Supporting Information

Control over Energy Transfer between Fluorescent BODIPY Dyes in a Strongly Coupled Microcavity

Kyriacos Georgiou,[1] Paolo Michetti,[2] Lizhi Gai,[3] Marco Cavazzini,[4] Zhen Shen,[3] and David G. Lidzey*,[1]

[1]Department of Physics and Astronomy, The University of Sheffield, Sheffield S3 7RH, United Kingdom

[2]Institute of Theoretical Physics, Technische Universität Dresden, 01062 Dresden, Germany

[3]State Key Laboratory of Coordination and Chemistry, School of Chemistry and Chemical Engineering, Nanjing University, Nanjing 210046, China

[4]Istituto di Scienze e Tecnologie Molecolari del Consiglio Nazionale delle Ricerche (ISTM-CNR), Via Golgi 19, I-20133 Milano, Italy

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Four level coupled oscillator model

The four level coupled oscillator model used to describe strong coupling in the hybrid microcavities.

$$\begin{pmatrix} E_\gamma(\theta) & \hbar\Omega_1/2 & \hbar\Omega_2/2 & \hbar\Omega_3/2 \\ \hbar\Omega_1/2 & E_{Br} & 0 & 0 \\ \hbar\Omega_2/2 & 0 & E_{R1} & 0 \end{pmatrix} \begin{pmatrix} \alpha_\gamma \\ \alpha_{Br} \\ \alpha_{R1} \end{pmatrix} = E_p \begin{pmatrix} \alpha_\gamma \\ \alpha_{Br} \\ \alpha_{R1} \end{pmatrix}$$

Diagonalization of the Hamiltonian gives rise to four unique solutions for the four polariton branches with the energy of the LPB at $k_{//}$=0 and the coupling constants tabulated bellow.

Cavity	**LPB energy $k_{//}$=0**	**$\hbar\Omega_1$ (meV)**	**$\hbar\Omega_2$ (meV)**	**$\hbar\Omega_3$ (meV)**
M_p	1.92481	185	23	90
M_n	1.75045	180	110	168
B_p	1.90440	185	50	160
B_n	1.75047	210	110	168

Table S1. Energy of the LPB at zero in-plane momentum and coupling constants for the four cavities M_p, M_n, B_p and B_n.

Polariton population and polariton PL

The photoluminescence measured by a lens *P* is the light collected in the solid angle which subtends the lens itself, i.e.

$$P_\theta = \iint_{lens} d\, cos\theta\, d\varphi\, I(\theta, \varphi), \tag{1}$$

where P_θ is the solid angle PL density for an isotropic MC.

In order to convert polariton PL into polariton population we need to use the reciprocal space, i.e. the number of polariton states for unit surface D_Ω corresponding to a certain wave vector region Ω

$$D_\Omega = \frac{1}{(2\pi)^2} \int_\Omega d^2\, \boldsymbol{k}. \tag{2}$$

Given a certain polariton population density f_k, the polariton population per unit surface is given by

$$N_\Omega = \frac{1}{(2\pi)^2} \int_\Omega d^2\, \boldsymbol{k}\, f_{\boldsymbol{k}}. \tag{3}$$

Assuming isotropic conditions, it is useful to introduce cylindrical coordinates. In this case, the integral of the polariton population density over the LPB can be written as

$$N_{LP} = \frac{1}{(2\pi)^2} \int_0^{k_{max}} dk\, k \int_0^{2\pi} d\varphi\;\; f_k. \tag{4}$$

Similarly, we could express the total photoluminescence of the LPB as

$$P_{LP} \propto \frac{1}{(2\pi)^2} \int_0^{k_{max}} dk\, k \int_0^{2\pi} d\varphi \frac{|a_\gamma|^2}{\tau_{cav}}\; f_k, \tag{5}$$

where $|a_\gamma|^2$ is the amplitude of the photon part of polariton of wave vector k and τ_{cav} is the confinement time of the microcavity. Assuming as usual that the in-plane wave vector is a conserved quantity, we can pass from the modulus of the wave vector k to the inclination angle θ by using the following formula

$$k = E_k \frac{\eta}{\hbar c} sin\theta, \tag{6}$$

with η the refractive index.

The PL emission can be then expressed as

$$P_{LP} \propto \frac{1}{(2\pi)^2} \int_0^{\theta_{max}} dsin\theta\, E_{k(\theta)}^2 \frac{\eta^2 sin\theta}{\hbar^2 c^2} \int_0^{2\pi} d\varphi \frac{|a_\gamma|^2}{\tau_{cav}}\; f_{k(\theta)}. \tag{7}$$

Comparing this expression with (1) we conclude that exists the follow proportionality relation between the observed PL I(θ) and the polariton population density $f_{k(\theta)}$

$$P_\theta \propto E_{k(\theta)}^2 \cos\theta\, |a_\gamma|^2\, f_{k(\theta)}. \tag{8}$$

Therefore $f_{k(\theta)}$ can be expressed as

$$f_{k(\theta)} \propto \frac{P_\theta}{E_{k(\theta)}^2 \cos\theta\, |a_\gamma|^2}. \tag{9}$$

We indeed employ the approximate proportionality $P_\theta \propto I(\theta)$, with the justified assumption for a smooth enough PL density.