 kg/m^3, 0.958 cP $< \mu <$ 515 cP, and 1.79 mN/m $< \sigma <$ 45.4 mN/m.

Except the necking phenomena is not considered in Tate's law, the difference between the existing drop size predictions[1–3] basically comes from the three added terms with Q in (S1); it is thus very interesting to know how the liquid feeding rate Q quantitatively influences the drop formation in (S1). Based on Scheele & Meister's model of (S1), Fig. S1 shows the weighting contribution of each term, i.e., volume ratio, which is defined as the liquid volume calculated from each term normalized by the generated drop volume for the water-air system ($\Delta\rho$ = 998 kg/m^3, σ = 72 mN/m, μ is 18.6 μPa·s). The weighting ratio of each term is used to evaluate its individual effect under different operation conditions. Taking three nozzle examples with D_{nozzle} = 0.05 mm, 0.5 mm, and 5 mm, with various flow rates for comparison, it is found that the static term, i.e., the 1st-term on the right-hand side of (S1), marked as [1] and shown as the black long-dash line, has a constant and overwhelming contribution to drop formation. The dynamic term, the 4th-term caused by the additional volumetric flow into drops during necking, marked as [4] and shown as the blue centre line, plays a more important role than the other two terms, playing the second most important role. The weighting of the drag term, the 2nd-term (marked as [2]) shown as the green dotted line, is almost constantly the smallest one (< 0.01%) in the application range for $Q < Q_c$, where Q_c is the critical flowrate determined by the upper bound of the simple drop dripping mode (unshaded region) without generation of a stream of drops (shaded region for $Q > Q_c$ in Fig. S1), according to Ambravaneswaran et al.[4]

In Figs. S1(*a*–*c*), it is revealed that the application range of the simple dripping mode (unshaded areas) decreased with the decrease of nozzle diameter. To further evaluate the Q effect on the generated drop, the dimensionless drop diameter predicted by (S1), $d^{\#}$ (= $d(Q)/d(Q=0)$), which is normalized by the static term [1], is used as the test parameter and plotted as the purple solid line. It apparently shows that the drop diameter is only weakly dependent (< 0.6 %) of all liquid feeding rates in the application range for D_{nozzle} = 0.05 mm in Fig. S1(*a*). However, as the nozzle diameter increases, e.g., D_{nozzle} = 0.5 mm in Fig. S1(*b*), the value of $d^{\#}$ significantly varies with liquid feeding rate for $Q \geq 0.1$ mL/min and the maximum deviation of $d^{\#}$ becomes about 5%. Furthermore, by using D_{nozzle} = 5 mm, the maximum deviation of $d^{\#}$ can reach from about 2.3% for Q = 1 mL/min to 25% for Q = 49.3 mL/min in Fig. S1(*c*), which indicates the importance of liquid feeding rate for using large nozzles. However, the maximum variations of $d^{\#}$ for the liquid feeding rate of Q_1 (one dripping drop per minute, shown as red solid line) and Q_2 (two dripping drops per minute, shown as blue solid line) are all within 0.4 % and 0.6 %, respectively. This means that the generated drop size is insensitive to liquid feeding rate of small Q in practical use, e.g., for drop impact experiments. As a result, the four terms on the right-hand side of (S1) could be significantly simplified to be only one term by setting Q = 0 without noticeable deviation.

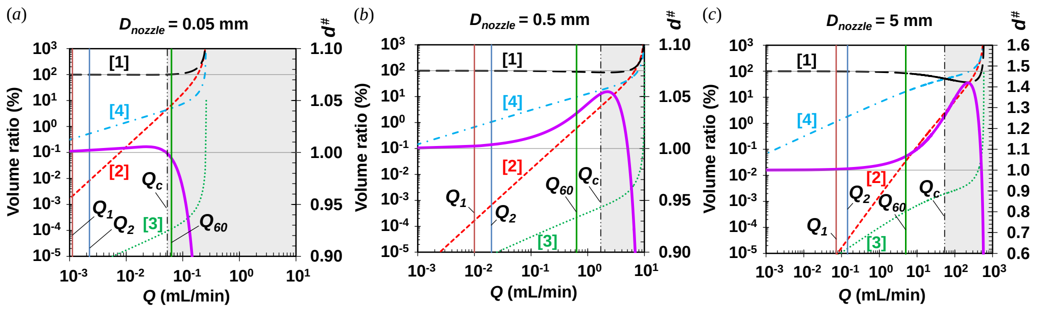

Figure S1. Dependence of volume ratio and predicted drop diameters ($d^{\#}$) on flow rates by Scheele & Meister's model, for three nozzle diameters of D_{nozzle} = (*a*) 0.05 mm, (*b*) 0.5 mm, and (*c*) 5 mm, respectively. $d^{\#}$ is defined as the drop diameter ratio of $d(Q)$ to $d(Q = 0)$ and marked by the purple line. The volume ratios of 1st term, 2nd term, 3rd term and 4th term in the model are illustrated by the lines of — —, – – –, ····· and –·–, respectively.

Dimensionless phase diagram of predicted drop diameter, d^*:

To broaden the drop size prediction of different fluids for a simple comparison, equation (S1) could be further derived as a dimensionless form, and expressed as

$$d^* = \left[6F^{'} D^*_{nozzle} + \left(30F^{'} We^{1/2} Oh D^{*3}_{nozzle} \right)^{3/5} - 2F^{'} We D^*_{nozzle} + 0.9\left(F^{'} We D^{*5}_{nozzle} \right)^{1/3} \right]^{1/3}, \qquad \text{(S2)}$$

where, $F^{'}$ is the Harkins-Brown correction factor, which can be obtained from the data of previous investigations[5–7] and is shown as $F' = -0.0061 D^{*3}_{nozzle} + 0.0758 D^{*2}_{nozzle} - 0.2812 D^*_{nozzle} + 0.9163$. Fig. S2 shows the dimensionless diagram of the predicted drop size d^* ($= d/\lambda$) in terms of Weber number We ($=16\rho Q^2/\pi^2 \sigma D_{nozzle}^3$) and the wetting diameter of nozzle D_w^* ($= D_w/\lambda$) for a wide range of $10^{-8} < We < 10^2$ and $10^{-2} < D_w^* < 10$ that could be possibly used in practice. It was concluded that the generated drop size is positively related to the wetting diameter of nozzle but insensitive to the influence of liquid feeding rate for small nozzles, i.e., Weber number We.

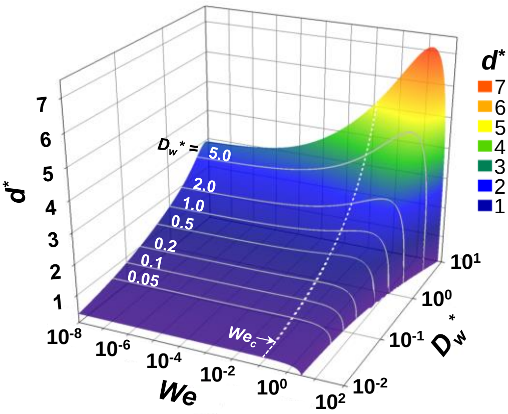

Figure S2. Dimensionless phase diagram of predicted drop diameter in terms of D_w^* and We.

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