Supporting information for:

The Effects of the Interplay between Motor and Brownian Forces on the Rheology of Active Gels

Andrés Córdoba*

Department of Chemical Engineering, Universidad de Concepción, Concepción, Chile

E-mail: ancordoba@udec.cl

Phone: +56 41 220 43 76

Examples for the explicit forms of the transition rate matrices for $N = 2$ and $N = 3$ are given below for both the general case as well as the strong attachment case, $\tau_a/\tau_d \ll 1$. The transition matrices are constructed using eqs. (4) and (5) in the main text.

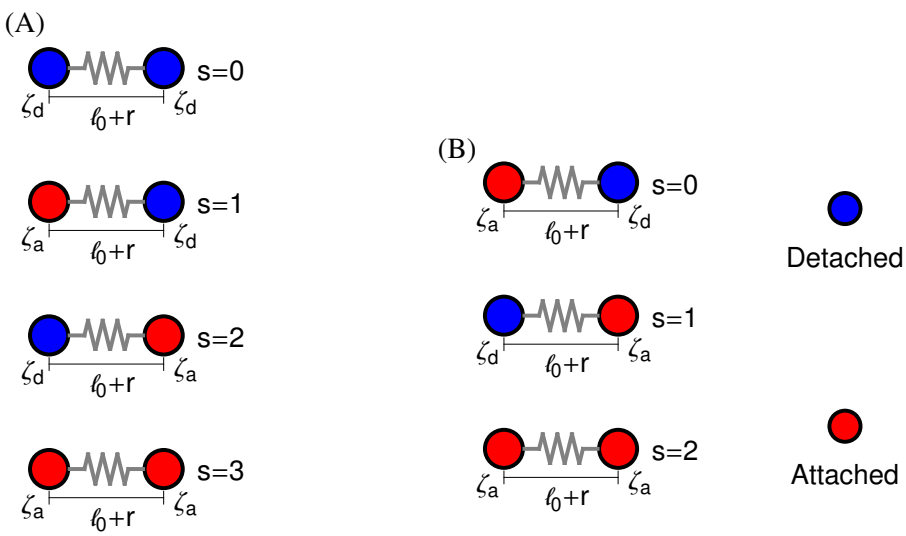

Figure S1: Attachment states for the active single-chain model with $N = 2$. (A) All possible attachment states for this N. (B) Attachment states considered in the strong attachment case, $\tau_a/\tau_d \ll 1$, for this N.

The transition rate matrix for $N = 2$ in the case in which all attachment states are accounted for, Fig. S1A, is given by,

$$\mathbb{W}(\hat{\omega}'|\hat{\omega}) = \begin{pmatrix} * & \frac{\delta(F_1')\delta(F_2')}{\tau_d} & \frac{\delta(F_1')\delta(F_2')}{\tau_d} & 0 \\ \frac{p(F_1')\delta(F_2')}{\tau_a} & * & 0 & \frac{\delta(F_1')\delta(F_2')}{\tau_d} \\ \frac{\delta(F_1')p(F_2')}{\tau_a} & 0 & * & \frac{\delta(F_1')\delta(F_2')}{\tau_d} \\ 0 & \frac{\delta(F_1')p(F_2')}{\tau_a} & \frac{p(F_1')\delta(F_2')}{\tau_a} & * \end{pmatrix}.$$

Where $\delta(...)$ is the Dirac delta function, $p(...)$ is the motor force probability distribution and the diagonal elements of the transition matrix $(*)$ are given by $\mathbb{W}(\hat{\omega}|\hat{\omega}) = -\sum_{s'(\neq s)=0}^{3} \mathbb{W}(\hat{\omega}'|\hat{\omega})$.

The transition rate matrix for $N = 2$ in the strong attachment case, Fig. S1B, is given by,

$$\mathbb{W}(\hat{\omega}'|\hat{\omega}) = \begin{pmatrix} * & 0 & \frac{\delta(F_1')\delta(F_2')}{\tau_d} \\ 0 & * & \frac{\delta(F_1')\delta(F_2')}{\tau_d} \\ \frac{\delta(F_1')p(F_2')}{\tau_a} & \frac{p(F_1')\delta(F_2')}{\tau_a} & * \end{pmatrix}.$$

Where $\delta(...)$ is the Dirac delta function, $p(...)$ is the motor force probability distribution and the diagonal elements $(*)$ of the transition matrix are given by $\mathbb{W}(\hat{\omega}|\hat{\omega}) = -\sum_{s'(\neq s)=0}^{2} \mathbb{W}(\hat{\omega}'|\hat{\omega})$.

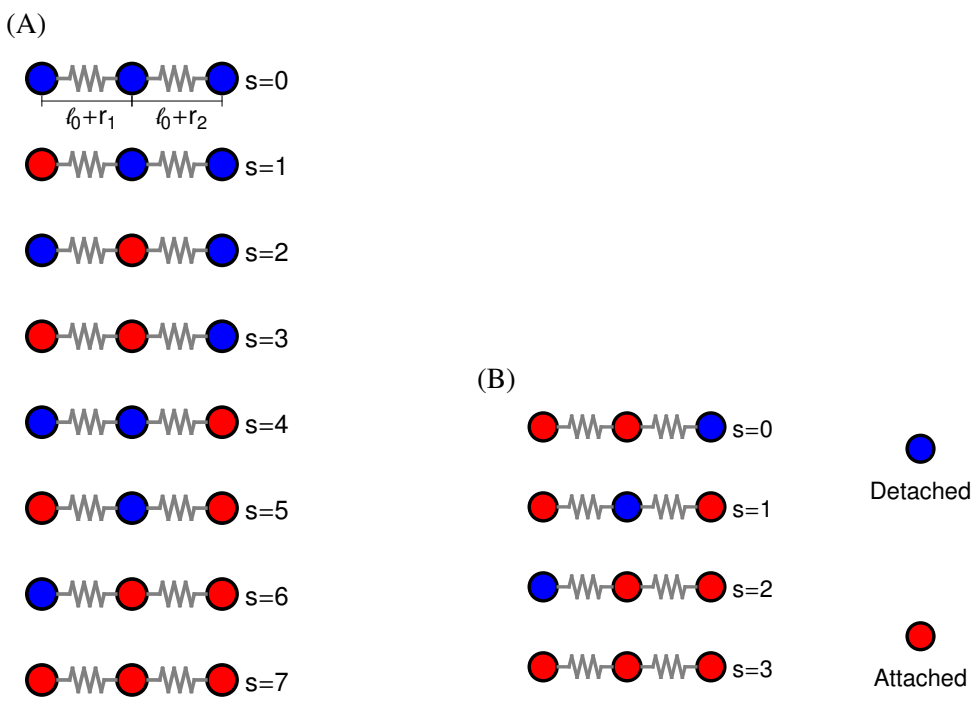

Figure S2: Attachment states for the active single-chain model with $N = 3$. (A) All possible attachment states for this N. (B) Attachment states considered in the strong attachment case, $\tau_a/\tau_d \ll 1$, for this N.

The transition rate matrix for $N = 3$ in the case in which all attachment states are accounted

for, Fig. S2A, is given by,

$$\mathbb{W}(\hat{\omega}'|\hat{\omega}) = \begin{pmatrix} * & \frac{X_0}{\tau_d} & \frac{X_0}{\tau_d} & 0 & \frac{X_0}{\tau_d} & 0 & 0 & 0 \\ \frac{X_1}{\tau_a} & * & 0 & \frac{X_0}{\tau_d} & 0 & \frac{X_0}{\tau_d} & 0 & 0 \\ \frac{X_2}{\tau_a} & 0 & * & \frac{X_0}{\tau_d} & 0 & 0 & \frac{X_0}{\tau_d} & 0 \\ 0 & \frac{X_3}{\tau_a} & \frac{X_3}{\tau_a} & * & 0 & 0 & 0 & \frac{X_0}{\tau_d} \\ \frac{X_4}{\tau_a} & 0 & 0 & 0 & * & \frac{X_0}{\tau_d} & \frac{X_0}{\tau_d} & 0 \\ 0 & \frac{X_5}{\tau_a} & 0 & 0 & \frac{X_5}{\tau_a} & * & 0 & \frac{X_0}{\tau_d} \\ 0 & 0 & \frac{X_6}{\tau_a} & 0 & \frac{X_6}{\tau_a} & 0 & * & \frac{X_0}{\tau_d} \\ 0 & 0 & 0 & \frac{X_7}{\tau_a} & 0 & \frac{X_7}{\tau_a} & \frac{X_7}{\tau_a} & * \end{pmatrix}$$

Where $\delta(...)$ is the Dirac delta function, $p(...)$ is the motor force probability distribution, and $X_0 = \delta(F_1')\delta(F_2')\delta(F_3')$, $X_1 = p(F_1')\delta(F_2')\delta(F_3')$, $X_2 = \delta(F_1')p(F_2')\delta(F_3')$, $X_3 = p(F_1')p(F_2')\delta(F_3')$, $X_4 = \delta(F_1')\delta(F_2')p(F_3')$, $X_5 = p(F_1')\delta(F_2')p(F_3')$, $X_6 = \delta(F_1')p(F_2')p(F_3')$, $X_7 = p(F_1')p(F_2')p(F_3')$. The diagonal elements of the transition matrix ($*$) are given by $\mathbb{W}(\hat{\omega}|\hat{\omega}) = -\sum_{s'(\neq s)=0}^{7} \mathbb{W}(\hat{\omega}'|\hat{\omega})$.

The transition rate matrix for $N = 3$ in the strong attachment case, Fig. S2B, is given by,

$$\mathbb{W}(\hat{\omega}'|\hat{\omega}) = \begin{pmatrix} * & 0 & 0 & \frac{X_0}{\tau_d} \\ 0 & * & 0 & \frac{X_0}{\tau_d} \\ 0 & 0 & * & \frac{X_0}{\tau_d} \\ \frac{X_7}{\tau_a} & \frac{X_7}{\tau_a} & \frac{X_7}{\tau_a} & * \end{pmatrix}$$

Where $\delta(...)$ is the Dirac delta function, $p(...)$ is the motor force probability distribution, and $X_0 = \delta(F_1')\delta(F_2')\delta(F_3')$, $X_7 = p(F_1')p(F_2')p(F_3')$. The diagonal elements of the transition matrix ($*$) are given by $\mathbb{W}(\hat{\omega}|\hat{\omega}) = -\sum_{s'(\neq s)=0}^{3} \mathbb{W}(\hat{\omega}'|\hat{\omega})$.