## In-Plane Optical Anisotropy and Linear Dichroism in

## Low-Symmetry Layered TlSe

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Figure S1. (a) XRD pattern of TISe single crystals. (b) XPS full spectrum of bulk TISe. The high-solution XPS signals of Tl 4 f (c) and Se 3d (d).


Figure S2. (a) The conventional cell of bulk TlSe. Top view (b) and side views (c-e) of TISe crystal structure.


Figure S3. (a) TEM image of a thin area of the mechanically exfoliated TlSe flake. (b) SADP, (c) HRTEM image and (d) FFT image of this flake.


Figure S4. Atomic-resolution HAADF (a) and BF STEM (b) images viewed from [110] direction.


Figure S5. (a) Band structure and (b) partial density of state (PDOS) of bulk TlSe.


Figure S6. (a) The first Brillouin zone (BZ) of bulk TlSe with high-symmetry points. Fermi surface around VBM (b) and CBM (c) is about 0.2 eV . The left of the figure shows the Fermi surface in the reciprocal unite cell and the right of the figure is the first BZ.


Figure S7. (a) The optical image of few-layer TlSe flakes on a Si wafer with a 300 $\mathrm{nm} \mathrm{SiO}_{2}$ layer. (b) AFM image of a typical TlSe flake. (c) The corresponding cross-section scan along the black line in b .


Figure S8. (a) Unpolarized Raman spectrum of an 18-nm TlSe flake at excitation wavelength of 633 nm . (b) The Raman intensity of bulk TlSe calculated by the DFT methods.


Figure S9. Character table of $\mathrm{D}_{4 \mathrm{~h}}$ and its subgroups.


Figure S10. TlSe phonon dispersion with the correction of long range interaction of macroscopic electric field induced by polarization of collective ionic motion near Gamma point.


Figure S11. Ato moving
Figure S11. Atom moving directions of phonon modes at Gamma point for bulk TlSe.


Figure 12. (a) Optical image of a $2.6-\mathrm{nm}$ TlSe flake. (b) AFM image of this flake. (c) The comparison of Raman spectra for two flakes with different thicknesses (18 nm and 2.6 nm ).


Figure S13. Possible patterns of polar plots for anisotropic Raman intensities simulated according to the Raman tensor.


Figure S14. HSE06 obtained band structure of TISe, the red lines denote the possible [001] polarized light jump, while green lines denote the jump of [1 10 ] polarized light.

Table S1 Selection rule for $\mathrm{A}_{2 \mathrm{u}}$ Polarization light, [001]-direction.

| $\mathrm{A}_{2 \mathrm{u}}, H_{\mathrm{oe}}$ |  | $\mathrm{E}_{\mathrm{g}}$ phonon |  | $\mathrm{B}_{2 \mathrm{~g}}$ phonon |  | $\mathrm{A}_{1 \mathrm{~g}}$ phonon |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $H_{\text {ep }}\left(\mathrm{E}_{\mathrm{g}}\right)$ | $H_{\text {oe }}\left(\mathrm{A}_{2 \mathrm{u}}\right)$ | $H_{\text {ep }}\left(\mathrm{B}_{2 \mathrm{~g}}\right)$ | $H_{\text {oe }}\left(\mathrm{A}_{2 \mathrm{u}}\right)$ | $H_{\text {ep }}\left(\mathrm{A}_{1 \mathrm{~g}}\right)$ | $H_{\mathrm{oe}}\left(\mathrm{A}_{2 \mathrm{u}}\right)$ |
| $\|i\rangle$ | $\left\|m^{\prime}\right\rangle$ | $\|m\rangle$ | $\|f\rangle$ | $\|m\rangle$ | $\|f\rangle$ | $\|m\rangle$ | $\|f\rangle$ |
| $\mathrm{A}_{1 \mathrm{~g}}(\Gamma 1)$ | $\mathrm{A}_{2 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{B}_{1 \mathrm{u}}$ | $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{A}_{2 \mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{~g}}$ |
| $\mathrm{A}_{2 \mathrm{~g}}(\Gamma 2)$ | $\mathrm{A}_{1 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{B}_{2} \mathrm{u}$ | $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{A}_{1 \mathrm{u}}$ | $\mathrm{A}_{2 \mathrm{~g}}$ |
| $\mathrm{B}_{1 \mathrm{~g}}(\Gamma 3)$ | $\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{Eg}_{\mathrm{g}}$ | $\mathrm{A}_{1 \mathrm{u}}$ | $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{B}_{1 \mathrm{~g}}$ |
| $\mathrm{B}_{2 \mathrm{~g}}(\Gamma 4)$ | $\mathrm{B}_{1 \mathrm{u}}$ | $\mathrm{E}_{u}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{A}_{2 \mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{~g}}$ | $\mathrm{B}_{1 \mathrm{u}}$ | $\mathrm{B}_{2 \mathrm{~g}}$ |
| $\mathrm{Eg}_{\mathrm{g}}($ Г5) | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{E}_{u}$ | $\mathrm{Eg}_{\mathrm{g}}$ |
| $\mathrm{A}_{1 \mathrm{u}}($ ( 6$)$ | $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{A}_{1 \mathrm{u}}$ |
| $\mathrm{A}_{2 \mathrm{u}}(\Gamma 7)$ | $\mathrm{A}_{1 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{B}_{1 \mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{~g}}$ | $\mathrm{A}_{2 \mathrm{u}}$ |
| $\mathrm{B}_{14}(\Gamma 8)$ | $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{~g}}$ | $\mathrm{A}_{2 \mathrm{u}}$ | $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{B}_{1 \mathrm{u}}$ |
| $\mathrm{B}_{2 \mathrm{u}}($ Г9) | $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{E}_{u}$ | $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{A}_{1 \mathrm{u}}$ | $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{B}_{2 \mathrm{u}}$ |
| $\mathrm{E}_{\mathrm{u}}($ Г10) | $\mathrm{Eg}_{\mathrm{g}}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{Eg}_{\mathrm{g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{Eg}_{\mathrm{g}}$ | $\mathrm{E}_{u}$ |

Table S2 Selection rule for $\mathrm{E}_{\mathrm{u}}$ Polarization light, $\left[\begin{array}{ll}1 & -1\end{array} 0\right]$ direction.

| $\mathrm{E}_{\mathrm{u}}, H_{\mathrm{oe}}$ |  | $\mathrm{E}_{\mathrm{g}}$ phonon |  | $\mathrm{B}_{2 \mathrm{~g}}$ phonon |  | $\mathrm{A}_{1 \mathrm{~g}}$ phonon |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | $H_{\text {ep }}\left(\mathrm{E}_{\mathrm{g}}\right)$ | $H_{\mathrm{oe}}\left(\mathrm{E}_{\mathrm{u}}\right)$ | $H_{\text {ep }}\left(\mathrm{B}_{2 \mathrm{~g}}\right)$ | $H_{\mathrm{oc}}\left(\mathrm{E}_{\mathrm{u}}\right)$ | $H_{\text {ep }}\left(\mathrm{A}_{1 \mathrm{~g}}\right)$ | $H_{\mathrm{oe}}\left(\mathrm{E}_{\mathrm{u}}\right)$ |
| $\|i\rangle$ | $\left\|m^{\prime}\right\rangle$ | $\|m\rangle$ | $\|f\rangle$ | $\|m\rangle$ | $\|f\rangle$ | $\|m\rangle$ | $\|f\rangle$ |
| $\mathrm{A}_{1 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}$ |
| $\mathrm{A}_{2 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}$ |
| $\mathrm{B}_{1 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}$ |
| $\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{E}_{u}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{E}_{u}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}$ |
| Eg | $\begin{gathered} \mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+ \\ \mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}} \\ \hline \end{gathered}$ | $\mathrm{E}_{\mathrm{u}}$ | $\begin{gathered} \mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+ \\ \mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+ \\ \mathrm{B}_{2 \mathrm{u}} \end{gathered}$ | $\mathrm{Eg}_{\mathrm{g}}$ | $\begin{gathered} \mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+ \\ \mathrm{B}_{2 \mathrm{u}} \end{gathered}$ | $\mathrm{Eg}_{\mathrm{g}}$ |
| $\mathrm{A}_{1 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}$ |
| $\mathrm{A}_{2 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}$ |
| $\mathrm{B}_{1 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{E}_{\mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}$ |
| $\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}}$ | $\mathrm{E}_{u}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}$ | $\mathrm{E}_{\mathrm{g}}$ | $\mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+\mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}}$ |
| $\mathrm{E}_{\mathrm{u}}$ | $\begin{gathered} \mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+ \\ \mathrm{B}_{1 \mathrm{~g}}+\mathrm{B}_{2 \mathrm{~g}} \end{gathered}$ | $\mathrm{Eg}_{\mathrm{g}}$ | $\begin{gathered} \mathrm{A}_{1 \mathrm{u}}+\mathrm{A}_{2 \mathrm{u}}+ \\ \mathrm{B}_{1 \mathrm{u}}+\mathrm{B}_{2 \mathrm{u}} \\ \hline \end{gathered}$ | $\begin{gathered} \mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+ \\ \mathrm{B}_{2 \mathrm{~g}} \\ \hline \end{gathered}$ | $\mathrm{E}_{\mathrm{u}}$ | $\begin{gathered} \mathrm{A}_{1 \mathrm{~g}}+\mathrm{A}_{2 \mathrm{~g}}+\mathrm{B}_{1 \mathrm{~g}}+ \\ \mathrm{B}_{2 \mathrm{~g}} \end{gathered}$ | $\mathrm{E}_{\mathrm{u}}$ |



Figure S15. Schematic diagram of the ADRDM measurement.


Figure S16. The optical image of a typical TlSe two-terminal device.


Figure S17. Experimental data fitting of photocurrent curves as changing of light intensities shown in Figure 5b.

## Analysis of Raman anisotropy

The one-step Raman spectroscopy contains three micro-steps: (1) An electron absorbs a photon and jumps into a high energy level state. The scattering matrix of this process can be described as $\langle m| H_{e o}|i\rangle$, where $H_{e o}$ is the Hamiltonian of electron-photon interaction. At first-order approximation, $H_{e o}$ can be described as

$$
\begin{equation*}
H_{e o}=-\frac{e}{m c} \mathbf{A}(\mathbf{r}) \not \mathbf{P} \tag{1}
\end{equation*}
$$

where $c$ is the speed of light, $\mathbf{A}(\mathbf{r})$ is the vector potential of optical electromagnetic field and $\mathbf{P}$ is the momentum of electron. As the vector potential is an external field, the Schrödinger group is determined by $\mathbf{P}$; (2) An excitation electron interacts with a phonon. In one-step Raman spectroscopy, only the phonon nearby the center zone can be involved into the electron-photon interaction. The scattering matrix of this process can be described as $\left\langle m^{\prime}\right| H_{e p}|m\rangle$, where $H_{e p}$ is the Hamiltonian of electron-phonon interaction, which can be expressed as

$$
\begin{equation*}
H_{e p}=\Delta V(\mathbf{r}) \tag{2}
\end{equation*}
$$

where $\Delta V$ is the change of potential caused by the phonon vibration. It is obvious that the Schrödinger group of this Hamiltonian follows the symmetry of corresponding phonon mode; (3) The last step is also an electron-photon interaction process, in which the excitation electron emits a photo when jumping into a low energy level state. The expression of Hamiltonian is same as the first step. In summary, the whole process of one-step Raman spectroscopy based on the Fermi-Gold rule can be described as,

$$
\begin{equation*}
I_{v}\left(E_{L}\right)=\left|\sum_{i, m, m^{\prime}} \frac{\langle f| H_{o p}\left|m^{\prime}\right\rangle\left\langle m^{\prime}\right| H_{e p}^{v}|m\rangle\langle m| H_{o p}|i\rangle}{\left(E_{L}-\Delta E_{m i}\right)\left(E_{L}-\hbar \omega_{v}-\Delta E_{m^{\prime} i}\right)}\right|^{2} \tag{3}
\end{equation*}
$$

where $I$ is the scattering strength, $\Delta E_{m i}=E_{m}-E_{i}-i \Gamma, m\left(m^{\prime}\right)$ represents the intermediate state and $\Gamma$ is a broadening factor, corresponding to the lifetime of photo-excited electrons.

For the active Raman modes in TlSe, their Raman tensor $R$ can be expressed as follows

$$
\begin{gather*}
R\left(\mathrm{~A}_{1 \mathrm{~g}}\right)=\left(\begin{array}{ccc}
a \cdot e^{i \phi_{a}} & 0 & 0 \\
0 & b \cdot e^{i \phi_{g_{b}}} & 0 \\
0 & 0 & \mathrm{~b} \cdot e^{i \phi_{D^{\prime}}}
\end{array}\right) ;  \tag{4}\\
R\left(\mathrm{~B}_{2 g}\right)=\left(\begin{array}{ccc}
0 & 0 & d \cdot e^{i \phi_{d}} \\
0 & d \cdot e^{i \phi_{d}} & 0 \\
0 & 0 & 0
\end{array}\right) ;  \tag{5}\\
R_{1}\left(\mathrm{E}_{g}\right)=\left(\begin{array}{ccc}
0 & f \cdot e^{i \phi_{e}} & 0 \\
f \cdot e^{i \phi_{e}} & 0 & 0 \\
0 & 0 & 0
\end{array}\right) . \tag{6}
\end{gather*}
$$

where $\phi_{a}, \phi_{b}, \phi_{d}, \phi_{e}$ are phases of Raman tensor elements. The Raman intensity $I(j)$ of the mode $j$ can be expressed as

$$
\begin{equation*}
I(j) \propto\left|\widehat{\mathbf{g}}_{s} \cdot \overline{\bar{R}}(j) \cdot \widehat{\mathbf{g}}_{i}^{T}\right|, \tag{7}
\end{equation*}
$$

where $\hat{g}_{i}$ and $\hat{g}_{s}$ are the polarized direction of the incident and scattered lights, respectively.

In the parallel configuration, since the angle between $\hat{g}_{i}$ and $\hat{g}_{s}$ is $0^{\circ}$, the Raman intensity for $\mathrm{A}_{\mathrm{g}}$ mode is described as

$$
\begin{equation*}
I^{\text {Para }}\left(\mathrm{A}_{1 \mathrm{~g}}\right) \propto\left(\cos ^{2}(\theta)+\frac{|a|}{|b|} \cdot \sin ^{2}(\theta) \cdot \cos \phi_{a b}\right)^{2}+\frac{|a|}{|b|} \sin ^{4}(\theta) \sin ^{2}\left(\phi_{a b}\right) ; \tag{8}
\end{equation*}
$$

for $\mathrm{B}_{2 \mathrm{~g}}$ mode, it is described as

$$
\begin{equation*}
I^{\text {Para }}\left(\mathrm{B}_{2 \mathrm{~g}}\right) \propto|d|^{2} \cos ^{4}(\theta) ; \tag{9}
\end{equation*}
$$

and for $\mathrm{E}_{\mathrm{g}}$ mode, it is described as

$$
\begin{equation*}
I^{\text {Para }}\left(\mathrm{E}_{g}\right) \propto 4|f|^{2} \sin ^{2}(2 \theta) . \tag{10}
\end{equation*}
$$

In the cross configuration, the angle between $\hat{g}_{i}$ and $\hat{g}_{s}$ is $90^{\circ}$, and the Raman intensity for $\mathrm{A}_{\mathrm{g}}$ mode is described as

$$
\begin{equation*}
I^{\text {Cross }}\left(\mathrm{A}_{1 \mathrm{~g}}\right) \propto(|a|-|b|)^{2} \sin ^{2}(\theta) \cos ^{2}(\theta) ; \tag{11}
\end{equation*}
$$

for $\mathrm{B}_{2 \mathrm{~g}}$ mode, it is described as

$$
\begin{equation*}
I^{\text {Cross }}\left(\mathrm{B}_{2 \mathrm{~g}}\right) \propto|d|^{2} \sin ^{2}(2 \theta) ; \tag{12}
\end{equation*}
$$

and for $\mathrm{E}_{\mathrm{g}}$ mode, it is described as

$$
\begin{equation*}
I^{\text {Cross }}\left(\mathrm{E}_{g}\right) \propto|f|^{2} \cos ^{2}(2 \theta) \tag{13}
\end{equation*}
$$

In all these equations, $\theta$ is the angle between the probe direction and x -axis direction of polar plots, and $\phi_{a b}=\phi_{a}-\phi_{b}$ defines phase difference. From Eqs. (8) to (13), one can notice that the polar plots depend not only on the configuration but also on the phase difference of Raman tensor. We simulated the polarized Raman intensity based on the Raman tensor, and the possible patterns of polar plots are shown in Figure S12. The calculated curves match well with the experimental ones (Figure 2c). The polar plots of $\mathrm{E}_{\mathrm{g}}$ modes are 4-leaf patterns, and the pattern for the cross configuration seems to be rotated by $45^{\circ}$ in comparison to that in the parallel configuration. In the parallel configuration, the polar plots of $\mathrm{A}_{1 \mathrm{~g}}$ and $\mathrm{B}_{2 \mathrm{~g}}$ modes are 2-leaf patterns, while they are changed to 4 -leaf patterns under cross configuration.

According to Eq. (8), the $\frac{|a|}{|b|}$ value determines the direction of the main axis: if $\frac{|a|}{|b|}$
$>1$, the main axis is along the y -direction.

