Numerical implementation of the poroelastic sliding model for Pe>1

For Pe>1, a numerical solution for the contact line and the associated pore pressure was obtained from Eq. 30 assuming that the contact line has the general form $\overline{r} = f(\theta)$ where f is an even function of θ

$$f(|\theta| \leq \pi/2) = \overline{a} \tag{SI2.1}$$

Consistently with the experimental observations, the curve f is assumed to be described by an ellipse for $|\theta| \geq \pi/2$

$$\begin{aligned} \overline{x} =& \zeta\overline{a}\cos\theta \quad |\theta| \geq \pi/2 \\ \overline{y} =& \overline{a}\sin\theta \end{aligned} \tag{SI2.2}$$

where $0 < \zeta < 1$ is a numerical parameter describing the contact asymmetry. For each velocity $v > v_c$, the two unknowns $\overline{a}$ and ζ are determined from an iterative procedure using Eqs. 30 and 32 for the pore pressure and contact stress, respectively.

We increase $\overline{v}$ from 1 to $1+\Delta\overline{v}$ where $\Delta\overline{v}$ is a small positive number. Then, the condition $\overline{\sigma} > 0$ is no longer satisfied by setting $\overline{\alpha}_1 = 1$, $\overline{\alpha}_0 = 0$, $\overline{\alpha}_2 = \overline{\alpha}_3 = ... = 0$ in Eq. 32 inside the circle $\overline{a} = 1$. Indeed, if we set $\overline{\alpha}_1 = 1$, $\overline{\alpha}_0 = 0$, $\overline{\alpha}_2 = \overline{\alpha}_3 = ... = 0$ when $\overline{v} = 1 + \Delta\overline{v}$, the pressure given by Eq. 30 is no longer vanishing on the circle $\overline{a} = 1$ and the boundary where $\overline{\sigma} = 0$ is given by

$$0 = \overline{r}\cos\theta\left(1 - \overline{r}^2\overline{v}\right) + \left(1 - \overline{r}^2\right) \tag{SI2.3}$$

which defines a new contact line.

Accordingly, we decrease ζ from 1 to $1 - \Delta\zeta$ where $\Delta\zeta$ is a small positive number to obtain a new contact line. From Eq. 30, we determine the set of $\overline{\alpha}_n$ parameters ensuring $\overline{p} = 0$ on this newly defined contact line. Using this new set of $\overline{\alpha}_n$ parameters, the contact stress is calculated from Eq. 32. Then, we calculate the resulting normal force from numerical integration of the contact stress (Eq. 32):

$$\int\int_A \sigma dA = F_n \tag{SI2.4}$$

where A is the contact area defined by $\zeta = 1 - \Delta\zeta$ and $\overline{a} = 1$. If the condition $F_n = F_{imp}$ is violated (F_{imp} is the imposed force), then we have to reduce $\overline{a}$ from 1 to $1 - \Delta\overline{a}$ where $\Delta\overline{a}$ is a small positive number. This defines a new ζ' parameter

$$\zeta' = \frac{1 - \Delta\zeta}{1 - \Delta\overline{a}} \tag{SI2.5}$$

Then we calculate again the set of $\overline{\alpha}_n$ parameters ensuring $\overline{p} = 0$ on the updated contact line and keep iterating until both conditions

$$\begin{aligned} &\overline{p} = 0 \text{ on the contact line } f(\theta) \\ &\int\int_A \sigma dA = F_n \end{aligned} \tag{SI2.6}$$

are satisfied.

The calculated values of $\overline{a} = a/a_0$ and ζ are reported as a function of Pe in Fig. 1. Figure 2 shows an example of the calculated contact stress distribution within the contact for Pe=30. Profiles of contact stress $\overline{\sigma}$, pore pressure $\overline{p}$ and network stress $\overline{\sigma} = \overline{a}^2 - \overline{r}^2$ are also detailed in Fig. 3.

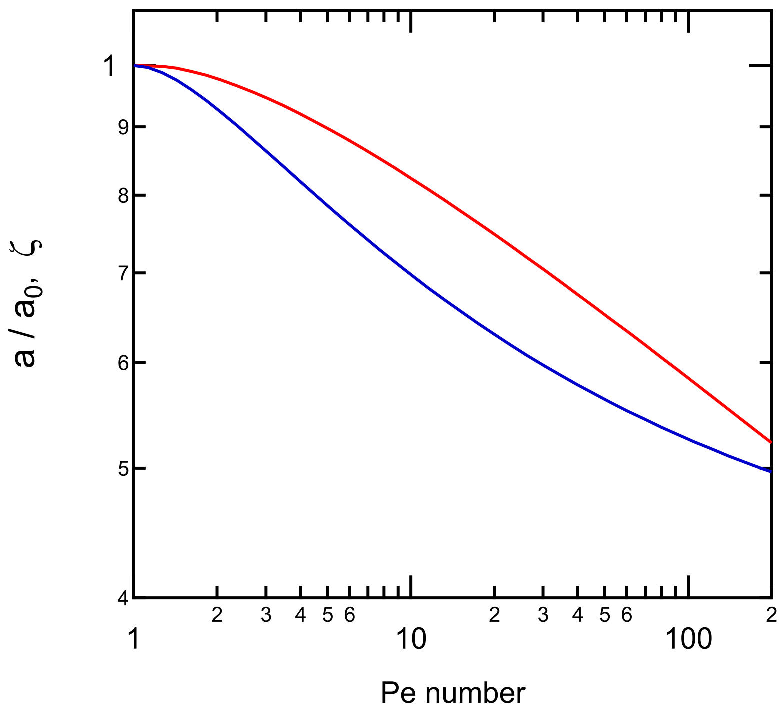

Figure 1: Calculated relative changes in contact radius a/a_0 (red) and in asymmetry parameter ζ (blue) as a function of Pe number. a_0 is the equilibrium contact radius achieved for Pe$\leq$1.

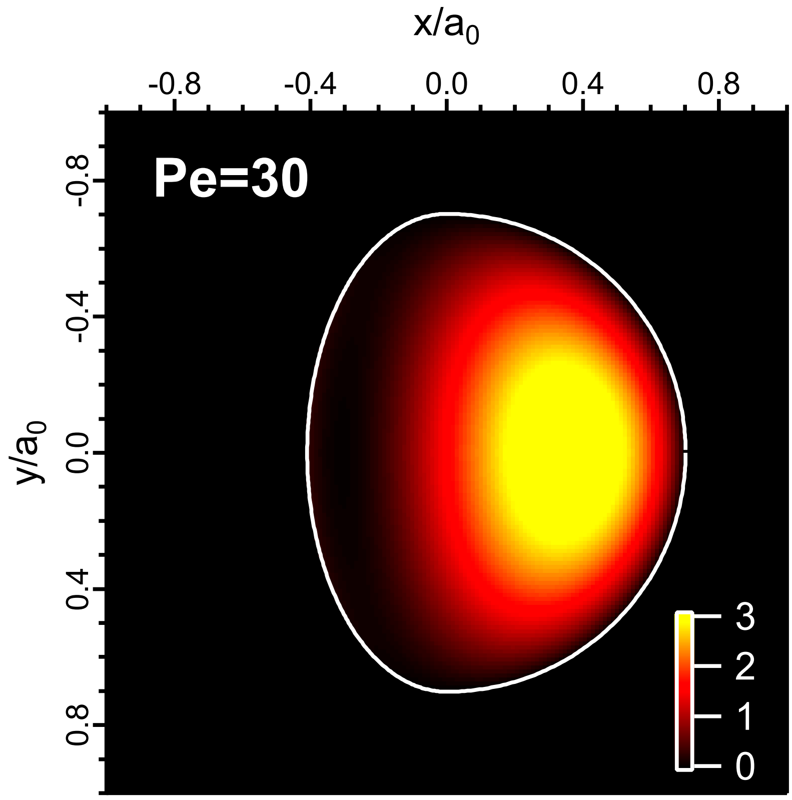

Figure 2: Calculated normalized contact stress distribution for Pe = 30. The white line represents the contact line where $\overline{p} = 0$.

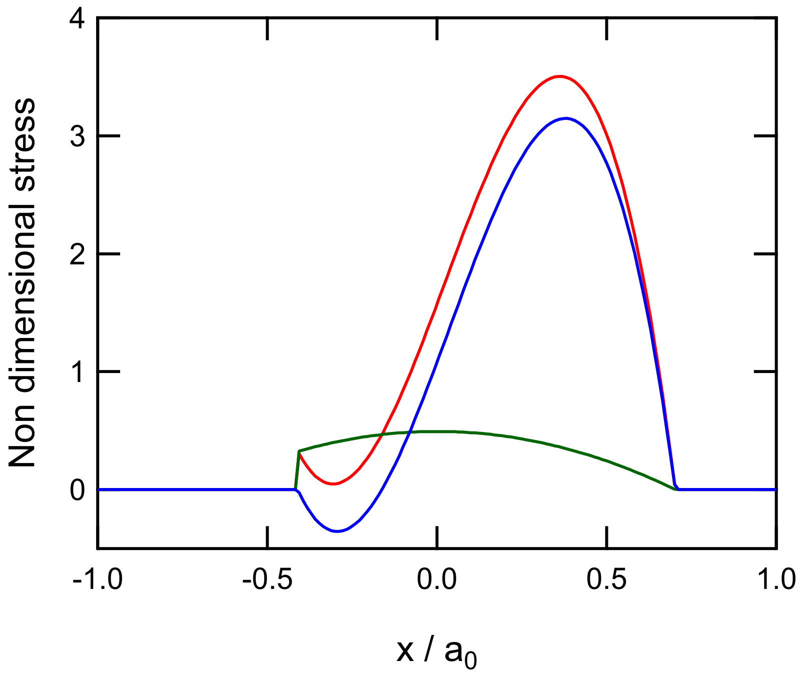

Figure 3: Profiles of calculated normalized contact stress $\overline{\sigma}$ (red), pore pressure $\overline{p}$ (blue) and network stress $\overline{\sigma} = \overline{a}^2 - \overline{r}^2$ (green) along x direction ($y = 0$) for Pe = 30.